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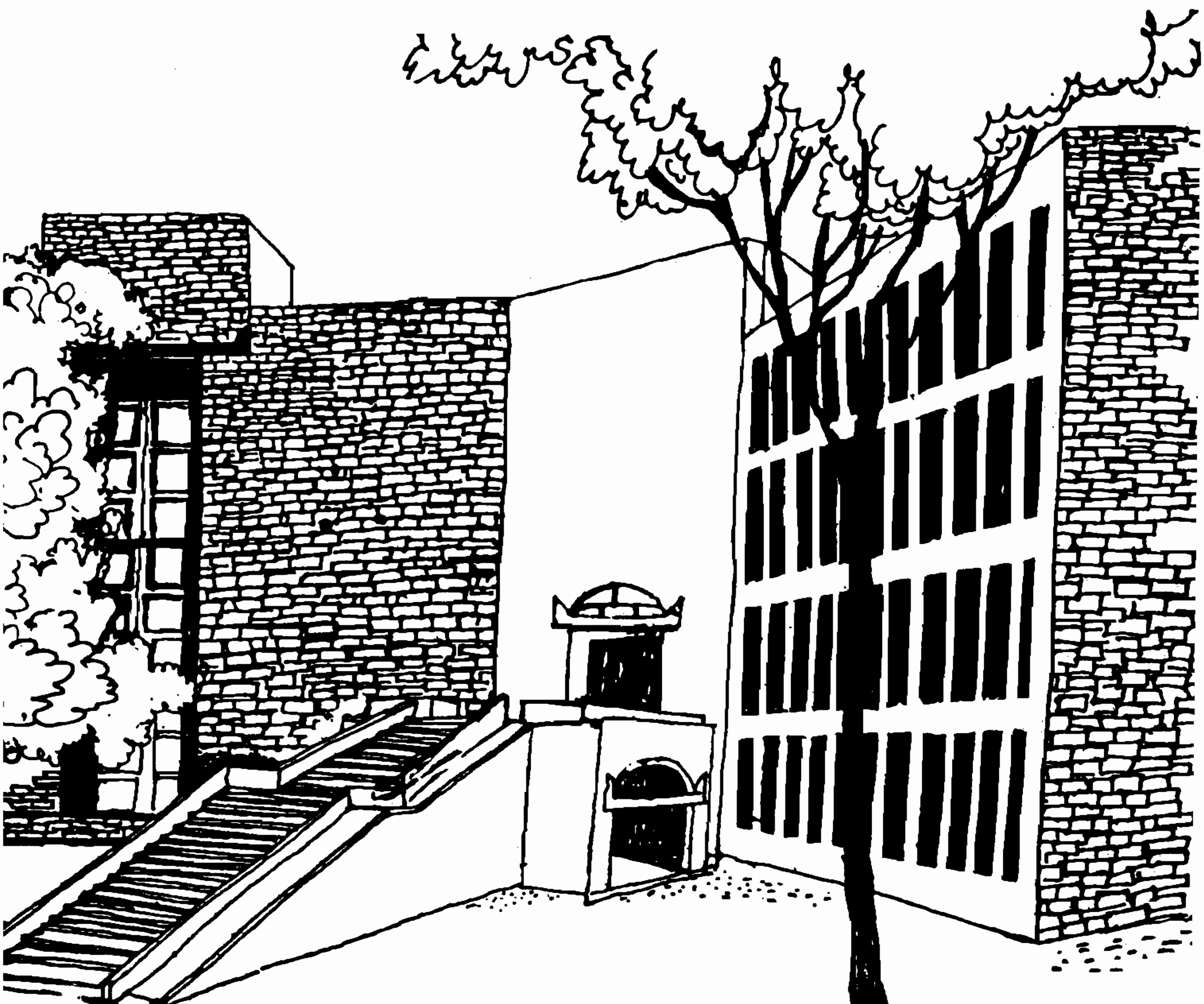

THREAT BARGAINING PROBLEMS WITH CORRELATED BELIEFS

By

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ABSTRACT

In this paper we provide a general framework for studying threat bargaining games with correlated beliefs. In this framework we obtain a characterization of the Kalai-Smorodinsky solution without any monotonicity assumption. The approach adds a dose of realism to the already existing literature on threat bargaining games.

1. Framework and Definitions :

In a pure bargaining problem between a group of two participants there is a set of feasible outcomes, any one of which will result if it is specified by the unanimous agreement of all participants. In the event that no unanimous agreement is reached, a given disagreement outcome obtains. We shall assume that the utility space or the set of possible payoffs is R^2 i.e. a two person bargaining problem is a pair (H,d) of a subset H of R^2 and of a point $d \in H$. H is the feasible set, and d is the disagreement (or threat) point.

The class of bargaining problems we consider is given by the following definitions:

Definition 1 :- The pair $\Gamma = (H,d)$ is a two-person fixed threat bargaining game if $H \subseteq R^2$ is compact, convex, comprehensive with non-empty interior, $d \in H$, and H contains atleast one element u such that $u \gg d$. (Note: $H \subseteq R^2$ is said to be comprehensive if $y \in R^2$, $x \geqslant y \geqslant d$ for some $x \in H$ implies $y \in H$).

Definition 2 :- The set of two-person fixed threat bargaining games is denoted W.

For the purpose of this paper we define a solution to bargaining problems in W as follows:

Definition 3 :- A solution is a function $F : W \rightarrow R^2_+$ satisfying

(i) $F(H,d) \in H \;\; \forall (H,d) \in W$ (feasibility)

(ii) $y \in H$, $y \geqslant F(H,d)$ implies $y = F(H,d)$ (Pareto optimality)

(iii) $F(H,d) \geq d \; \forall \; (H,d) \in W$ (individual rationality)

(iv) If $(a_1,a_2) >> 0$, $(b_1,b_2) \in R^2$; $H' = \{y \in R^2 / y_i = a_i x_i + b_i$, $i = 1,2$, $y = (y_1,y_2)$, $x = (x_1,x_2) \in H\}$ and $d_i' = a_i d_i + b_i$, $i = 1,2$, $d' = (d_1',d_2')$, then $F_i(H', d') = a_i F_i(H,d) + b_i$, $i = 1,2$.

(Independence with respect to affine utility transformations)

The conditions we impose on a solution to bargaining problems are st ndard and are satisfied by the more well known solutions to bargaining problems (e.g. Nash (1950), Kalai-Smorodinsky (1975)).

We now make an assumption which is satisfied by most familiar solu ons to bargaining problems and which will be required significantly by us.

<u>Assumption</u> (FUD) :- Let $(H,d) \in W$ and $P(H_d) = \{(x_1,x_2) \in H / x = (x_1,x_2)$, $x_i \geq d_i$, $i = 1,2$ and $y_i \geq x_i$ $y \in H$ implies $y = x\}$

Then $\forall \; (x_1,x_2) \in P(H_d)$, $\exists$ $d_1' \geq d_1$, or $d_2' \geq d_2$ such that

(i) $F(H; d_1', d_2) = (x_1,x_2)$

or (ii) $F(H; d_1,d_2') = (x_1,x_2)$

(fullness through unilateral deviations).

This assumption requires that unilateral deviation from the given disagreement payoffs yield any Pareto Optimal and individually rational outcome. As mentioned earlier this property is satisfied by all the more well known solutions to bargaining problems, including some of those which may not satisfy some of the conditions of Definition 3 (e.g. the Proportional Solution of Kalai [1977]).

Our analysis requires the notion of a true bargaining problem, which in view of the above and following Anbar and Kalai (1978) may be defined as follows:

Definition 4 :- A true bargaining problem H is a compact, convex subset of the unit square containing (0,0), (1,0) and (0,1).

The interpretation of such a bargaining game is that the true disagreement point of the players have been set equal to (0,0) and the game has been normalized in such a way that the utility demands of the players belong to the closed interval [0,1]. Let us call the set of all true bargaining problems $\bar{W}$.

Every member $H \in \bar{W}$ defines uniquely a monotone non-increasing concave function $\phi_H : [0,1] \to [0,1]$ by $\phi_H(x_1) = \max \{x_2 / (x_1,x_2) \in H\}$. Conversely every monotone non-increasing concave function $\phi : [0,1] \to [0,1]$ such that $\phi(0) = 1$ determines uniquely a set $H_\phi \in \bar{W}$ by $H_\phi = \{(x_1,x_2) : 0 \leq x_1 \leq 1, \ 0 \leq x_2 \leq \phi(x_1)\}$. For every such function ϕ we define the (generalized) inverse $\phi^{-1} : [0,1] \to [0,1]$ by $\phi^{-1}(x_2) = \max \{x_1 / (x_1,x_2) \in H_\phi\}$.

Let $G_i : [0,1] \times [0,1] \to [0,1]$ be the conditional distribution function which summarizes the belief of player i about player $j \neq i$ (i's opponent) accepting a utility outcome, given player i's utility outcome, $i = 1,2$. Thus, $G_1(x_2|x_1)$ is player 1's assessment of the probability of player 2 accepting a utility outcome x_2 or less, given that player 1's utility outcome is x_1.

The non-cooperative game we have in mind is the following. The underlying <u>true bargaining problem</u> $H \in \bar{W}$ being given each player i announces a disagreement utility d_i. The pair (H,d), $d = (d_1,d_2)$ is a fixed threat bargaining problem in W. Based on the information announced by the players the arbitrator using a solution F selects an outcome $F(H,d)$ which each player accepts with a probability determined by G_1 and G_2 respectively. In the event that the outcome is rejected, by any one or both the players, the participants settle down for their true disagreement payoffs $0 = (0,0)$.

Let $(d_1,d_2) \in H$ be the announced disagreement payoffs of the respective players. If F is the solution being used by the arbitrator, the expected payoff of player 1 is

$$P_1(d_1,d_2) = F_1(H;d_1,d_2) \cdot G_1(F_2(H;d_1,d_2)|F_1(H;d_1,d_2)).$$

The expected payoff of player 2 is

$$P_2(d_1,d_2) = F_2(H;d_1,d_2) \cdot G_2(F_1(H;d_1,d_2)|F_2(H;d_1,d_2))$$

<u>Definition 5</u> :- A threat bargaining game with correlated beliefs equipped with a solution F is an ordered triplet (H, F, G) where

(i) $H \in \overline{W}$ is a true bargaining problem

(ii) $F : W \rightarrow R^2$ is a bargaining solution

(iii) $G = (G_1, G_2)$ is a pair of conditional probability distribution functions on $[0,1]$.

The notion of an equilibrium that we adopt in this paper is given by the following definition.

<u>Definition 6</u> :- An equilibrium for a threat bargaining game with correlated beliefs equipped with a solution F, i.e. (H, F, G) is an ordered pair $(d_1^*, d_2^*) \in H$ such that

(i) $p_1(d_1^*, d_2^*) \geq p_1(d_1, d_2^*) \; \forall \; d_1 \in [0,1]$

(ii) $p_2(d_1^*, d_2^*) \geq p_1(d_1^*, d_2) \; \forall \; d_2 \in [0,1]$

This is the familiar Nash equilibrium which by dint of its self enforceability finds a distinguished placed as a solution concept. In the case of threat bargaining problems, the relationship between a Nash equilibrium and well known solutions to bargaining problems have been studied earlier.

2. Main Theorem :

In this section we shall try to impose conditions under which truthful revelation of disagreement utility will be guaranteed by a bargaining solution.

The main theorem of this paper is the following:

Theorem 1 :- Let

$$G_1(x_2 \mid x_1) = \frac{\min\{x_1, x_2\}}{x_1} \quad \text{if } x_1 > 0$$

$$= 1 \quad \text{if } x_1 = 0$$

and

$$G_2(x_1 \mid x_2) = \frac{\min\{x_1, x_2\}}{x_2} \quad \text{if } x_2 > 0$$

$$= 1 \quad \text{if } x_2 = 0$$

Then (0,0) is an equilibrium of the threat bargaining game with correlated beliefs Γ = (H,F,G) equipped with a solution F is and only if F is the Kalai-Smorodinsky [1975] solution i.e.

$$F(S) = \underset{0 \leq x_1 \leq 1}{\arg\max} \left\{\min (x_1, \varphi_s(x_1))\right\} = \underset{0 \leq x_2 \leq 1}{\arg\max} \left\{\min (x_2, \varphi_s^{-1}(x_2))\right\}$$

$\forall S \in \bar{W}$.

Proof :- Given G_1 and G_2,

$$P_1(d_1, d_2) = \min\left\{F_1(H;d_1,d_2), \varphi_H(F_1(H;d_1,d_2))\right\}$$

$$P_2(d_1, d_2) = \min\left\{F_2(H;d_1,d_2), \varphi_H^{-1}(F_2(H;d_1,d_2))\right\}.$$

Observe that by property (i) of a solution $F_2(H;d_1,d_2) = \varphi_H(F_1(H;d_1,d_2))$ and $F_1(H;d_1,d_2) = \varphi_H^{-1}(F_2(H;d_1,d_2))$

Suppose $F = (F_1,F_2)$ is the Kalai-Smorodinsky [1975] solution.

$$P_1(H;0,0) = \min\{F_1(H;0,0), \varphi_H(F_1(H;0,0))\} \geq \min\{x_1, \varphi_H(x_1)\} \forall 0 \leq x_1 \leq 1,$$

by definition of the solution.

Since $P_1(H;d_1,0) = \min\{x_1, \varphi_H(x_1)\}$ for some $x_1 \in [0,1]$,

we get,

$$P_1(H;0,0) \geq P_1(H;d_1,0) \forall\ d_1 \in [0,1].$$

By a similar argument it follows that

$$P_2(H;0,0) \geq P_2(H;0,d_2) \forall\ d_2 \in [0,1].$$

Hence (0,0) is an equilibrium for (H,F,G).

Conversely suppose that (0,0) is an equilibrium for (H,F,G), but F is not the Kalai-Smorodinsky [1975] solution. Let $(x_1^*, \varphi_H(x_1^*))$ be the Kalai-Smorodinsky solution outcome for $H \in \bar{W}$. By assumption (FUD) and without loss of generality $\exists\ d_1' \geq 0$, such that

$$F(H;d_1',0) = (x_1^*, \varphi_H(x_1^*))$$

Hence

$$P_1(H;d_1',0) = \min\{x_1^*, \varphi_H(x_1^*)\} > \min\{F_1(H;0,0), \varphi_H(F_1(H;0,0))\}$$
$$= P_1(H;0,0),$$

contradicting that (0,0) is an equilibrium. Hence the theorem.

Q.E.D.

3. Conclusion :

Apart from achieving a characterization of the Kalai-Smorodinsky (1975) solution without a monotonicity assumption, we have also extended the framework of threat bargaining games to include correlated beliefs in our model. This accounts for additional realism in our study.

The structure of the beliefs used to characterize the Kalai-Smorodinsky solution is not as straightforward as the uniform distribution used in characterizing the Nash (1950) solution (see Lahiri (1989)). None the less it is generated by a genuine distribution function and one that can arise very naturally in the presence of incomplete information.

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