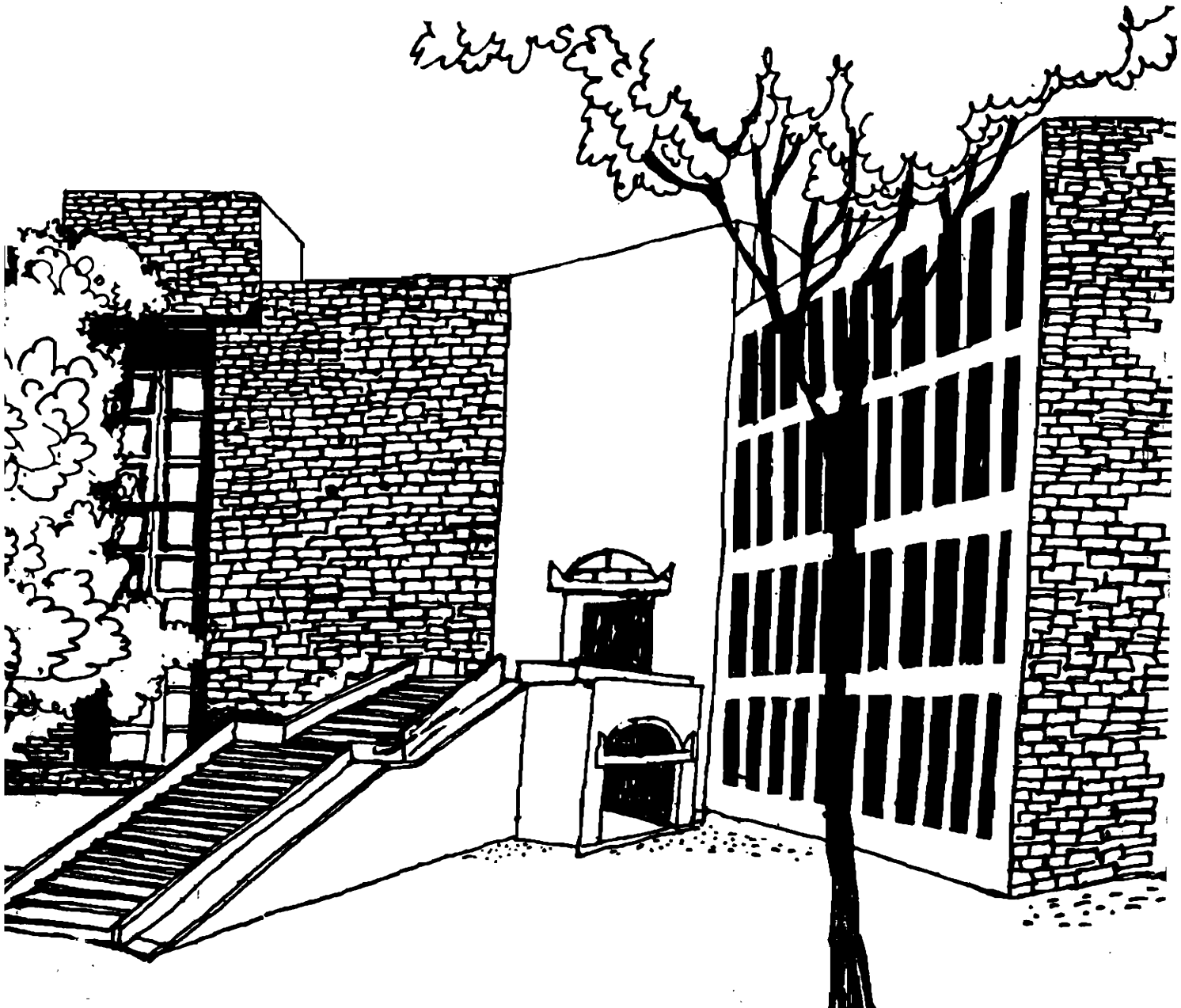




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**JUSTIFIABLE PREFERENCES FOR
FREEDOM OF CHOICE**

By

Somdeb Lahiri

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Abstract of
Justifiable Preferences for Freedom of Choice

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In this paper we say that a preference for freedom of choice is justifiable if there exists a reflexive and complete binary relation on the set of alternatives, such that one opportunity is atleast as good as a second, if and only if there is at least one alternative from the first set which is no worse than any alternative of the two sets combined together, with respect to the binary relation on the alternatives. In keeping with the revered tradition set by von Neumann and Morgenstern we call a reflexive and complete binary relation, an abstract game (note : strictly speaking von Neumann and Morgenstern refer to the asymmetric part of a reflexive and complete binary relation as an abstract game; hence our terminology though analytically equivalent, leads to a harmless corruption of the original meaning). It turns out that if a preference for freedom of choice is justifiable, then the base relation with respect to which it is justifiable, is simply the restriction of the preference for freedom of choice, to the set of all singletons. Our main result is about the justifiability of transitive preferences for freedom of choice. It says that such preferences are justifiable if and only if they satisfy Monotonicity and Concordance. Concordance says that if one opportunity set is at least as desirable as a second then it should also be the case that the first opportunity set is at least as desirable as the union of the two. Since, for the case of transitive preferences for freedom of choice, our notion of justifiability coincides with that of Arrow and Malishevsky, our axiomatic characterization can throw some light on properties of indirect utility functions.

Justifiable Preferences for Freedom of Choice

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1. **Introduction** :- The dominant paradigm of welfare economics is one of choice, where an individual chooses one or more alternatives from a non empty given set of alternatives. There may or may not be an underlying preference structure with respect to which the decision is arrived at. However, welfare economics exhibits a preferential bias towards choice situations where decisions are determined by some underlying preference structure. This is what is commonly known as rational choice theory.

Some problems of decision making are naturally two stage procedures, as for instance when in a first round of interview we short list a set of candidates for a second round of interview. One way in which such procedures can be viewed is the final choices are determined by the lexicographic composition of a first round ranking followed by a second round ranking. The composition defines a binary relation on the set of alternatives. Study of such objects are available in Aizerman and Malishevsky (1986), Aizerman and Aleskerov (1995), Aleskerov (1999), Lahiri (2000) and more recently Lahiri (2001). An alternative way of viewing such two stage decision problems is to conceive the first stage as the selection of a feasible set, to which the second step choices are reflected. The first stage decision is often decided by what is known as preference for flexibility or preference for freedom of choice. Some of the very early discussions of this concept can be found in Koopmans (1964), Kreps (1979) and Sen (1988). The problem received fresh impetus with subsequent contributions by Pattanaik and Xu (1990, 1997, 1998), Klemisch – Ahlert (1993), Arrow (1994), Puppe (1995, 1996), Sen (1990, 1991), to mention a few. The most emphatic statement of the problem discussed in this paper can be traced to the paper by Arrow (1994). The standpoint that Arrow adopted was the following: There are many important situations where it makes sense to say that one opportunity set is atleast as good as another if and only if there exists a reflexive, complete and transitive binary relation, with respect to which the best element of the first set is no worse than the best element of the second. Such preferences for freedom of choice are similar to the concept of indirect utility functions of rational economic choice theory. Malishevsky (1997) provides axiomatic characterizations of such preferences for freedom of choice. It turns out that the axiomatic characterizations make significant use of a property called Monotonicity, which says that if one subset contains another subset then the bigger subset is no worse than the second. While Monotonicity is a doubtful assumption to make for a choice between being served a morning cup of tea and the opportunity set which includes the possibility of being “beheaded at dawn”

as well, such is not the case if the possibility of being beheaded at dawn is replaced by being served coffee at dawn. The context determines the validity of an axiom and in this sense Monotonicity is no different.

In this paper we say that a preference for freedom of choice is justifiable if there exists a reflexive and complete binary relation on the set of alternatives, such that one opportunity is at least as good as a second, if and only if there is at least one alternative from the first set which is no worse than any alternative of the two sets combined together, with respect to the binary relation on the alternatives. In keeping with the revered tradition set by von Neumann and Morgenstern we call a reflexive and complete binary relation, an abstract game (note: strictly speaking von Neumann and Morgenstern refer to the asymmetric part of a reflexive and complete binary relation as an abstract game; hence our terminology though analytically equivalent, leads to a harmless corruption of the original meaning). It turns out that if a preference for freedom of choice is justifiable, then the base relation with respect to which it is justifiable, is simply the restriction of the preference for freedom of choice, to the set of all singletons. Our main result is about the justifiability of transitive preferences for freedom of choice. It says that such preferences are justifiable if and only if they satisfy Monotonicity and Concordance. Concordance says that if one opportunity set is at least as desirable as a second then it should also be the case that the first opportunity set is at least as desirable as the union of the two. Since, for the case of transitive preferences for freedom of choice, our notion of justifiability coincides with that of Arrow and Malishevsky, our axiomatic characterization can throw some light on properties of indirect utility functions.

2. **The Model** :- Let X be a non-empty finite set of alternatives containing at least two elements. Let $[X]$ be the set of all non empty subsets of X . Let $\Delta(X) = \{(x,x)/x \in X\}$ and $\Delta([X]) = \{(A,A)/A \in [X]\}$. $\Delta(X)$ is called the diagonal of X and $\Delta([X])$ is called the diagonal of $[X]$.

A binary relation R on X is said to be :

- (a) reflexive, if $\Delta(X) \subset R$;
- (b) complete, if given $x,y \in X$, with $x \neq y$, either $(x,y) \in R$ or $(y,x) \in R$;
- (c) an abstract game if it is both reflexive and complete;

Given a binary relation R on X , let $P(R) = \{(x,y) \in R / (y,x) \notin R\}$ denote the asymmetric part of R and let $I(R) = \{(x,y) \in R / (y,x) \in R\}$ denote the symmetric part of R .

A binary relation R on X is said to be acyclic if given any positive integer n and elements $x(1), \dots, x(n)$ in X : $[(x(i), x(i+1)) \in P(R) \forall i \in \{1, \dots, n-1\}]$ implies $[(x(n), x(1)) \notin P(R)]$.

A binary relation R on X is said to be transitive if $\forall x,y,z \in X$: $[(x,y), (y,z) \in R]$ implies $[(x,z) \in R]$.

A binary relation \mathfrak{R} on $[X]$ is said to be :

- (d) reflexive, if $\Delta([X]) \subset \mathfrak{R}$;
- (e) complete, if given $A,B \in [X]$, with $A \neq B$, either $(A,B) \in \mathfrak{R}$ or $(B,A) \in \mathfrak{R}$;
- (f) a preference for freedom of choice (PFC) if it is both reflexive and complete;

Given a binary relation \mathfrak{R} on $[X]$, let $P(\mathfrak{R}) = \{(A,B) \in \mathfrak{R} / (B,A) \notin \mathfrak{R}\}$ denote the asymmetric part of \mathfrak{R} and let $I(\mathfrak{R}) = \{(A,B) \in \mathfrak{R} / (B,A) \in \mathfrak{R}\}$ denote the symmetric part of \mathfrak{R} .

A binary relation \mathfrak{R} on $[X]$ is said to be acyclic if given any positive integer n and elements $A(1), \dots, A(n)$ in $[X] : [(A(i), A(i+1)) \in P(\mathfrak{R}) \forall i \in \{1, \dots, n-1\}]$ implies $[(A(n), A(1)) \notin P(\mathfrak{R})]$.

A binary relation \mathfrak{R} on $[X]$ is said to be transitive if $\forall A, B, C \in [X] : [(A,B), (B,C) \in \mathfrak{R}]$ implies $[(A,C) \in \mathfrak{R}]$.

Given a PFC \mathfrak{R} , we define the abstract game revealed by \mathfrak{R} , to be the binary relation $R(\mathfrak{R}) = \{(x,y) / (\{x\}, \{y\}) \in \mathfrak{R}\}$.

Given an abstract game R and $A \in [X]$, let $G(A,R) = \{x \in A / (x,y) \in R, \text{ whenever } y \in A\}$.

The following well known result can be found in Kreps [1988]:

Proposition 1: Let R be an abstract game. Then, $G(A,R) \neq \emptyset$ whenever $A \in [X]$ if and only if R is acyclic.

We say that a PFC \mathfrak{R} is justifiable if there exists an abstract game R such that $\forall A, B \in [X] : [(A,B) \in \mathfrak{R}]$ if and only if $[G(A \cup B, R) \cap A \neq \emptyset]$.

It is easy to see that if a PFC is justifiable by R , then $R = R(\mathfrak{R})$. Hence we have the following proposition :

Proposition 2 : If a PFC \mathfrak{R} is justifiable by an abstract game R , then $R = R(\mathfrak{R})$. Further, in such a situation $R(\mathfrak{R})$ is acyclic.

Proof : The acyclicity of $R(\mathfrak{R})$ is required by Proposition 1 and the definition of justifiability.
Q.E.D.

However, there are extremely well behaved PFC's which are not justifiable.

A PFC \mathfrak{R} is said to be monotonic if $\forall A, B \in [X] : [A \subset B]$ implies $[(A,B) \in \mathfrak{R}]$. Monotonicity is a very reasonable assumption to make on a PFC. It simply says that a set which contains another set should be at least as desirable as the set it contains.

Proposition 3: Let $X = \{x, y, z\}$ with $x \neq y \neq z \neq x$. Then there exists a PFC \mathfrak{R} , which is transitive and monotonic, but not justifiable.

Proof : Let $\mathfrak{R} = \Delta([X]) \cup \{(\{x\}, \{y\}), (\{y\}, \{z\}), (\{x\}, \{z\})\} \cup \{(A,B) \in [X] \times [X] / \#(A) > \#(B)\} \cup \{(A,B) \in [X] \times [X] / \#(A) = \#(B) = 2\}$. Clearly, $R(\mathfrak{R}) = \Delta(X) \cup \{(x,y), (y,z), (x,z)\}$. It is easy to observe that \mathfrak{R} is not justifiable : $(\{y,z\}, \{x\}) \in \mathfrak{R}$, although $G(X, R(\mathfrak{R})) \cap \{y,z\} = \emptyset$. However, \mathfrak{R} is reflexive since $\Delta([X]) \subset \mathfrak{R}$, and monotonic since $B \subset A$ implies $\#(A) > \#(B)$, which leads to $(A,B) \in \mathfrak{R}$. Let $A, B \in [X] \times [X]$ with $A \neq B$. If $A = X$, then $(A,B) \in \mathfrak{R}$. Hence suppose neither A nor B is equal to X . If $\#(A) > \#(B)$, then $(A,B) \in \mathfrak{R}$. Hence suppose, $\#(A) = \#(B)$. If $\#(A) = \#(B) = 1$

and $(A,B) \notin \mathfrak{R}$, then $(B,A) \in \mathfrak{R}$. If $\#(A) = \#(B) = 2$ then $(A,B) \in \mathfrak{R}$. Thus, \mathfrak{R} is complete. In a similar fashion it can be shown that \mathfrak{R} is transitive.

Q.E.D.

The following proposition is easy to establish.

Proposition 4 : Let \mathfrak{R} be a PFC. If \mathfrak{R} is justifiable, then it satisfies Monotonicity.

A PFC \mathfrak{R} is said to be weakly justifiable if there exists an abstract game R such that $\forall A,B \in [X]: [(A,B) \in \mathfrak{R}]$ if and only if [there exists $x \in A$ so that $x \in G(\{x\} \cup B, R)$]. It is easy to see that if \mathfrak{R} is justifiable then it is weakly justifiable (: if $G(A \cup B, R) \cap A \neq \emptyset$, then whenever $x \in G(A \cup B, R) \cap A$, we have $x \in G(\{x\} \cup B, R)$). Further the following is easily verified :

Proposition 5 : If a PFC \mathfrak{R} is weakly justifiable by an abstract game R , then $R = R(\mathfrak{R})$. Further, in such a situation $R(\mathfrak{R})$ is acyclic.

However, the following is also true:

Proposition 6: Let $X = \{x,y,z,w\}$ consist of four distinct elements. Then there exists a PFC \mathfrak{R} , which is weakly justifiable, but not justifiable.

Proof: Let $R = \Delta(X) \cup \{(x,y),(y,z),(z,y),(z,x),(w,x),(z,w),(w,z),(w,y),(y,w)\}$. R is acyclic. Define $\mathfrak{R}(R)$ as follows: $\forall A,B \in [X]: [(A,B) \in \mathfrak{R}(R)]$ if and only if [there exists $x \in A$ so that $x \in G(\{x\} \cup B, R)$]. Clearly $\mathfrak{R}(R)$ is a weakly justifiable PFC. However, since $y \in G(\{y,z,w\}, R)$, we have $(\{x,y\}, \{z,w\}) \in \mathfrak{R}$, although $G(\{x,y,z,w\}, R) \cap \{x,y\} = \emptyset$, implies that \mathfrak{R} is not justifiable.
Q.E.D.

The above proof indicates the following result whose proof is immediate:

Proposition 7: Let R be an acyclic abstract game. Then,

- (a) $\mathfrak{R} = \{(A,B) \in [X] \times [X] / G(A \cup B, R) \cap A \neq \emptyset\}$ implies that \mathfrak{R} is justifiable by R ;
- (b) $\mathfrak{R} = \{(A,B) \in [X] \times [X] / \text{there exists } x \in A \text{ such that } x \in G(\{x\} \cup B, R)\}$ implies that \mathfrak{R} is weakly justifiable by R .

3. **Justifiability of Transitive PFC's :** If \mathfrak{R} is a transitive PFC then weak justifiability implies justifiability as is easily verified.

Let \mathfrak{R} be a transitive PFC and for a non empty subset \mathfrak{I} of $[X]$, let $\Pi(\mathfrak{I}, \mathfrak{R}) = \{A \in \mathfrak{I} / \forall B \in \mathfrak{I}: (A,B) \in \mathfrak{R}\}$.

A PFC \mathfrak{R} is said to satisfy Concordance if $\forall A,B \in [X]: [(A,B) \in \mathfrak{R}]$ implies $[(A, A \cup B) \in \mathfrak{R}]$.

A PFC \mathfrak{R} is said to satisfy Strong Concordance if $\forall A,B \in [X]: [(A,B) \in \mathfrak{R}]$ implies $[(A, A \cup B) \in I(\mathfrak{R})]$.

Proposition 8: (a) Strong Concordance implies Concordance; (b) Concordance plus Monotonicity implies Strong Concordance; (c) There exists a transitive PFC which satisfies Concordance but does not satisfy Strong Concordance. Hence it does not satisfy Monotonicity. Thus, it is not justifiable.

Proof: (a) and (b) are easy. Hence let us establish (c). Let $\mathfrak{R} = \Delta([X]) \cup \{(\{x\}, \{y\}), (\{y\}, \{z\}), (\{x\}, \{z\})\} \cup \{(A, B) \in [X] \times [X] / \#(A) < \#(B)\} \cup \{(A, B) \in [X] \times [X] / \#(A) = \#(B) = 2\}$. Clearly, $R(\mathfrak{R}) = \Delta(X) \cup \{(x, y), (y, z), (x, z)\}$. It is easy to observe that \mathfrak{R} is not justifiable : $(\{y\}, \{x, z\}) \in \mathfrak{R}$, although $G(X, R(\mathfrak{R})) \cap \{y\} = \emptyset$. However, \mathfrak{R} is reflexive since $\Delta([X]) \subset \mathfrak{R}$. Let $A, B \in [X] \times [X]$ with $A \neq B$. If $B = X$, then $(A, B) \in \mathfrak{R}$. Hence suppose neither A nor B is equal to X . If $\#(A) < \#(B)$, then $(A, B) \in \mathfrak{R}$. Hence suppose $\#(A) = \#(B)$. If $\#(A) = \#(B) = 1$ and $(A, B) \notin \mathfrak{R}$, then $(B, A) \in \mathfrak{R}$. If $\#(A) = \#(B) = 2$ then $(A, B) \in \mathfrak{R}$. Thus, \mathfrak{R} is complete. In a similar fashion it can be shown that \mathfrak{R} is transitive. However, \mathfrak{R} is not monotonic since $\{x\} \subset \subset \{x, y\}$ but $(\{x, y\}, \{x\}) \notin \mathfrak{R}$. Neither does it satisfy Strong Monotonicity: $(\{x\}, \{y\}) \in \mathfrak{R}$, but $(\{x\}, \{x, y\}) \notin \mathfrak{R}$.
Q.E.D.

Proposition 9 : Let \mathfrak{R} be a transitive PFC. Then \mathfrak{R} satisfies Concordance if and only if the following condition holds: $\forall n \in \mathbb{N}$, and $A(1), \dots, A(n) \in [X]$, there exists $i \in \{1, \dots, n\}$ such that $(A(i), \cup\{A(j)/j=1, \dots, n\}) \in \mathfrak{R}$.

Proof : Let \mathfrak{R} satisfy the condition and let $A, B \in [X]$ with $(A, B) \in \mathfrak{R}$. If $(A, A \cup B) \in \mathfrak{R}$, then we are done. If not, then by the above condition, we have $(B, A \cup B) \in \mathfrak{R}$. By transitivity,

$(A, A \cup B) \in \mathfrak{R}$. Thus \mathfrak{R} satisfies Concordance.

Now suppose, \mathfrak{R} satisfies Concordance. Let $\mathfrak{A}_1 = \{A(j)/j=1, \dots, n\}$. Without loss of generality suppose $A(1) \in \Pi(\mathfrak{A}_1, \mathfrak{R})$. Thus, by Concordance $(A(1), A(1) \cup A(j)) \in \mathfrak{R} \forall j \in \{2, \dots, n\}$. Let $\mathfrak{A}_2 = \{A(1) \cup A(j)/j=2, \dots, n\}$. Without loss of generality suppose $A(1) \cup A(2) \in \Pi(\mathfrak{A}_2, \mathfrak{R})$. Thus, by Concordance $(A(1) \cup A(2), A(1) \cup A(2) \cup A(j)) \in \mathfrak{R} \forall j \in \{3, \dots, n\}$. By transitivity of \mathfrak{R} , $(A(1), A(1) \cup A(2) \cup A(j)) \in \mathfrak{R} \forall j \in \{3, \dots, n\}$.

Suppose that in this manner we have arrived at $(A(1), A(1) \cup A(2) \cup \dots \cup A(i-1) \cup A(j)) \in \mathfrak{R} \forall j \in \{i, \dots, n\}$. Let $\mathfrak{A}_i = \{A(1) \cup A(2) \cup \dots \cup A(i-1) \cup A(j)/j=i, \dots, n\}$. Without loss of generality suppose $A(1) \cup A(2) \cup \dots \cup A(i-1) \cup A(i) \in \Pi(\mathfrak{A}_i, \mathfrak{R})$. By Concordance, $(A(1) \cup A(2) \cup \dots \cup A(i-1) \cup A(i), A(1) \cup A(2) \cup \dots \cup A(i-1) \cup A(i) \cup A(j)) \in \mathfrak{R} \forall j \in \{i+1, \dots, n\}$. By transitivity, of \mathfrak{R} we get $(A(1), A(1) \cup A(2) \cup \dots \cup A(i-1) \cup A(i) \cup A(j)) \in \mathfrak{R} \forall j \in \{i+1, \dots, n\}$. Proceeding thus we get, $(A(1), A(1) \cup A(2) \cup \dots \cup A(n)) \in \mathfrak{R}$. This proves the proposition.

Q.E.D.

A similar result is available for transitive PFC's in the case of Strong Concordance.

Proposition 10 : Let \mathfrak{R} be a transitive PFC. Then \mathfrak{R} satisfies Strong Concordance if and only if the following condition holds: $\forall n \in \mathbb{N}$, and $A(1), \dots, A(n) \in [X]$, there exists $i \in \{1, \dots, n\}$ such that (a) $A(i) \in \Pi(\{A(1), \dots, A(n)\}, \mathfrak{R})$; (b) $(A(i), \cup\{A(j)/j=1, \dots, n\}) \in \mathfrak{R}$.

Proof : Let \mathfrak{R} satisfy the condition and let $A, B \in [X]$ with $(A, B) \in \mathfrak{R}$. If $(A, A \cup B) \in I(\mathfrak{R})$, then we are done. If by the above condition, we have $(B, A) \in \mathfrak{R}$ and $(B, A \cup B) \in I(\mathfrak{R})$, then we have $(A, B) \in I(\mathfrak{R})$ and consequently $(A, A \cup B) \in I(\mathfrak{R})$ by the transitivity of \mathfrak{R} . Thus \mathfrak{R} satisfies Strong Concordance.

Now suppose, \mathfrak{R} satisfies Strong Concordance. Let $\mathfrak{I}_1 = \{A(j)/j=1, \dots, n\}$. Without loss of generality suppose $A(1) \in \Pi(\mathfrak{I}_1, \mathfrak{R})$. Thus, by Strong Concordance $(A(1), A(1) \cup A(j)) \in I(\mathfrak{R}) \forall j \in \{2, \dots, n\}$. Let $\mathfrak{I}_2 = \{A(1) \cup A(j)/j=2, \dots, n\}$. Without loss of generality suppose $A(1) \cup A(2) \in \Pi(\mathfrak{I}_2, \mathfrak{R})$. Thus, by Strong Concordance $(A(1) \cup A(2), A(1) \cup A(2) \cup A(j)) \in I(\mathfrak{R}) \forall j \in \{3, \dots, n\}$. By transitivity of \mathfrak{R} , $(A(1), A(1) \cup A(2) \cup A(j)) \in I(\mathfrak{R}) \forall j \in \{3, \dots, n\}$. Suppose that in this manner we have arrived at $(A(1), A(1) \cup A(2) \cup \dots \cup A(i-1) \cup A(j)) \in I(\mathfrak{R}) \forall j \in \{i, \dots, n\}$. Let $\mathfrak{I}_i = \{A(1) \cup A(2) \cup \dots \cup A(i-1) \cup A(j)/j=i, \dots, n\}$. Without loss of generality suppose $A(1) \cup A(2) \cup \dots \cup A(i-1) \cup A(i) \in \Pi(\mathfrak{I}_i, \mathfrak{R})$. By Strong Concordance, $(A(1) \cup A(2) \cup \dots \cup A(i-1) \cup A(i), A(1) \cup A(2) \cup \dots \cup A(i-1) \cup A(i) \cup A(j)) \in I(\mathfrak{R}) \forall j \in \{i+1, \dots, n\}$. By transitivity, of \mathfrak{R} we get $(A(1), A(1) \cup A(2) \cup \dots \cup A(i-1) \cup A(i) \cup A(j)) \in I(\mathfrak{R}) \forall j \in \{i+1, \dots, n\}$. Proceeding thus we get, $(A(1), A(1) \cup A(2) \cup \dots \cup A(n)) \in I(\mathfrak{R})$. This proves the proposition.

Q.E.D.

Proposition 11: Let \mathfrak{R} be a PFC. If \mathfrak{R} is justifiable, then it satisfies Concordance.

Proof : Let $A, B \in [X]$ and suppose $(A, B) \in \mathfrak{R}$. Since it is justifiable, $G(A \cup B, R(\mathfrak{R})) \cap A \neq \emptyset$. Hence, $(A, A \cup B) \in \mathfrak{R}$.

Q.E.D.

The following is the main theorem of this paper:

Theorem 1: Let \mathfrak{R} be a transitive PFC. Then \mathfrak{R} is justifiable if and only if \mathfrak{R} satisfies Concordance and Monotonicity.

Proof : That the justifiability of a PFC \mathfrak{R} implies Concordance and Monotonicity has been observed in Propositions 4 and 11. Hence let us assume that \mathfrak{R} is a transitive PFC satisfying Monotonicity and Concordance.

Suppose $A, B \in [X]$ and $(A, B) \in \mathfrak{R}$. Towards a contradiction suppose that $G(A \cup B, R(\mathfrak{R})) \cap A = \emptyset$. Since $R(\mathfrak{R})$ is transitive we thus get that $\emptyset \neq G(A \cup B, R(\mathfrak{R})) \subset B$. Further the transitivity of $R(\mathfrak{R})$ implies that $G(A, R(\mathfrak{R})) \neq \emptyset$. Let $x \in G(A \cup B, R(\mathfrak{R}))$. Thus, $x \in B$. By Monotonicity, $(B, \{x\}) \in \mathfrak{R}$. By Concordance and Proposition 9, there exists $y \in A : (\{y\}, A) \in \mathfrak{R}$. This is because $A = \cup\{z/z \in A\}$. Thus, $(x, y) \in P(R(\mathfrak{R}))$. Hence, $(\{x\}, \{y\}) \in P(\mathfrak{R})$. Thus, by transitivity of \mathfrak{R} , $(B, A) \in P(\mathfrak{R})$, contradicting $(A, B) \in \mathfrak{R}$. Thus, $G(A \cup B, R(\mathfrak{R})) \cap A \neq \emptyset$.

Now, suppose that $A, B \in [X]$ and $G(A \cup B, R(\mathfrak{R})) \cap A \neq \phi$. By completeness of \mathfrak{R} , $(B, A) \in \mathfrak{R}$. By Concordance and Proposition 9, there exists, $y \in B : (\{y\}, B) \in \mathfrak{R}$. This is because $B = \cup \{ \{z\} / z \in B \}$. Let $x \in G(A \cup B, R(\mathfrak{R})) \cap A$. Thus, $(A, \{x\}) \in \mathfrak{R}$ by Monotonicity. Further, $(\{x\}, \{y\}) \in \mathfrak{R}$, since $(x, y) \in R(\mathfrak{R})$. By transitivity of \mathfrak{R} , we get $(A, B) \in \mathfrak{R}$. Thus \mathfrak{R} is justifiable.
Q.E.D.

Given a PFC \mathfrak{R} and sets $A, B \in [X]$, say that A is Revealed Weakly Superior (RWS) to B if $G(A, R(\mathfrak{R})) \subset G(A \cup B, R(\mathfrak{R}))$. Thus, A is RWS to B, if while choosing from the union of B to A, we are not lead to the omission of elements already chosen from A. We now state a Lemma whose obvious proof is being omitted.

Lemma 1: Let \mathfrak{R} be a transitive PFC and let $A, B \in [X]$. Then A is RWS to B if and only if $G(A \cup B, R(\mathfrak{R})) \cap A \neq \phi$.

A PFC \mathfrak{R} is said to satisfy the Weak Revealed Preference Property if $\forall A, B \in [X]$: $(A, B) \in \mathfrak{R}$ if and only if A is RWS to B.

In view of Lemma 1, the following theorem stands established.

Theorem 2 : Let \mathfrak{R} be a transitive PFC. Then \mathfrak{R} is justifiable if and only if \mathfrak{R} satisfies Weak Revealed Preference Property.

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