

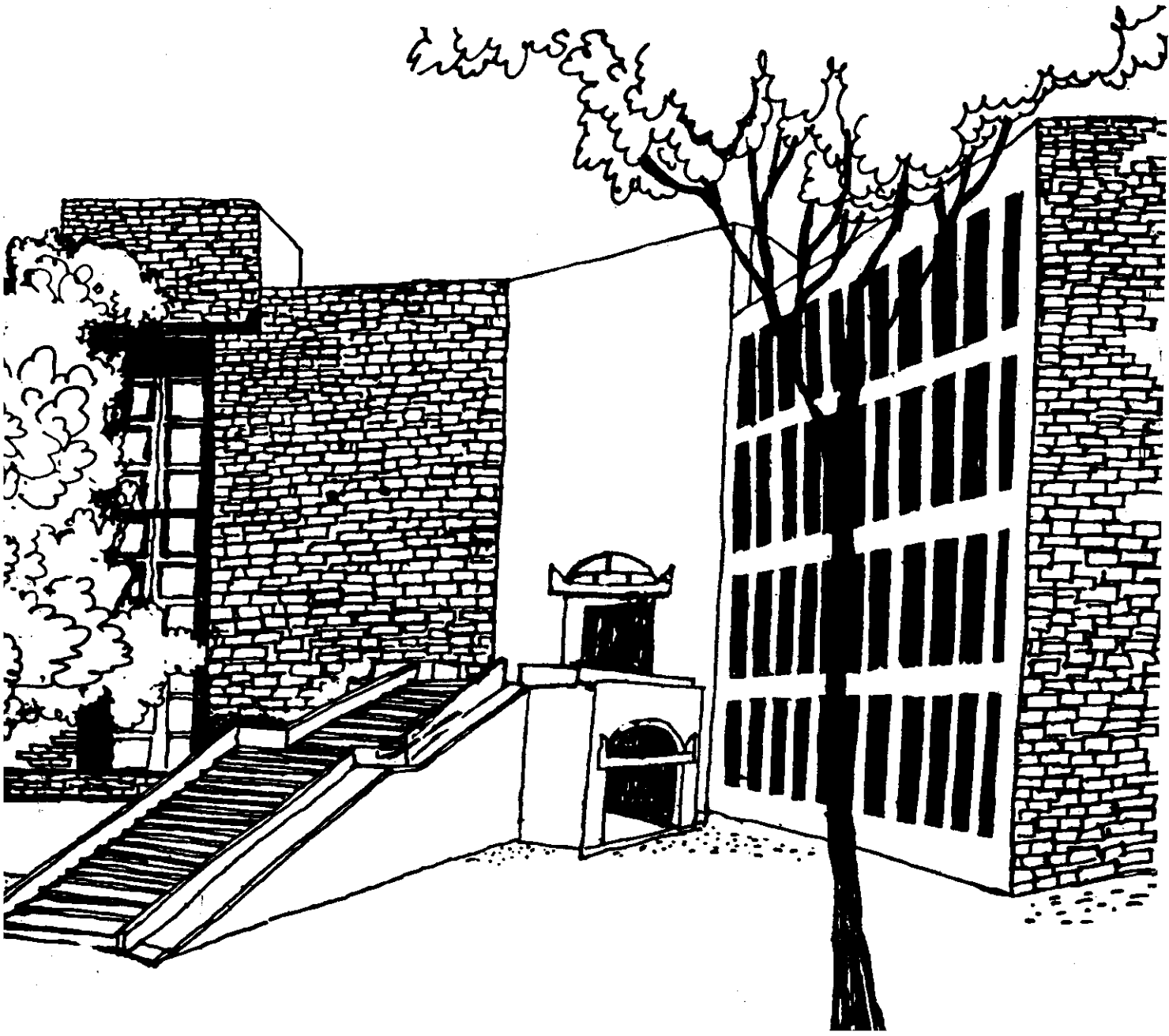


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Working Paper



**An Optimization-Based Decision Support System
for Strategic and Operational Planning in Process Industries**

**Goutam Dutta
Robert Fourer**

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An Optimization-Based Decision Support System for Strategic and Operational Planning in Process Industries

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Abstract. We describe how a generic multi-period optimization-based decision support system (DSS) can be used for strategic and operational planning in process industries. Built on five fundamental elements — materials, facilities, activities, time periods and storage areas — this DSS requires little direct knowledge of optimization techniques to be used effectively. Results based on real data from an American integrated steel company demonstrate significant potential for improvement in revenues and profits.

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1. Introduction and Motivation

This research is motivated by our previous experience in modeling integrated steel plants in Asia and the North America. A series of publications (Dutta *et al.*, 1994; Sinha *et al.*, 1995) reports the conceptualization, development and implementation of one of the largest strategic planning models in India, whose development took about 20 person-years. This work resulted in a 58 percent increase in profitability (or a direct financial benefit of 73 million dollars) during the last six months of fiscal year 1986-87, and accrued similar proportional benefit in later years. During the same time an initiative at Northwestern University (Fourer, 1997) with the support of the American Iron and Steel Institute resulted in the development of a generic model for strategic and operational planning in steel plants. In both cases, however, the models were single-period and did not discuss the effect of time-varying parameters in the decision making.

The application of linear programming in integrated steel companies was first reported at Kaiser Steel in the United States (Fabian, 1958). Other applications (Beale, Coen and Flowerdew, 1965; Bandopadhyay, 1969; Bielefeld, Klaus and Wartmann, 1986; Hanai, 1994, Sarma, 1995; Sharma and Sinha, 1991) have since been reported in the United Kingdom, Japan, Germany, and other developed countries. However, a comprehensive survey of mathematical programming models (Dutta and Fourer, 2002) indicates that very little work has been done in the area of multi-period linear programming models. In North America, for example, most of the applications of linear programming in steel plants have been in short-term scheduling models for individual facilities.

Against this background, we discuss in this paper the development of a generic model that can be customized by a variety of industries for multi-period planning. The focus of our research is in the steel industry, but our model can be used as well for different process industries. In section 2, we discuss the basic approach of modeling a process industry. The elements of the database required to define the mathematical model are discussed in section 3. In section 4, we discuss the application of the model in a steel company in North America, with some examples using real data to show the possible impact. The paper concludes by describing the scope of further work in section 5. A mathematical formulation of our model is provided in the Appendix.

2. Modeling a Process Industry

The material flow diagram of an integrated steel plant is shown in Figure 1. From this diagram, we see the basic structure of a steel or process industry. Normally raw materials (such as coal, ore, and limestone) can only be bought, while finished products (pipes, axles, wheels) can only be sold. Intermediates can often neither be bought nor sold. Practically all material can be inventoried. However, for practical purposes, the inventory of hot metal (liquid steel) is bounded by the mixer capacity. At any time, we can set products bought, sold or inventoried to zero to indicate that no buying, selling or inventorying is possible.

For each material the model also specifies a list of conversions to other materials.

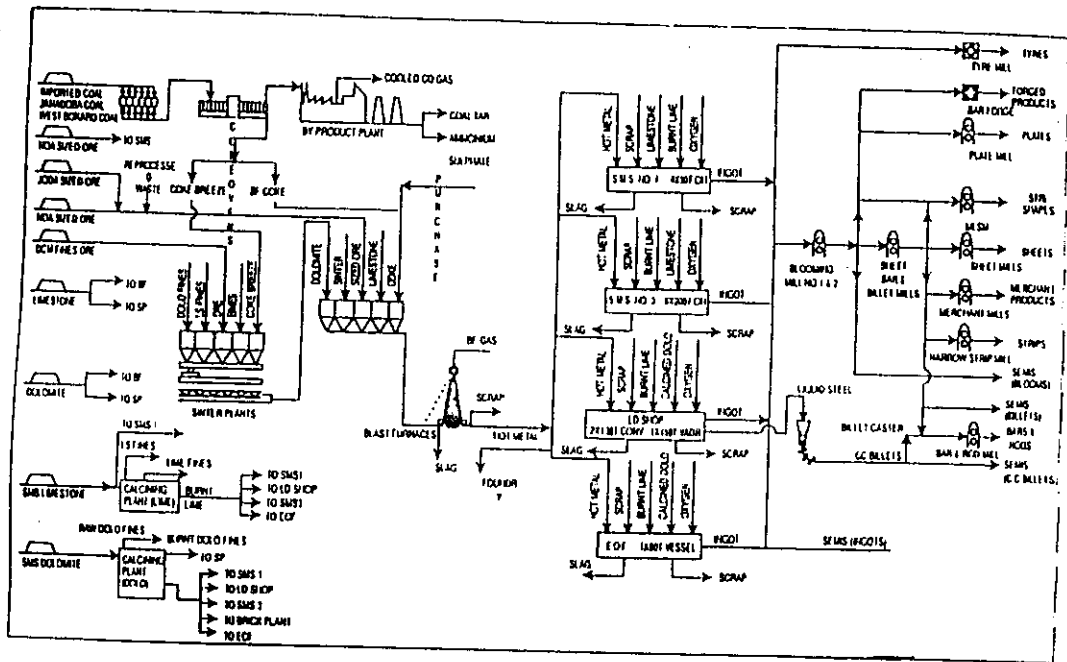


Figure 1: Material flow in an integrated steel plant.

Each conversion has a yield and cost at any time. Conversion takes care of recycling of coke-oven gas and scrap.

The creation of any product is much more difficult than a simple conversion. We define a collection of facilities at which production occurs. At any time, each facility houses one or more activities, which use and produce material in certain proportions. For long-term planning purposes we can reasonably view the production system as continuous and linear, so that a linear programming model makes sense.

The following information is provided for *each activity* at each facility at each time:

- ◊ The amount of each input required for the activity.
- ◊ The amount of each output resulting from the activity.
- ◊ The cost per unit of the activity.
- ◊ Upper and lower limits on the number of units of the activity.
- ◊ The number of units of activity that can be accommodated by one unit of the facility's capacity — the *facility-activity ratio*.

In defining an activity we have two different cases. For a rolling mill where we produce various outputs like billets and bars in the same facility, the rolling of each product is modeled as a separate activity, since each activity produces a separate output. The units of each activity are tons, but the capacity of each facility is in hours. Thus the model specifies the facility-activity ratio as tons per hour. If two products are produced in the same facility at different rolling rates, we have two different activities and two different facility-activity ratios; for example, the capacity of a rolling mill may be 50 tons/hour for billet production and 40 tons per hour for angle production. In the other case, where a facility has essentially only one activity,

both the activity and the facility capacity may be in the same units. For example at a blast furnace that produces liquid hot metal, both production and capacity are measured in tons. The facility-activity ratio must be 1 in this case.

Another important factor in our model is the definition of time. We take the time unit to be flexible. For long-term capital budgeting and strategic planning, we could use a year, month or quarter as the unit of time, whereas for shorter-term operational planning we could use a month or even a week. With respect to strategic planning, the DSS is intended to help answer questions such as the following:

- ▷ What is the optimal product mix and how does it compare to the current product mix?
- ▷ Which products should be chosen for diversification?
- ▷ How does the nominal optimum vary from the discounted one?
- ▷ What is the effect of cost or price changes, of raw materials or finished products, on the product mix?

In operational planning, the DSS would be able to help with questions like these:

- ▷ Should external scrap be purchased as a substitute for hot metal? If so, at what price?
- ▷ In response to a shortage of liquid steel, which downstream finishing mills should be idled?

In the long-term case the discounted cash flow and the interest rate are an important factor.

3. Optimization Model Structure

We optimize a generalization of a network-flow linear program, based on five fundamental elements: materials, facilities, activities, storage areas and time periods. The details of the computer implementation are beyond the scope of this paper, but are discussed in a related paper by the same authors (Dutta and Fourer, 2000). An appendix to this paper shows the model's complete formulation.

3.1 Definitions

The fundamental elements are defined as follows:

Times: These are the periods of the planning horizon, represented by discrete numbers $1, 2, 3, \dots, T$.

Materials: Any product in the steel mill at any stage of production — input, intermediate, output — is regarded as a material.

Facilities: A facility is a collection of machines which produce some materials from others. For example, a hot mill that produces sheets from slabs is a facility.

Activities: At any time, each facility houses one or more activities, which use and produce material in certain proportions. In each activity at each time, we have one

or more input materials being transformed to various output materials. Production of hot metal at a blast furnace and production of billets at a rolling mill are examples of activities.

Storage areas: These are the warehouses where raw materials, intermediate products, and finished products can be stored.

The model is a generalized network-flow model that maximizes the contribution to profit (nominal or discounted) of a company subject to the following categories of constraints for all time periods:

- ◊ Material balances
- ◊ Facility capacities — optionally “soft” capacities that may be exceeded at some cost in outsourcing
- ◊ Storage area capacities
- ◊ Bounds — on materials (bought, sold, inventoried), on facility inputs and outputs, and on activities

Owing to the generality of the model, each material can potentially be bought, sold, or inventoried at each time period. Thus any material can be modeled as a raw material, intermediate, or finished product depending on the circumstances being considered.

3.2 Assumptions

The model we describe is general enough to accommodate facilities that are in series, in parallel, or in some more complex configuration. As previously remarked, each facility can have one or more activities.

There can be purchase, sale and storage of materials at the raw materials stage, at the finishing stage, or at the intermediate processing stages. Moreover the purchase price of raw materials, the selling price of finished goods, and the inventory carrying costs may vary over time.

At any time, one or more materials may be used as inputs or outputs at a facility. Generally more than one input material is used to produce one output. The relative proportions of inputs and outputs (the technological coefficients) of an activity remain the same in a period. Technological coefficients may vary with time.

The capacity of each facility and each storage area is finite. Since the facilities will have different patterns of preventive maintenance schedules, the capacities of the facilities will vary over time.

Costs and production amounts are considered to vary proportionately with activity levels. Thus essential features of the production planning problem can be captured in a deterministic, linear optimization model.

3.3 Implementation

Our model is implemented within 4th Dimension (Adams and Beckett, 1999), a relational database management system. Other database systems such as Access or Oracle could be used just as well.

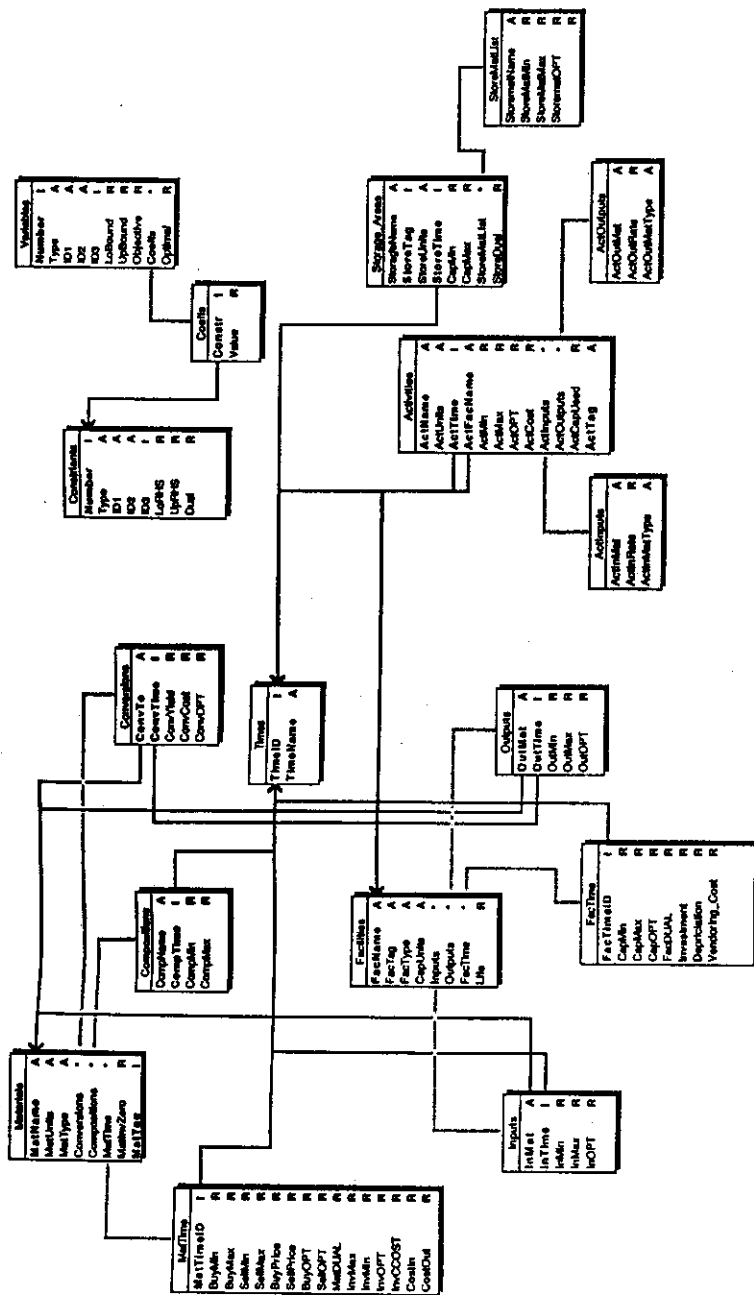


Figure 2: Database structure for the general planning model.

Figure 2 shows the structure of the database as expressed within 4th Dimension. The five boxes labeled Materials, Facilities, Activities, Times, and Storage_Areas correspond to the five major elements or *files* of the database. Items within each box denote the file's data fields and subfiles, with the subfile entries distinguished by a light-shaded line that runs to the top of a separate box in which the subfile data fields are listed. The smaller, independent database structure in the upper right of the diagram holds a generated linear program as described in the next subsection.

Following 4th Dimension's notation, we use bracketed names to denote files and apostrophes to separate subfile and field names. Thus [Facilities] is the database file of facilities, [Facilities]Inputs is the subfile of facility inputs, and [Facilities]Inputs'InMin is a data field of the subfile. Further details can be found in the discussion of the one-period model in an earlier publication (Fourer, 1997).

3.4 Optimization Steps

Once the data of the five database files and their respective subfiles are entered, they are validated by a series of simple tests, for example to insure that every material can be either used somewhere in production or sold. The principal steps of the subsequent optimization process are as follows:

1. The data describing the production scenario at different time periods are collected and stored in the database.

2. The constraints associated with the linear program are generated. The constant terms of the constraint equations or inequalities, LoRHS and HiRHS, are extracted from the database and stored in the [Constraints] file.

3. The variables of the associated linear program are determined, along with their coefficients in the constraints. Variables are stored in a separate [Variables] file and coefficients in its [Variables]Coeff subfile. This step gives the user a choice of discounted or undiscounted optimization. If the latter is chosen then it prompts for an interest rate and all cost, price, and revenue data are converted to their discounted values in the objective function.

4. The [Constraints] and [Variables] files are scanned and all of the essential information about the linear program is written to an ordinary textfile in a compact format. This textfile is the input file to our solver.

5. The text file is read by a linear programming solver — we used XMP (Martsen, 1981) — which solves the indicated linear program and then writes the optimal values of the variables to a second text file.

6. The second text file is read and the optimal values are placed in appropriate fields of the [Materials], [Facilities], [Activities], and [Storage_Areas] files and their subfiles.

To support these activities the database offers three modes of display. The Data mode is primarily for entering data describing the operations to be modeled. The Optimal mode shows also the fields for the optimal values, and hence is intended for examination of results. Finally, an Update mode allows small changes to be made to the data without a time-consuming re-generation of the [Constraints] and [Variables] files.

4. Application to an American Steel Company

We have customized our DSS for strategic and operational planning of an American steel plant. At the time of our study this was a \$1.4 billion (sales revenue per year) company producing 860,000 tons of marketable steel annually. The company

had two manufacturing facilities, one located in the eastern part of the United States and another in the midwest. In this section we discuss how we set up an instance of our model using real data of the company. The specific questions we are interested in studying include:

- ▷ What is the opportunity for increasing the profit of the company?
- ▷ What facilities have the most costly capacity limitations?
- ▷ What are the products that need the attention of the management?

We also discuss the difficulties involved and indicate the impact the DSS can make in improving the bottom line of a company.

4.1 Conversion of the company's data to model data

The following agreement was reached with the company representatives regarding confidentiality of the data.

- ◊ The financial figures of the company, such as the purchase and sales prices of materials and the costs of activities, were multiplied by a factor. This factor was not disclosed.
- ◊ The exact input and output materials of each facility, yields for each activity, and capacities of different facilities were supplied.
- ◊ The actual names of the products and facilities were replaced by code-names.
- ◊ The yield, capacity and facility activity ratios were the annual average values for the previous year.

All of the following results are based on the data presented to us in this disguised form.

Since the route of each product is different, products at different stages were distinctly identified as different materials, hence as different records in the [Materials] file. Suppose for example that a product P1 is processed at F4 and then at F5. Then product P1 after facility F4 was identified as "P1 after F4" and the same product after facility F5 was identified as "P1 after F5." Although the chemical composition of the material is the same in both cases, "P1 after F4" is a different material in the [Materials] file from "P1 after F5," as they are physically different and their dimensions are different.

The facilities included electric steelmaking units, coiling machines, secondary steelmaking units, continuous casting machines, hot mills, pickling machines, coating machines, and finish trimmers. Typical activities were production of steel, production of slabs, hot rolling of slabs, cold rolling of slabs, trimming of sheets, and pickling of sheets.

We decided to keep units of all materials in tons. The capacity of each facility was in hours and the capacity of each activity was in tons, so that the facility-activity ratio was expressed in tons per operating hour. For example, if 100 tons of cold rolled steel could be galvanized in 2 hours then the facility-activity ratio was

set to 50 tons per operating hour. The company did not supply a corresponding minimum number of operating hours, so [Facilities]FacTime'CapMin was set to zero for all facilities.

It is expected that in a company where there are many selling prices, yields, rolling rates, and capacities, a linear programming based DSS will be more useful where these figures are not very close to each other. For example, a company whose products have yields from 89% to 89.5%, selling prices from \$100 to \$101 per ton, and rolling rates from 50 to 51 tons/hour will not receive nearly as much benefit from an LP-based DSS as a company whose products have yields from 50% to 100%, selling prices from 50 to 100, and yields from \$100 to \$200. In the case of the American company we studied, the yield values of activities varied between 81% and 100%. The ratio of maximum selling price to minimum selling price was 5.7, and the analogous ratio for raw materials was 127.

Although our DSS was multi-period, the studied company decided to test it for one year and supplied the data for one period. Therefore issues regarding inventories and discounting could not be tested.

The [Materials] file had altogether 443 records, of which 104 were marketable products and 97 were raw materials. There were 58 facilities and 560 activities describing the production details of 3 product types and 23 product groups. This data translated to 1909 constraints and 2838 variables in the production planning LP.

4.2 Activity definition and the scrap rate

Our generic model can accommodate a variety of approaches to handling scrap. We followed the practice of the studied company, in which one ton of input was considered to be the unit of activity.

As an example, consider a hot mill operating with 100 tons of SLAB as input and producing 96 tons of finished PRIME plates and 4 tons of SCRAP. The company treated 50% of the SCRAP as recoverable and 50% as not recoverable. Therefore the activity input rate (or [Activities]ActInputs'ActInRate in the database) was 1.00 for SLAB, and the activity output rate (or [Activities]ActOutputs'ActOutRate) was 0.96 for PLATE and 0.02 for SCRAP. The unit of activity is one ton of the input of SLAB to be rolled.

In the context of the material balance equations, we could equivalently say that 1.00/0.96 tons of input yields 1.00 ton of PLATE and 0.02/0.96 tons of SCRAP. However, in this case the unit of activity would be defined to be one ton of PLATE produced as output.

4.3 Multiple processing in the same facility

Multiple processing in the same facility raised another modeling issue. As an example, in Figure 3 showing the processing of product CS7, the facility CM1 is used twice, once after P1 and again after P4. Thus two *stages* of the same product are competing for the same facility capacity.

This situation can be incorporated into our model by defining different activities at facility CM1 for the different processing stages. In the first-stage activ-

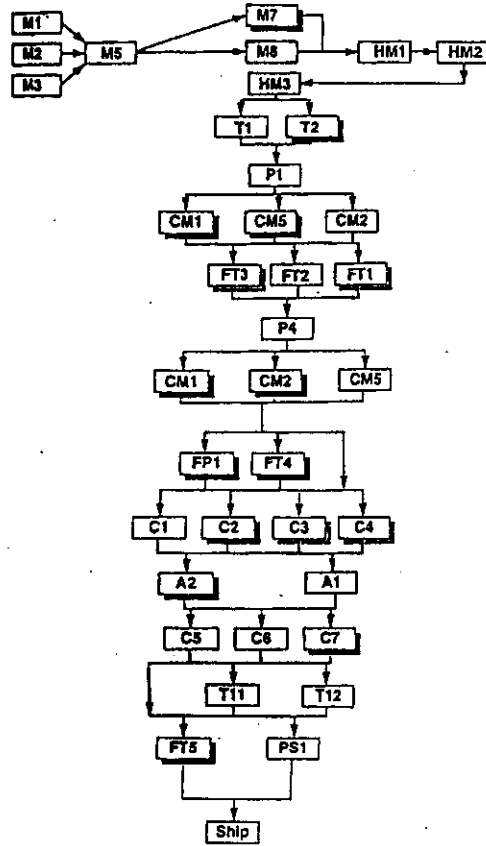


Figure 3: Process flow for product CS7, showing two different stages of processing at facility CM1.

ity, the input material is “CS7 after P1” and the output material is “CS7 after CM1/CM2/CM5-STAGE1.” In the second-stage activity, the input is “CS-7 after P4” and the output is “CS-7 after CM1/CM2/CM5-STAGE2.” The interrelationship of these activities is handled implicitly in the overall solution of the LP.

4.4 Modeling re-treatment

We define a product to be PRIME if it satisfies all required physical and metallurgical characteristics and is sold at the best price in the market. A non-prime or DEFECTIVE product can be sent for reprocessing (or re-treatment) once in the same facility.

To see how re-treatment might be handled, consider a facility where an activity takes 100 tons of input, producing 100 tons per hour of PRIME product. Since all products are good there is no re-treatment and the capacity of the facility is not affected. If the capacity is $(100 + d)$ tons per hour, $(100 + d)$ tons of PRIME product will be produced, as shown in Figure 4.

Now consider a similar situation, but with re-treatment required for some fraction $1/d$ of the product. In one hour, 100 tons of input produces $(100 - d)$ tons of output of PRIME product and d tons of DEFECTIVE product that must be re-treated. Thus at a steady state, as shown in Figure 5, $(100 + d)$ tons of input — 100 of new

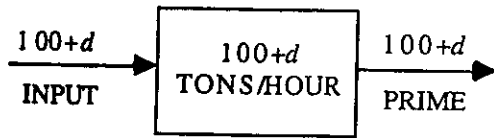


Figure 4: Material flow with no re-treatment needed.

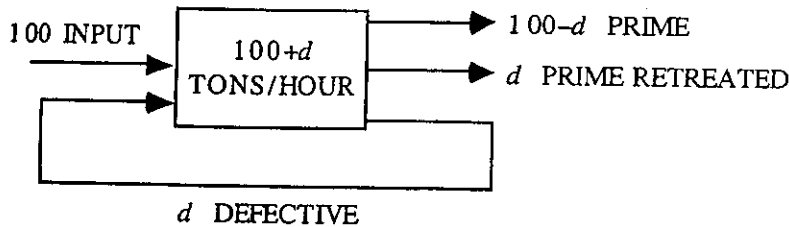


Figure 5: Material flow with a fraction d of output requiring re-treatment.

input and d of DEFECTIVE — yields $(100-d)$ tons of PRIME that was never re-treated, d tons of PRIME that was re-treated, and d tons of newly produced DEFECTIVE. Thus, although the capacity of the facility is $(100+d)$ tons per hour, it is working at 100 tons of PRIME output per hour. This is equivalent to reducing the capacity of the machine from $(100+d)$ tons per hour to 100 tons per hour. So the re-treatment can be effectively handled by multiplying the capacity by a factor of $100/(100+d)$.

This approach proved sufficient for facilities in the company we were modeling. In more complex cases that involve multiple re-treatments, possibly with varying rates of scrap production, it may be necessary to add DEFECTIVE product for a particular facility as a separate material in the database, and to define one or more additional activities to explicitly represent re-treatment.

4.5 Verification

To check that the model was sufficiently consistent with reality, the following key totals were identified:

- ◇ Total production of the steelmaking units (tons)
- ◇ Total production of the hot mills (tons)
- ◇ Total production of Product Group ES (tons)
- ◇ Total production of Product Group SS (tons)
- ◇ Total production of Product Group CS (tons)
- ◇ Total revenue (dollars)
- ◇ Total cost of purchases (dollars)
- ◇ Total cost of activities (dollars)
- ◇ Total profit (dollars)

These quantities are functions of the variables in the model and at the same time measurable quantities in actual operation. By comparing them, we were able to verify that the results of the model were in line with the capabilities of the plant.

4.6 Experiments on the model and results

We optimized with and without the company's actual production limits for financial year 1994. Thus we had four cases as follows:

Case 0: With the company's upper and lower bounds

Case 1: With the company's upper bounds, but with its lower bounds removed

Case 2: With the company's lower bounds, but with its upper bounds removed

Case 3: With the company's lower and upper bounds both removed

The results of the four cases are listed in Table 1. When all the bounds of the company are removed, the net revenue of the company is increased by 31%. The cost of purchases increases by 52% and the cost of activity increases by 11%, giving a profit improvement of 30%. Total production of the company also increases, from about 867000 to 973000 tons.

Since all the figures of production, profit and revenue have gone up from case 0 to case 3, it may be misleading to compare only the total figures. Thus the optimal results were also converted to per-unit results, to show the impact of optimization on the basis of results per ton of liquid steel. As seen in the lower portion of Table 1, there is a potential of 16% profit improvement and 17% percent revenue improvement per ton of steel.

We also analyzed the results by identifying the final products for which optimal values [Materials]MatTime'SellOPT were at their upper limits as given by [Materials]MatTime'SellMax. We re-ran the model three times, with the upper limits on all such products increased by 5% each time. The results, as seen in Table 2, show that — with the same level of liquid steel production — the profit and

Variables	Unit	Case 0	Case 1	Case 2	Case 3	Change
Revenue	\$ (millions)	977.86	1,037.07	1,176.56	1,289.20	+31.84%
Cost of Purchases	\$ (millions)	270.30	306.50	322.46	412.84	+52.73%
Cost of Activities	\$ (millions)	245.80	250.40	290.40	274.20	+11.55%
Net Profit	\$ (millions)	461.76	480.17	563.70	602.16	+30.41%
Total Steel	Tons	869685	906229	972203	973154	+11.90%
Total Hot Mill	Tons	638109	636652	697584	573793	-10.08%
Total ES	Tons	422601	451451	351127	304504	-27.95%
Total CS	Tons	235516	205905	284105	203738	-13.49%
Total SS	Tons	211568	463111	336744	463111	+118.89%
Unit Revenue	\$/Ton	1,124.38	1,144.38	1,210.20	1,324.76	+17.82%
Unit Purchase Cost	\$/Ton	310.80	338.21	331.68	424.23	+36.49%
Unit Activity Cost	\$/Ton	282.63	276.31	298.70	281.76	-0.31%
Net Profit	\$/Ton	530.95	529.86	579.82	618.77	+16.54%

Table 1. Overall results, for the four cases described in the text. Case 0 retains the company's bounds, cases 1 and 2 relax the company's lower and upper bounds, respectively, and case 3 relaxes all bounds. The "change" shows the value in case 3 relative to case 0.

Variables	Unit	Case 0	Case A	Case B	Case C	Change
Revenue	\$ (millions)	977.86	979.61	981.49	983.36	+0.56%
Cost of Purchases	\$ (millions)	270.30	270.30	270.30	270.30	0.00%
Cost of Activities	\$ (millions)	245.80	245.93	245.07	246.21	+0.17%
Net Profit	\$ (millions)	461.76	463.38	465.07	465.84	+0.88%

Table 2. Optimization at the finishing mills. Each case represents a 5% increase in final production limits over the preceding case, as explained in the text.

revenue can be improved by changing the final production limits. In other words, even if the same amount of steel is made in the steel melting shop, optimization at the finishing mills can have a substantial impact on the organization. Indeed three compounded 5% increases (about 15.8% total) lead to a profit increase of 0.88% and a revenue increase of 0.56% without any change in cost of purchases. The improvement comes entirely from using different production routes through the plant.

A comparison of facility production levels and upper limits identified a number of bottlenecks. We found that the facilities HM3 and T3 were the bottlenecks having the highest dual values. They would be the best candidates to investigate for further investment.

Our results confirm that a simple deterministic linear programming model can have significant impact in an integrated steel company. We can also draw a comparison with the previously mentioned study (Dutta *et al.*, 1994; Sinha *et al.*, 1995) in India. Whereas the primary focus of the work reported here is in the area of product-mix optimization, the focus of the work in India was in the area of optimal power distribution. In the Indian case, profit actually obtained (and verified by annual reports) with an optimization model was 2.25 times that of profit without an optimization model. The production of steel improved by 42 percent, resulting in a per ton profit improvement of 58 percent. In the case of the American steel company the revenue and profit improvement potentials are only 17 and 16 percent, respectively. But for a company of \$1.4 billion in annual revenue, this is a very significant improvement.

5. Directions for Additional Study

We have studied a generic model for process industries that is multi-material, multi-facility, and multi-activity, and that optimizes the nominal or discounted net profit of the company subject to the constraints of the industry. This problem can be visualized as multi-period single-scenario or (with a somewhat different interpretation) as single-period multi-scenario. We would like to extend it to a multi-period, multi-scenario model. This requires that we define some of the data as stochastic with probability distributions. This will be a harder problem as the constraint generation and variable generation time will increase proportionally with the number of scenarios. Moreover, we need to study the issues related to the interface of database and optimization in the stochastic case.

A second extension would introduce non-linearity to the model. Most of the in-

dustrial cost curves are non-linear or at best can be represented as having a piecewise linear behavior. It will be interesting to study how to represent this non-linearity while retaining the model's user-friendliness.

A third extension would be to have multiple-objective linear programs and represent them in the database. This can be done by changing the model management system. For example, the current model can be changed to cost minimization, revenue maximization, maximization of marketable products (revenue or production), or maximization of the utilization of the facilities. It is possible to have a menu-driven interface in the DSS which allows for optimization over different objectives.

A fourth and particularly interesting extension would be to study the paradigm neutrality (Geoffrion, 1989) of this data structure for the multiple period model. Although the model is designed for the mathematical programming paradigm, we can extend it for inventory control and also for scheduling, vehicle routing and queuing applications. We have parameters for all materials at all times; we can determine the ordering and holding cost for all material and hence try to find optimal order quantities. However, the batch size will be decided by practical considerations like the heat size of the steelmaking shop, the capacity of the vehicle carrying the products, and the capacity of the loading and unloading facility. If we can obtain the batch size and lead-time of all materials, the present model can then be extended to a scheduling model of each product in each time period.

Appendix: Model Formulation

We first define the data, in five parts: times, materials, facilities, activities, and storage-areas. The notation for the decision variables is then presented. Finally the objective and constraints are described, in both words and formulas.

All quantities of materials are taken to be in the same units, such as tons.

Time data

$T = \{1, \dots, T\}$ is the set of time periods

ρ is the interest rate per period, taken as 0 if there is no discounting

Materials data

\mathcal{M} is the set of materials

l_{jt}^{buy} = lower limit on purchases of material j , for each $j \in \mathcal{M}, t \in T$

u_{jt}^{buy} = upper limit on purchases of material j , for each $j \in \mathcal{M}, t \in T$

c_{jt}^{buy} = cost per unit of material j purchased, for each $j \in \mathcal{M}, t \in T$

l_{jt}^{sell} = lower limit on sales of material j , for each $j \in \mathcal{M}, t \in T$

u_{jt}^{sell} = upper limit on sales of material j , for each $j \in \mathcal{M}, t \in T$

c_{jt}^{sell} = revenue per unit of material j sold, for each $j \in \mathcal{M}, t \in T$

l_{jt}^{inv} = lower limit on inventory of material j , for each $j \in \mathcal{M}, t \in \mathcal{T}$

u_{jt}^{inv} = upper limit on inventory of material j , for each $j \in \mathcal{M}, t \in \mathcal{T}$

v_{j0} = initial inventory of material j , for each $j \in \mathcal{M}$

c_{jt}^{inv} = holding cost per unit of material j , for each $j \in \mathcal{M}, t \in \mathcal{T}$

$\mathcal{M}^{conv} \subseteq \{j \in \mathcal{M}, j' \in \mathcal{M} : j \neq j'\}$ is the set of conversions:

$(j, j') \in \mathcal{M}^{conv}$ means that material j can be converted to material j'

$a_{jj't}^{conv}$ = number of units of material j' that result from converting one unit of material j , for each $(j, j') \in \mathcal{M}^{conv}, t \in \mathcal{T}$

$c_{jj't}^{conv}$ = cost per unit of material j of the conversion from j to j' , for each $(j, j') \in \mathcal{M}^{conv}, t \in \mathcal{T}$

Facilities data

\mathcal{F} is the set of facilities

l_{it}^{cap} = the minimum amount of the capacity of facility i that must be used, for each $i \in \mathcal{F}, t \in \mathcal{T}$

u_{it}^{cap} = the capacity of facility i , for each $i \in \mathcal{F}, t \in \mathcal{T}$

c_{it}^{cap} = the cost of vendoring (outsourcing) a unit of capacity at facility i , for each $i \in \mathcal{F}, t \in \mathcal{T}$

$\mathcal{F}^{in} \subseteq \mathcal{F} \times \mathcal{M}$ is the set of facility inputs:

$(i, j) \in \mathcal{F}^{in}$ means that material j is used as an input at facility i

l_{ijt}^{in} = the minimum amount of material j that must be used as input at facility i , for each $(i, j) \in \mathcal{F}^{in}, t \in \mathcal{T}$

u_{ijt}^{in} = the maximum amount of material j that may be used as input at facility i , for each $(i, j) \in \mathcal{F}^{in}, t \in \mathcal{T}$

$\mathcal{F}^{out} \subseteq \mathcal{F} \times \mathcal{M}$ is the set of facility outputs:

$(i, j) \in \mathcal{F}^{out}$ means that material j is produced as an output at facility i

l_{ijt}^{out} = the minimum amount of material j that must be produced as output at facility i , for each $(i, j) \in \mathcal{F}^{out}, t \in \mathcal{T}$

u_{ijt}^{out} = the maximum amount of material j that may be produced as output at facility i , for each $(i, j) \in \mathcal{F}^{out}, t \in \mathcal{T}$

Activities data

$\mathcal{F}^{act} \subseteq \{(i, k) : i \in \mathcal{F}\}$ is the set of activities:

$(i, k) \in \mathcal{F}^{act}$ means that k is an activity available at facility i

- l_{ikt}^{act} = the minimum number of units of activity k that may be run at facility i ,
 for each $(i, k) \in \mathcal{F}^{\text{act}}, t \in \mathcal{T}$
- u_{ikt}^{act} = the maximum number of units of activity k that may be run at facility i ,
 for each $(i, k) \in \mathcal{F}^{\text{act}}, t \in \mathcal{T}$
- c_{ikt}^{act} = the cost per unit of running activity k at facility i ,
 for each $(i, k) \in \mathcal{F}^{\text{act}}, t \in \mathcal{T}$
- r_{ikt}^{act} = the number of units of activity k that can be accommodated
 in one unit of capacity of facility i , for each $(i, k) \in \mathcal{F}^{\text{act}}, t \in \mathcal{T}$
- $\mathcal{A}^{\text{in}} \subseteq \{(i, j, k, t) : (i, j) \in \mathcal{F}^{\text{in}}, (i, k) \in \mathcal{F}^{\text{act}}, t \in \mathcal{T}\}$ is the set of activity inputs:
 $(i, j, k, t) \in \mathcal{A}^{\text{in}}$ means that input material j is used
 by activity k at facility i during time period t
- a_{ijkt}^{in} = units of input material j required by one unit of activity k at facility i
 in time period t , for each $(i, j, k, t) \in \mathcal{A}^{\text{in}}$
- $\mathcal{A}^{\text{out}} \subseteq \{(i, j, k, t) : (i, j) \in \mathcal{F}^{\text{out}}, (i, k) \in \mathcal{F}^{\text{act}}, t \in \mathcal{T}\}$ is the set of activity outputs:
 $(i, j, k, t) \in \mathcal{A}^{\text{out}}$ means that output material j is produced
 by activity k at facility i during time period t
- a_{ijk}^{out} = units of output material j produced by one unit of activity k at facility i
 in time period t , for each $(i, j, k, t) \in \mathcal{A}^{\text{out}}$

Storage-areas data

\mathcal{S} is the set of storage areas

l_{st}^{stor} = lower limit on total material in storage area s , for each $s \in \mathcal{S}, t \in \mathcal{T}$

u_{st}^{stor} = upper limit on total material in storage area s , for each $s \in \mathcal{S}, t \in \mathcal{T}$

Variables

x_{jt}^{buy} = units of material j bought, for each $j \in \mathcal{M}, t \in \mathcal{T}$

x_{jt}^{sell} = units of material j sold, for each $j \in \mathcal{M}, t \in \mathcal{T}$

x_{jst}^{stor} = units of material j in storage area s , for each $j \in \mathcal{M}, s \in \mathcal{S}, t \in \mathcal{T}$

x_{jt}^{inv} = total units of material j in inventory (storage), for each $j \in \mathcal{M}, t \in \mathcal{T}$

x_{j0}^{inv} = initial inventory of material j , for each $j \in \mathcal{M}$

$x_{jj't}^{\text{conv}}$ = units of material j converted to material j' , for each $(j, j') \in \mathcal{M}^{\text{conv}}, t \in \mathcal{T}$

x_{ijt}^{in} = units of material j used as input by facility i , for each $(i, j) \in \mathcal{F}^{\text{in}}, t \in \mathcal{T}$

x_{ijt}^{out} = units of material j produced as output by facility i , for each $(i, j) \in \mathcal{F}^{\text{out}}, t \in \mathcal{T}$

x_{ikt}^{act} = units of activity k operated at facility i , for each $(i, k) \in \mathcal{F}^{\text{act}}, t \in \mathcal{T}$

x_{it}^{cap} = units of capacity vendored at facility i , for each $i \in \mathcal{F}, t \in \mathcal{T}$

Objective

Maximize the sum over all time periods of revenues from sales less costs of purchasing, holding inventories, converting, operating activities at facilities, and vendoring:

$$\sum_{t \in \mathcal{T}} (1 + \rho)^{-t} \left\{ \sum_{j \in \mathcal{M}} c_{jt}^{\text{sell}} x_{jt}^{\text{sell}} - \sum_{j \in \mathcal{M}} c_{jt}^{\text{buy}} x_{jt}^{\text{buy}} - \sum_{j \in \mathcal{M}} c_{jt}^{\text{inv}} x_{jt}^{\text{inv}} \right. \\ \left. - \sum_{(j,j') \in \mathcal{M}^{\text{conv}}} c_{jj't}^{\text{conv}} x_{jj't}^{\text{conv}} - \sum_{(i,k) \in \mathcal{F}^{\text{act}}} c_{ikt}^{\text{act}} x_{ikt}^{\text{act}} - \sum_{i \in \mathcal{F}} c_{it}^{\text{cap}} x_{it}^{\text{cap}} \right\}$$

Constraints

For each $j \in \mathcal{M}$ and $t \in \mathcal{T}$, the amount of material j made available by purchases, production, conversions, and beginning inventory must equal the amount used for sales, production, conversions, and ending inventory:

$$x_{jt}^{\text{buy}} + \sum_{(i,j) \in \mathcal{F}^{\text{out}}} x_{ijt}^{\text{out}} + \sum_{(j',j) \in \mathcal{M}^{\text{conv}}} a_{j'jt}^{\text{conv}} x_{j'jt}^{\text{conv}} + x_{j,t-1}^{\text{inv}} \\ = x_{jt}^{\text{sell}} + \sum_{(i,j) \in \mathcal{F}^{\text{in}}} x_{ijt}^{\text{in}} + \sum_{(j,j') \in \mathcal{M}^{\text{conv}}} x_{jj't}^{\text{conv}} + x_{jt}^{\text{inv}}$$

For each $(i,j) \in \mathcal{F}^{\text{in}}$ and $t \in \mathcal{T}$, the amount of input j used at facility i must equal the total consumption by all activities at facility i :

$$x_{ijt}^{\text{in}} = \sum_{(i,j,k,t) \in \mathcal{A}^{\text{in}}} a_{ijkt}^{\text{in}} x_{ikt}^{\text{act}}$$

For each $(i,j) \in \mathcal{F}^{\text{out}}$ and $t \in \mathcal{T}$, the amount of output j produced at facility i must equal the total production by all activities at facility i :

$$x_{ijt}^{\text{out}} = \sum_{(i,j,k,t) \in \mathcal{A}^{\text{out}}} a_{ijkt}^{\text{out}} x_{ikt}^{\text{act}}$$

For each $i \in \mathcal{F}$ and $t \in \mathcal{T}$, the capacity used by all activities at facility i must be within the range given by the lower limit and by the upper limit plus the amount of capacity vendored:

$$l_{it}^{\text{cap}} \leq \sum_{(i,k) \in \mathcal{F}^{\text{act}}} x_{ikt}^{\text{act}} / r_{ikt}^{\text{act}} \leq u_{it}^{\text{cap}} + x_{it}^{\text{cap}}$$

For each $j \in \mathcal{M}$, the amount of material j inventoried before the first time period is defined to equal the specified initial inventory:

$$x_{j0}^{\text{inv}} = r_{j0}$$

For each $j \in \mathcal{M}$ and $t \in \mathcal{T}$, the total amount of material j inventoried is defined as the sum of the inventories over all storage areas:

$$\sum_{s \in \mathcal{S}} x_{jst}^{\text{stor}} = x_{jt}^{\text{inv}}$$

For each $s \in \mathcal{S}$ and $t \in \mathcal{T}$, the total of all materials inventoried in storage area s must be within the specified limits:

$$l_{st}^{\text{stor}} \leq \sum_{j \in \mathcal{M}} x_{jst}^{\text{stor}} \leq u_{st}^{\text{stor}}$$

All variables must lie within the relevant limits defined by the data:

$$\begin{aligned} l_{jt}^{\text{buy}} \leq x_{jt}^{\text{buy}} \leq u_{jt}^{\text{buy}}, & \text{ for each } j \in \mathcal{M} \text{ and } t \in \mathcal{T} \\ l_{jt}^{\text{sell}} \leq x_{jt}^{\text{sell}} \leq u_{jt}^{\text{sell}}, & \text{ for each } j \in \mathcal{M} \text{ and } t \in \mathcal{T} \\ l_{jt}^{\text{inv}} \leq x_{jt}^{\text{inv}} \leq u_{jt}^{\text{inv}}, & \text{ for each } j \in \mathcal{M} \text{ and } t \in \mathcal{T} \\ 0 \leq x_{jj't}^{\text{conv}}, & \text{ for each } (j, j') \in \mathcal{M}^{\text{conv}} \text{ and } t \in \mathcal{T} \\ 0 \leq x_{it}^{\text{cap}}, & \text{ for each } i \in \mathcal{F} \text{ and } t \in \mathcal{T} \\ 0 \leq x_{jst}^{\text{stor}}, & \text{ for each } j \in \mathcal{M}, s \in \mathcal{S} \text{ and } t \in \mathcal{T} \\ l_{ij}^{\text{in}} \leq x_{ij}^{\text{in}} \leq u_{ij}^{\text{in}}, & \text{ for each } (i, j) \in \mathcal{F}^{\text{in}} \text{ and } t \in \mathcal{T} \\ l_{ij}^{\text{out}} \leq x_{ij}^{\text{out}} \leq u_{ij}^{\text{out}}, & \text{ for each } (i, j) \in \mathcal{F}^{\text{out}} \text{ and } t \in \mathcal{T} \\ l_{ik}^{\text{act}} \leq x_{ik}^{\text{act}} \leq u_{ik}^{\text{act}}, & \text{ for each } (i, k) \in \mathcal{F}^{\text{act}} \text{ and } t \in \mathcal{T} \end{aligned}$$

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