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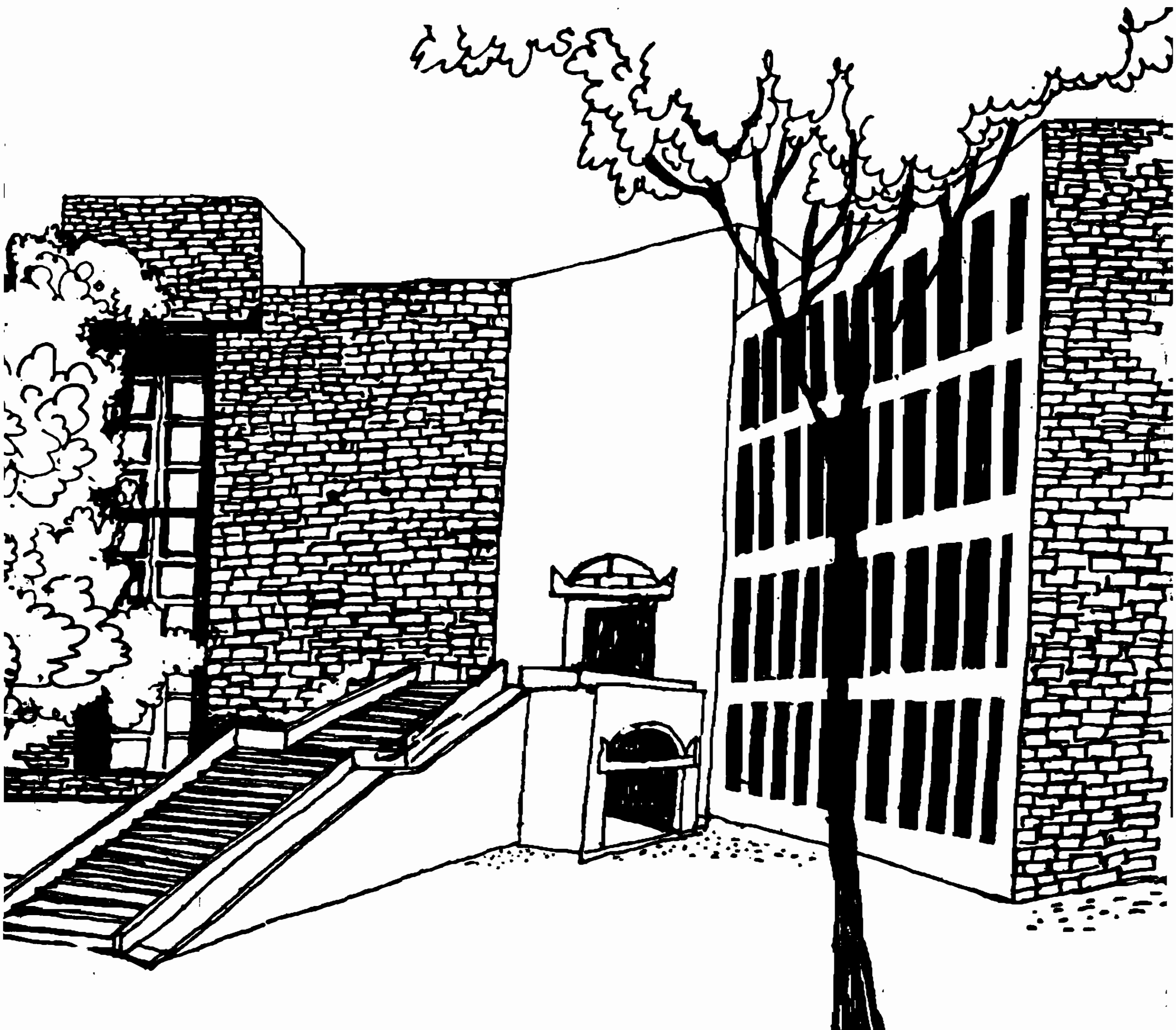

ABSTRACT GAMES ADMITTING STABLE SOLUTIONS

By

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1. **Introduction** :- An abiding problem in choice theory has been one of characterizing those choice functions which are obtained as a result of some kind of optimization. Specifically, the endeavour has concentrated largely on finding a binary relation (if there be any) whose best elements coincide with observed choices. An adequate survey of this line of research till the mid eighties is available in Moulin [1985].

The idea of a function which associates with each set and a binary relation a non-empty subset of the given set has a long history whose exact origin is very difficult to specify and in any case is unknown to the author. In Laslier [1997] can be found a very exhaustive survey of the related theory when binary relations are reflexive, complete and anti-symmetric.

In Lahiri [2000], we extend this set of binary relations to include those which are not necessarily anti-symmetric. Such binary relations are known as abstract games. An ordered pair comprising a non-empty subset of the universal set and an abstract game is referred to as a subgame. A (game)solution is a function which associates to all subgames of a given (nonempty) set of games, a nonempty subset of the set in the subgame. An important consequence of this framework is that often, a set may fail to have an element which is best with respect to the given binary relation. To circumvent this problem the concept of the top cycle set is introduced, which selects from among the feasible alternatives only those which are best with respect to the transitive closure of the given relation. The top cycle set is always non-empty and in Lahiri [2000],we provide an axiomatic characterization of the top-cycle solution. It is also observed there that the top cycle solution is the coarsest solution which satisfies two innocuous assumptions. In the same paper we also provide an axiomatic chracterization of the uncovered solution (where 'covering' is defined as a 'menu-based' concept as in Sen [1997]).The concept of the uncovered solution has been analyzed in great detail in Dutta and Laslier [1999].Our axiomatic characterization is different from the ones available there.

Among the many different solutions which have been prescribed for problems of choice, one of the most significant is the solution related to the (von

Neumann-Morgenstern)stable set. Lucas [1994] surveys the very large literature dealing with this concept, particularly in the context of co-operative games. A recent abstract approach to stable set theory and its connections to other solution concepts is given in Greenberg [1989,1990].

In this paper we characterize the maximal set of abstract games which admits a solution which always selects a stable set from every admissible subgame. This set is obviously the same as the maximal set of abstract games such that each subgame has atleast one stable set. It is proved in this paper that the maximal set of abstract games to satisfy this property are those which do not admit any strict preference cycle of length three and satisfy what we call stable set property. The stable set property is equivalent to the requirement that all subgames with exactly five elements have atleast one stable set. We also prove that these two properties are logically independent.

2. **Game Solutions** :- Let X be a finite, non-empty set and given any non empty subset A of X, let [A] denote the collection of all non-empty subsets of A. Thus in particular, [X] denotes the set of all non-empty subsets of X. If $A \in [X]$, then # (A) denotes the number of elements in A. Given a binary relation R on X, let $P(R) = \{(x, y) \in R \,/\, (y, x) \notin R\}$ and $I(R) = \{(x, y) \in R \,/\, (y, x) \in R\}$. P(R) is called the asymmetric part of R and I(R) is called the symmetric part of R. Given a binary relation R on X and $A \in [X]$, let $R|A = R \cap (A \times A)$.

A binary relation R on X is said to be (a) reflexive if $\forall\, x \in X : (x, x) \in R$; (b) complete if $\forall\, x, y \in X$ with $x \neq y$, either $(x, y) \in R$ or $(y, x) \in R$;(c) quasi-transitive if $\forall\, x, y, z \in X$:[$(x, y) \in P(R)$ & $(y, z) \in P(R)$] implies [$(x, z) \in P(R)$]. Let Π denote the set of all reflexive and complete binary relations. If $R \in \Pi$, then R is called an abstract game. An ordered pair $(A,R) \in [X]\times\Pi$ is called a subgame. Given a binary relation R on X and $A \in [X]$, let $G(A,R) =\{ x \in A / \forall y \in A :(x,y) \in R\}$. Given $A \in [X]$, let Δ (A) denote the diagonal of A i.e. $\Delta(A)=\{(x,x)/x\in A\}$.
Given a binary relation R on X and a positive integer K, R is said to admit a K-cycle if there exists $x_1,\ldots,x_K$ in X, all distinct such that (i) $(x_i, x_{i+1}) \in P(R)\ \forall\, i \in \{1,\ldots K\text{-}1\}$; and (ii) $(x_K, x_1) \in P(R)$. A binary relation R on X is said to be acyclic if given any positive integer K,R does not admit a K-cycle. Of particular interest to us in what follows is the set $\Pi(3)$ which is defined to be the set $\{ R \in \Pi / R$ does not admit a 3-cycle$\}$.
Any non-empty subset Λ of Π is called a domain. In the sequel whenever we mention Λ,it will be implicitly assumed that it is a domain.
A (game) solution on Λ is a function $S: [X]\times\Lambda \to [X]$ such that:
(i) $\forall (A,R)\in [X]\times\Lambda : S(A,R) \subset A$;
(ii) $\forall (A,R), (A,Q)\in [X]\times\Lambda : R|A=Q|A$ implies $S(A,R)=S(A,Q)$;
(iii) $\forall x,y\in X$ and $R\in \Lambda : x\in S(A,R)$ if and only if $(x,y) \in R$.

If $\forall(A,R)\in[X]\times\Lambda$, $G(A,R)$ is non-empty valued then the associated solution is called the best solution on Λ.
Rational choice theory till the mid eighties has concentrated on the following problem : given a domain Λ, specify a finite minimal set of (reasonable axioms) on solutions on Λ such that whenever a solution on Λ satisfies those properties it coincides with the best solution on Λ.This theory has been adequately surveyed in Moulin [1985].The following result is well known in the relevant literature:

Theorem 1:-Let R belong to Π.Then $\forall(A,R)\in[X]\times\Lambda$, $G(A,R)$ is non-empty valued if and only if R is acyclic.

Given $R\in\Pi$,let us say that G is well defined at R if $\forall A\in[X]$, $G(A,R)$ is non-empty valued.
Let (A,R) be a subgame. A set $B\in[A]$ is said to be a (von Neumann-Morgenstern) stable set for (A,R) if:(i)$\forall x,y\in A$: $(x,y)\in I(R)$ (i.e. B satisfies internal stability);(ii) $\forall x\in A\backslash B$, there exists $y\in B$:$(y,x)\in P(R)$.Given a subgame (A,R),let $\Psi(A,R)=\{B\in[A]/B$ is a stable set for $(A,R)\}$. Given $R\in\Pi$,let us say that Ψ is well defined at R if $\forall A\in[X]$, $\Psi(A,R)$ is non-empty.

Observation 1:-Let $A\in[X]$ and suppose $\#(A)\leq 2$. Then,$\forall R\in\Pi$:$[G(A,R)\neq\phi$ $\&\Psi(A,R)\neq\phi]$.
A solution $S:[X]\times\Lambda\to[X]$ is called a stable solution if $\forall(A,R)\in[X]\times\Lambda$:$S(A,R)$ $\Psi(A,R)$. Λ is said to admit a stable solution if there exists a stable solution on Λ.Clearly, Λ admits a stable solution if and only if $\forall(A,R)\in[X]\times\Lambda$: $\Psi(A,R)$ is non-empty. The following example shows that there are domains which do not admit any stable solution:
Example 1 : Let $X=\{x,y,z\}$ and let $R=\Delta\cup\{(x,y),(y,z),(z,x)\}$.Then $\Psi(X,R)=\phi$.Thus Ψ is not well defined at R. Hence any domain which contains R does not admit a stable solution.
In view of Example 1, the following question derives relevance: What is the largest domain which admits a stable solution? We try to answer this question in the following sections of this paper.

3. **Preliminary Results** :-In this section we obtain some preliminary results about stable sets.

Theorem 2:- Let (A,R) be a sub game such that $G(A,R)\neq\phi$. Then, $\Psi(A,R)\neq\phi$.
Proof:- Suppose $G(A,R)\neq\phi$ and towards a contradiction suppose $\Psi(A,R)=\phi$. Thus $G(A,R)\notin\Psi(A,R)$.Hence there exists $y_1\in A\backslash G(A,R)$ such that $\forall x\in G(A,R)$: $(y_1, x)\in R$. Now, $y_1\in A\backslash G(A,R)$ implies that there exists $z_1\in A\backslash G(A,R)$ such that $(z_1, y_1)\in P(R)$.Let $B_1=G(A,R)\cup\{y_1\}$.Since $\Psi(A,R)=\phi$, $B_1\notin\Psi(A,R)$.Suppose

that finite sequences { B_1 ,...., B_s} and { y_1 ,...., y_s} have been obtained such that
(i) $\forall$ i $\in$ {1,...,s}:[B_i=B_{i-1} $\cup${y_i} if i>1, B_1 = G(A,R) $\cup${y_1} if i = 1]
(ii) $\forall$ i $\in$ {1,...,s}; there exists z_i $\in$A\ B_i :(z_i , y_i) $\in$ P(R).
(iii) (y_i , x) $\in$ R $\forall$ x$\in B_{i-1}$ if i>1 and (y_1,x) $\in$ R$\forall$x$\in$G(A,R).
Since Ψ(A,R) = ϕ, B_i $\notin\Psi$(A,R) whenever i $\in$ {1,...,s}. Thus there exists y_{s+1} $\in$A\B_s:(y_{s+1}, x) $\in$R $\forall$ x$\in B_s$. Since A is finite, there exists a positive integer K: A=G(A,R)$\cup${y_1,...,y_K} with {y_1,...,y_K} $\subset$A\G(A,R).
However (y_K,x) $\in$R$\forall$x$\in$G(A,R) $\cup${y_1,...,y_{K-1}} and (y_K,y_K) $\in$R since R is reflexive. Thus $y_K\in$G(A,R) contradicting {y_1,...,y_K} $\subset$A\G(A,R). Hence Ψ(A,R)$\neq$ ϕ.

Q.E.D.

<u>**Example 2**</u> :- Let X = {x,y,z} and R = $\Delta\cup${(x,y), (y,z), (x,z), (z,x)}. G(X,R) = {x} and Ψ(X,R) = {{x,z}}. Thus G(X,R) is a proper subset of the only stable set of (X,R).
Example 2 shows that G(A,R) can be a proper subset of a stable set. In fact the following example shows that it is quite possible for G(A,R) to be empty and Ψ(A,R) to be non-empty.

<u>**Example 3**</u> :- Let X = {x,y,z,w} and let R = $\Delta\cup${(x,w), (y,z), (w,y), (z,x), (x,y), (y,x), (z,w), (w,z)}.Now G(X,R) = ϕ, but Ψ(X,R) = {{x,y}, {w,z}}.Further, G({x,y,w},R) = {x} but Ψ({x,y,w},R) = {x,y}; G({x,y,z},R) = {y} but Ψ({x,y,z},R) = {x,y};G({x,y,z},R) = {y} but Ψ({x,y,z},R) = {x,y};G({x,w,z},R) = {z} but Ψ({x,w,z},R) = {w,z} ;G({y,w,z},R) = {w} but Ψ({y,w,z},R) = {w,z}. Thus G is not well defined at R but Ψ is.

<u>**Theorem 3**</u> :- Let (A,R) $\in$ [X] x Π and suppose B $\in$ Ψ(A,R). Then G(A,R) $\subset$B.

<u>**Proof**</u> :- Let B$\in\Psi$(A,R) and towards a contradiction suppose x $\in$G(A,R)\B. Thus x$\in$A\B. Since B$\in\Psi$(A,R), there exists y$\in$B$\subset$A:(y,x)$\in$P(R). This contradicts x$\in$G(A,R) and proves the theorem.
Q.E.D.

The following result is of immense significance for quasi-transitive rational choice. Lahiri [1999] (and references there in) contain related results.

<u>**Theorem 4**</u> :- Let R$\in\Pi$. Then [Ψ(A,R)={G(A,R)}$\forall$A$\in$[X]] if and only if R is quasi-transitive.

<u>**Proof**</u> :- Suppose R is quasi-transitive. Clearly $\forall$ A$\in$[X]:G(A,R)$\in\Psi$(A,R). Let B$\in\Psi$(A,R). Towards a contradiction suppose x$\in$B\G(A,R). Then there exists y $\in$G(A,R) such that (y,x)$\in$P(R). Since B$\in\Psi$(A,R), y$\in$A\B. Since y$\in$G(A,R) there

does not exist z in A (and hence in B) such that $(z,y)\in P(R)$. This contradicts $B\in\Psi(A,R)$. Thus $B\subset G(A,R)$.

Now suppose $x\in G(A,R)\setminus B$. Thus there does not exist $z\in B:(z,x)\in P(R)$. Thus $B\notin\Psi(A,R)$. Thus $G(A,R)\subset B$. Hence $B=G(A,R)$.

Now suppose $\Psi(A, R) = \{G(A, R)\}\ \forall A \in [X]$.Towards a contraction suppose R is not quasi-transitive. Thus there exists $x, y, z \in X : (x, y) \in P(R), (y,z) \in P(R)$ and $(x,z) \notin P(R)$. Since, $\Psi(\{x,y,z\}, R) = \{G(\{x,y,z\}, R)\}$, $G(\{x,y,z\}, R) \neq \phi$. Hence $(z,x) \notin P(R)$. Thus $(x,z) \in I(R)$. Thus $\Psi(\{x,y,z\}, R) = \{\{x,z\}\} \neq \{G(\{x,y,z\}, R)\} = \{x\}$. Thus R must be quasi-transitive.

Q.E.D.

4. **<u>Three Cycles and Stable Set Property</u>** :- In this section we define two logically independent properties that all elements of a domain need to satisfy for it to admit a stable solution. These two properties are both necessary and sufficient for our desired result.

<u>Lemma 1</u> :- Let $R \in \Pi \setminus \Pi(3)$. Then Ψ is not well defined at R.

Proof :- Let $R \in \Pi \setminus \Pi(3)$. Then there exists $x,y,z \in X$ such that $(x,y), (y,z), (z,y) \in P(R)$. Clearly $\Psi(\{x,y,z\}, R) = \phi$. Thus Ψ is not well defined at R.

Q.E.D.

In what follows, for $(A, R) \in [X] \times \Pi$, let $W(A,R) = \{x\in A / \forall y\in A : (y,x)\in R\}$.

<u>Lemma 2</u> :- Let $R \in \Pi \setminus \Pi(3)$ and let $A \in [X]$ with $\#(A) \leq 4$. Then $\Psi(A,R) \neq \phi$.

<u>Proof</u> :- For # (A) equal to 1 or 2 there is nothing to prove and for # (A) equal to 3, $R \in \Pi \setminus \Pi(3)$ implies $G(A,R) \neq\phi$. Thus, by Theorem 2, $\Psi(A,R) \neq\phi$. Hence suppose $\#(A) = 4$. If $G(A,R) \neq \phi$, then by Theorem 2, $\Psi(A,R) \neq \phi$. If $G(A,R) = \phi$, $W(A,R) \neq \phi$, then let $y \in W(A,R)$. Thus $\#(A\setminus\{y\}) = 3$ and by the above $\Psi(A\setminus\{y\},R) \neq \phi$. Let $B\in\Psi(A\setminus\{y\},R)$. If there exists $x \in B : (x,y) \in P(R)$, then $B \in \Psi(A,R)$. If $\forall x\in B : (x,y) \in I(R)$, then $B \cup \{y\} \in \Psi(A,R)$. Thus $\Psi(A,R) \neq \phi$. Finally suppose $G(A,R) = W(A,R) = \phi$. Let $A = \{x,y,z,w\}$ where all elements are distinct. Since $G(A,R) = \phi$, there exists $a \in A : (a, x) \in P(R)$. Without loss of generality suppose $(y, x) \in P(R)$. Since $W(A,R) = \phi$, there exists $a \in A : (x, a) \in P(R)$. Without loss of generality suppose $(x,z) \in P(R)$. By the same reasoning as above and since $R \in \Pi \setminus \Pi(3)$ (: and hence $(z,y) \notin P(R)$) we must have $(z,w) \in P(R)$. Since $G(A,R) = \phi$, and since $(y,x) \in P(R)$ and $(z,y) \notin P(R)$ we must have $(w,y) \in P(R)$. If $(w,x) \in P(R)$ then along with $(x,z)\in P(R)$ and $(z,w)\in P(R)$ we get a 3-cycle. Hence $(w,x)\notin P(R)$. If $(x,w)\in P(R)$ then along with $(w,y) \in P(R)$ and $(y,x) \in P(R)$ we get a three cycle. Thus $(w.x) \in I(R)$. By identical reasoning we must have $(y,z) \in I(R)$. Thus $\Psi(A,R) =\{\{w,x\},\{y,z\}\}$. Thus $\Psi(A,R) \neq \phi$. This proves the Lemma.

Q.E.D.

However the conclusion of Lemma 2 does not hold if # (A) > 4.

<u>Example 4</u> :- Let X = {x,y,z,w,u} with all the five elements distinct. Let R = Δ $\cup$ {(x,y), (y,x), (x,z), (u,x), (y,u), (w,y), (w,x), (x,w), (u,z), (z,u), (w,u), (u,w), (z,w)}. Clearly R $\in \Pi$ - $\Pi(3)$. Towards a contradiction suppose $\Psi(X,R)$ = {B}. Suppose x $\in$ B. Thus z $\notin$ B and u $\notin$ B. Thus y $\in$ B. Thus w $\notin$ B. But B contains no element 'a' such that (a,w) $\in$ P(R). Thus x $\notin$ B. Hence u $\in$ B. Thus y $\notin$ B. Thus w $\in$ B. Thus z $\notin$ B. But then B contains no element 'a' such that (a,z) $\in$ P(R). Thus $\Psi(X,R) = \phi$.

Let R $\in \Pi$. R is said to satisfy the <u>stable set property</u> (SSP) if there does not exist u,w,x,y,z in X such that :

(i) (x,y), (z,y) $\in$ I(R) ;
(ii) {(x,z), (u,x), (y,u),(z,w), (w,y)} $\subset$ P(R).

<u>Lemma 3</u> :- Let R $\in \Pi \setminus \Pi(3)$ and suppose R does not satisfy SSP. Then Ψ is not well defined at R.

<u>Proof</u> :- Example 4.

Q.E.D.
<u>Lemma 4</u> :-Let R be an abstract game and suppose that $\forall$ A $\in$ [X], $\Psi(A,R) \neq \phi$, if # (A) = 1,...,n for some positive integer n. Let A $\in$ [X] with #(A) = n+1 and suppose that W(A,R) $\neq \phi$.Then, $\Psi(A,R) \neq \phi$.

<u>Proof</u> :- Let x $\in$ W(A,R). By the induction hypothesis, $\Psi(A\setminus\{x\}, R) \neq \phi$. Let B $\in$ $\Psi(A\setminus\{x\}, R)$. If there exists y $\in$ B such that (y,x) $\in$ P(R), then B $\in$ $\Psi(A,R)$. Otherwise $\forall$ y $\in$ B, (x,y) $\in$ I(R) (: since x $\in$ W(A,R)) and so B $\cup$ {x} $\in$ $\Psi(A,R)$. Thus W(A,R) $\neq \phi$ implies $\Psi(A,R) \neq \phi$.

Q.E.D.
Let Π^o = {R $\in \Pi \setminus \Pi(3)$ / R satisfies SSP}.
The following Lemma is rather enlightening :

<u>Lemma 5</u> :- Let R $\in \Pi^o$. Then Ψ is well defined at R.

<u>Proof</u> :- We have already seen in Lemma 2 that if R $\in \Pi^o \subset \Pi \setminus \Pi(3)$ then $\forall$ A $\in$ [X] with # (A) $\leq$4, $\Psi(A,R) \neq \phi$. Suppose R $\in \Pi^o$ and $\forall$ A $\in$ [X], $\Psi(A,R) \neq \phi$, if # (A) = 1,...,n for some positive integer n $\geq$ 4. Let A $\in$ [X] with #(A) = n+1. Towards a contradiction suppose, $\Psi(A,R) = \phi$.

Since $\Psi(A,R) \neq \phi$, by Theorem 2 $G(A,R) = \phi$,and by Lemma 4, $W(A,R) = \phi$. Let $x \in A$. By the induction hypothesis, $\Psi(A \setminus \{x\}, R) \neq \phi$. Let $B \in \Psi(A \setminus \{x\}, R)$.
<u>Step 1</u>:If $\forall a \in B$; $(a,x) \in I(R)$, then $B \cup \{x\} \in \Psi(A,R)$. If there exists $a \in B$: $(a,x) \in P(R)$, then $B \in \Psi(A,R)$. Since $\Psi(A,R) = \phi$, it must be the case that $\forall a \in B$, $(a,x) \in R$ and for some $a \in B$, $(x,a) \in P(R)$.
<u>Step 2</u>:Let $B_0 = \{a \in B/(a,x) \in I(R)\}$ and $B_1 = \{a \in B/ (x,a) \in P(R)\}$. Since $\Psi(A,R) = \phi$, $B_0 \cup \{x\} \notin \Psi(A,R)$. Hence there exists $b \in A \setminus B$: $\forall a \in B_0 \cup \{x\}$: $(b,a) \in R$. Let $A \setminus B = C_1 \cup C_2 \cup C_3$ where $C_1 = \{b \in A \setminus B/(x,b) \in P(R)\}$, $C_2=\{b \in A \setminus B/(b,x) \in P(R)\}$ and $C_3 = \{b \in A \setminus B/b \neq x \ \& \ (b,x) \in I(R)\}$. Since $G(A,R) = W(A,R) = \phi$, $C_1 \neq \phi$ and $C_2 \neq \phi$. Let $b \in C_2$. Since $B \in \Psi(A \setminus \{x\}, R)$, there exists $a \in B$:$(a,b) \in C_2$. If $a \in B_1$, then a,b and x form a 3-cycle contradicting $R \in \Pi \setminus \Pi(3)$. Thus $a \in B_0$. If $C_3=\phi$, then $B_0 \cup \{x\}$ is a stable set for (A,R) contradicting $\Psi(A,R)=\phi$. Thus $C_3 \neq \phi$.
<u>Step 3</u>:If $\forall\, b \in C_3$, there exists $a \in B_0 \cup \{x\}$:$(a,b) \in P(R)$ then again $B_0 \cup \{x\} \in \Psi(A,R)$ contradicting $\Psi(A,R)=\phi$. If $\forall$ $b \in C_3$ and $\forall a \in B_0 \cup \{x\}$:$(a,b) \in I(R)$ then $B_0 \cup C_3 \cup \{x\} \in \Psi(A,R)$ contradicting $\Psi(A,R)=\phi$. Hence there exists $b \in C_3$:$\forall a \in B_0 \cup \{x\}$, $(b,a) \in R$ and $(b,a) \in P(R)$ for some $a \in B_0 \cup \{x\}$. Let $\bar{B}$ be a maximal subset of A containing $B_0 \cup \{x\}$:$\forall a, b \in \bar{B}$, $(a,b) \in I(R)$. Since A is finite, $\bar{B}$ exists. Thus if $b \in C_3 \setminus \bar{B}$ then $(b,a) \in R \forall a \in \bar{B}$ and $(b,a) \in P(R)$ for some $a \in \bar{B}$. Further $B_1 \cup C_1 \cup C_2 \subset A \setminus \bar{B}$. Hence $\bar{B}$ is a proper subset of A.
<u>Step 4</u>:Let $\bar{B} \setminus \{x\} = B_2 \cup B_3$ where $B_2=\{a \in \bar{B} / \forall b \in C_3 \setminus \bar{B} : (a,b) \in I(R)\}$ and $B_3 = (\bar{B} \setminus \{x\}) \setminus B_2$. Suppose $[u \in C_2,\ a \in \bar{B}$ and $(a,u) \in P(R)]$ implies $[a \in B_2]$. Then $B_2 \cup (C_3 \setminus \bar{B}) \in \Psi(A,R)$ contradicting $\Psi(A,R)=\phi$. Hence there exists $u \in C_2$, $y \in \bar{B}$ and $w \in C_3 \setminus \bar{B}$ such that $(y,u) \in P(R)$ and $(w,y) \in P(R)$. Since, $(w,v) \in R \forall v \in \bar{B}$ and since,$w \in A \setminus B$, there exists $z \in B$, such that $(z,w) \in P(R)$. This contradicts $R \in \Pi^0$. Hence $\Psi(A,R) \neq \phi$. It follows by induction that Ψ is well defined at R.

Q.E.D.

In view of Lemmas 1,2,3 and 5 the following theorem stands established:

<u>**Theorem 5**</u> :- Let Λ be a domain. Then Λ admits a stable solution if and only if $\Lambda \subset \Pi^0$.

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