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A LINEAR PROGRAMMING MODEL FOR OPTIMAL WATER TRANSMISSION SYSTEM : A CASE STUDY FOR AHMEDABAD

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This paper presents a methodology for the optimal design of a water transmission system, given a source of supply and demand values for water from various zonal divisions in the city. The Linear Programming Model developed considers the detailed design of the water transmission system including the choice of pipe lengths and diameters, and the computation of pressure losses due to friction, and pumping head required to meet minimum allowable discharge pressure at each demand centre.

1. Introduction

Most cities in India are facing an ever increasing demand for water due to population and industrial growth and some of them have to go to considerable distances away from the city and spend large amounts for water development

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and transmission systems to obtain water supply for the urban population and industrial requirements. Water requirements in urban areas increase with the growth of the population, industries and commercial activities and also to the location of new industries and other economic activities. The population in the cities and towns in India are growing at a faster rate than the national rate of population growth and in spite of repeated assurances regarding decentralisation of industrial locations, still most of the new industries are being located in the urban areas which are already congested. To satisfy the additional water demand due to such population and industrial growth in the urban areas, there is always a need to find new sources of water near the urban areas, building of treatment facilities and laying up extensive and long water transmission and water distribution pipelines to cover such areas. Major water development projects involve high financial investments, long gestation periods of 5 - 6 years and therefore advanced planning as well as better economic analysis of alternative investment possibilities is very essential for any urban area facing the growing demand for water supply and waste disposal facilities.

A large quantity of water will be available to Ahmedabad city following the completion of Dharoi Dam project on the Sabarmati river. A new water transmission network will have to be constructed to connect the three existing head works at Acher, Bhadreswar and Dudheshar to receive water from the Dharoi reservoir and to distribute through the existing distribution system in the receiving areas. The costliest parts of the water supply system are the transmission facilities which include the pipeline network as well as the pumping and storage facilities. Ahmedabad does not have alternative sources of water except from the boreholes at the Sabarmati river bed and the various tubewells that has been sunk at various locations in the city. A Network programming model to evaluate alternatives for supplying water from various sources and to satisfy future demand for multiple water uses in an urban area at minimum annual cost has been presented elsewhere. (6,7). As in the case of Ahmedabad, such alternative sources do not exist, it was decided to concentrate on the optimisation of the water transmission system to receive and distribute the water available when the construction of the Dharoi project is completed.

This paper presents a methodology for the design of a water transmission system, given a source of supply and demand values for water from various zonal divisions in the city. The linear programming model developed considers the detailed design of the water transmission system including the choice of pipelength and diameter and computation of pressure losses due to friction. The pumping head required to meet minimum allowable discharge pressure at each demand centre need also be computed using this approach. The linear programming model is illustrated with a case study of the water transmission system that will be required for the distribution of water available from the Dharoi reservoir to various locations in the Western zone of Ahmedabad, on the West Bank of Sabarmati river. It was decided to study the pipeline network system because it contributes most to the total cost of the whole water supply system. This can be seen from the following allocation of Rs.16 crores for the ancillary works for Dharoi water project (1) as shown below:-

1. Cost of land	Rs 0.25 crores
2. Head works	Rs 5.00 crores
3. Transmission lines	Rs 6.00 crores
4. Distribution lines	Rs 4.5 crores

In this model we are considering the problem of determining the optimal diameters and lengths of pipes in the transmission system, which will maintain the required net head at demand points considering the frictional losses in the pipeline system, for a given pumping head and will also minimise the total cost of the pipeline system.

2. Description of the Water Transmission System

It was proposed that three headworks will be installed at Acher, Bhadreswar and Dudheshwar when water from the Dharoi reservoir will be available for supplying to the city of Ahmedabad. At each intake or headworks, a high capacity pump will be installed to deliver water to elevated reservoirs in each district through transmission lines. From the elevated reservoir water will be supplied to customers through distribution lines in each district. Each of the zones will have one headworks and separate transmission and distribution networks. A study of one such zone in Ahmedabad will be sufficient to illustrate the Linear Programming Methodology proposed in this paper. The Western Zone of the city on the right bank of Sabarmati river was selected for illustrating the model.

The water transmission system for this zone consists of the following elements: (a) A single source of supply from headworks at Acher with a booster pump, (b) multiple demand centres each with an elevated reservoir from which water will be supplied to adjoining areas in each district of western zone and (c) a water transmission system consisting of pipe line elements of different diameters and lengths connecting the source to the demand centres. A map of Ahmedabad showing the different water supply zones, the location of the three headworks and the Western Zone transmission system are shown in figure 1. A schematic diagram of the western zone water transmission system is shown in figure 2. The flow required at each of the 13 demand centres are shown in table-1.

At each of the demand points a minimum required pressure must be maintained at all times in addition to supplying the required quantity of water. The total head required for covering losses in the water distribution system depends on the topographic elevation at the service area as compared with the head works and the minimum pressure required at ultimate consumer supply points. Thus, for determining the pressure requirements at each demand centre from which the adjoining areas are supplied through the distribution system, sound engineering judgement must be exercised before the problem of computing optimal computer diameters and pipe layouts comprising transmission system could be taken up.

It is possible to formulate the minimum cost water transmission design problem as a linear programming problem. This is obtained from a mathematical model of the whole pipeline system which is used to choose the right combination of various pipe sizes and diameters to minimise the cost of lines and at the same time satisfying the demand for water delivery at required pressure. Hazen-Williams' formula is used to relate pressure losses in a pipeline as a function of its diameter, length and the friction factor. The pressure losses in any open loop of the transmission system is then expressed as a linear functional of the lengths of different diameter pipes used in loop. As the maximum pressure loss in an open loop is specified, this linear functionals become the constraints of the linear programming model along with the other constraints on the total length of each pipe which could be composed of different diameters. The objective function to be minimised is the total cost of pipelengths in the network which also includes the cost of laying down the pipeline transmission system. The Linear Programming approach would be highly efficient because of the high computational efficiency of modern linear programming codes. Another characteristic of the optimal solution obtained by the use of this model is that in the final solution water is available at each given demand centre only at the required minimum pressure and as such no energy is wasted.

The data required for illustrating this model was collected for transmission of water from the Dharoi project to various localities in the Western Zone of Ahmedabad. In the schematic diagram for the Western zone transmission system (Fig.2), generated head at source and net head required at different demand centres are specified. These data were obtained from the water supply department of Ahmedabad Municipal Corporation. Distances between the various demand points and the head works are taken from the relevant maps following the given lay out of the water transmission system. The flows required at each demand centre are shown in Table 1. The solution of the model required only minimal computer efforts and provided an acceptable design for the pipeline system. The total cost of the optimal pipeline system was much lower than the original system described in the project report (1).

3. Formulation of the Linear Programming Model for Water Transmission System

Various mathematical approaches for modelling water transmission system are reported in the literature (3,4,5,8). Due to linearity of the underlying system, a linear programming model of the water transmission system as presented by Gupta (3) was applied for the water transmission system in Ahmedabad. Gupta used a hypothetical example for illustrating his model while in the particular study the model was applied to the realistic system of water transmission at Ahmedabad.

A linear programming model for analysis of a water supply system presented by Gupta (3) was used for the study.

The model used the Hazen-Williams' formula to determine frictional loss.

The formula is given by

$$V = 131.8\ R^{0.63}\ S^{0.54} \qquad \text{---(1)}$$

also $$Q = V.A \qquad \text{---(2)}$$

$$A = (\pi D^2)/4 \qquad \text{---(3)}$$

$$R = D/4 \qquad \text{---(4)}$$

Where,

Q = Discharge, cubic feet per second

D = Internal pipe diameter, feet

A = Area of cross section of pipe, square feet

R = Mean hydraulic radius

S = Frictional loss of head in feet per foot length of pipe

V = Average velocity in feet per second

Using Equations 2,3 and 4, the Hazen-Williams' formula can be rewritten as

$$S = 0.887 \times 10^{-3} \frac{Q^{1.852}}{D^{4.870}} \qquad \text{----(5)}$$

For a given discharge and a constant diameter pipe, S remains constant. Thus head loss is linearly proportional to length

$$\text{or } H = S \times L \qquad \text{----(6)}$$

Where H = head loss between two ends of the pipe, feet

L = length of constant diameter pipe, feet.

An open loop is defined as imaginery pipeline from water source to an individual demand centre. A Line is defined as a part of the system connecting either two adjacent nodes, a node and an adjacent centre or the supply point and either an adjacent node or demand centre.

For a given pump at the source and pressure head required at each demand centre, the head loss in any open loop can be computed as -

$$H = H_p - (H_d \pm h) \qquad \text{---(7)}$$

where, H = maximum allowable friction loss in an open loop

H_p = pump head at source

H_d = minimum head required at demand point

h = is the difference in elevation between the demand centre and the source.

In each open loop, frictional head loss should not exceed H, which can be written mathematically as

$$\sum_{j=1}^{n} \sum_{i \in I_k} S_{ij} L_{ij} \leq H_k \qquad \text{---(8)}$$

where, S_{ij} = frictional head loss per foot length of pipe in line i, diameter j.

L_{ij} = Length of pipe section of diameter j in line i

n = number of various pipe diameter available

m = number of lines

A second restriction on the length of each line in any open loop must be given

$$\sum_{j=1}^{n} L_{ij} = L_i \text{ for each line } i = 1, 2, \ldots\ldots, m \qquad (9)$$

where, L_1, L_2 ———— L_n = length of lines in line numbers 1,2, ..., m respectively

Objective function is to minimize the total cost of pipes in all m lines and is given by

$$\text{Minimize } Z = \sum_{j=1}^{n} \sum_{i=1}^{m} C_j \, Lij \qquad \text{---(10)}$$

An open loop consist of one or more set of lines and a line consists of one or more pipes of different diameters. In the objective function, C_j represents per foot cost for pipe of diameter j.

4. Application of the Linear Programming Model

As seen from Fig.2, in the Western zone water transmission system there are 24 lines and 13 open loops. Flow rates throughout the system were computed from the data given in table 1 for demand at individual centres. Values of 'S' were computed by an auxillary programme. The cost of pipe lengths of different diameters from 6" to 36" as per laid foot length were obtained from a schedule of rates and this as shown in table.2. Using the above data in the equations of the Linear Programming Model and applying a computer code utilising the simplex method, a solution was obtained which provides values of optimal diameter and lengths and this is shown in table 3. The optimal solution is also shown in the schematic diagram in Fig.3.

The total cost of the optimal water transmission amounts to Rs.76,23,030 as compared to the total cost of Rs.2,66,68,000 mentioned in the project report based on the existing engineering design with cast iron pipes. It should be noted that in the optimal solution the pressure at all demand points remains the same and no pressure head was wasted at the supply side. This is not so as seen from an analysis of the design carried out by the Engineering department.

It is also possible to determine the optimal size of the pump at the supply end to minimise the total cost of the pipe-line and the present worth of the cost of pumping for overcoming friction losses in the pipeline system for its entire life. This is obtained by a minor modification of the model.

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TABLE - 1

Water Districts in Western Zone and Respective Demands[+]

Dist. No.	Name of District	Flow Required in cub.ft. /sec.
I	Sabarmati	4.81
II	New Vadaj	6.73
III	Vadaj	1.93
IV	Naranpura	6.19
V	Memnagar	6.24
VI	University	3.19
VII	Gulbai Tekra	2.78
VIII	Ambawadi	5.72
IX	Usmanpura	7.43
X	Stadium	8.97
XI	V.S. Hospital	9.75
XII	Paldi	5.26
XIII	Vasana	8.20

[+] These demands has been estimated on the basis of a projected population of 25 lacks for Ahmedabad to be reached by 1991 or earlier.

TABLE - 2

Labour and material cost of different diameter C.I. pipes per foot length+

S.No.	Diameter	Labour + material cost in Rs.
1	6"	12.55
2	9"	17.36
3	12"	24.73
4	15"	33.51
5	18"	43.83
6	21"	54.06
7	24"	65.54
8	27"	82.40
9	30"	95.46
10	33"	140.27
11	36"	157.97

+ Source : Schedule of Rate - year 1969-70
Executive Engineer's Public Health Department, Government of Gujarat.

TABLE - 3

Optimal IP Solution Showing Pipe Diameters for Each Line

Line No.	Total Length:ft.	Composition of the Line Length in ft.(Diameter in inches)
1	2079	962(27) + 1117 (30)
2	11913	11913 (27)
3	3399	3399 (24)
4	2640	2640 (36)
5	4983	3332(30) + 1651(36)
6	4686	4686 (30)
7	7326	7326 (24)
8	990	321 (6) + 669 (9)
9	924	415 (9) + 509 (12)
10	264	264 (6)
11	1188	512 (12) + 676 (15)
12	594	305 (12) + 289 (15)
13	500	96 (9) + 404 (12)
14	2970	28 (12) + 2942 (15)
15	528	528 (24)
16	18282	14002 (30) + 4280 (36)
17	12231	12231 (30)
18	6435	6435 (24)
19	3465	662 (21) + 2803 (24)
20	792	400 (9) + 392 (12)
21	3663	276 (12) + 3387 (15)
22	1782	1342 (15) + 440 (18)
23	1056	1056 (15)
24	264	264 (21)

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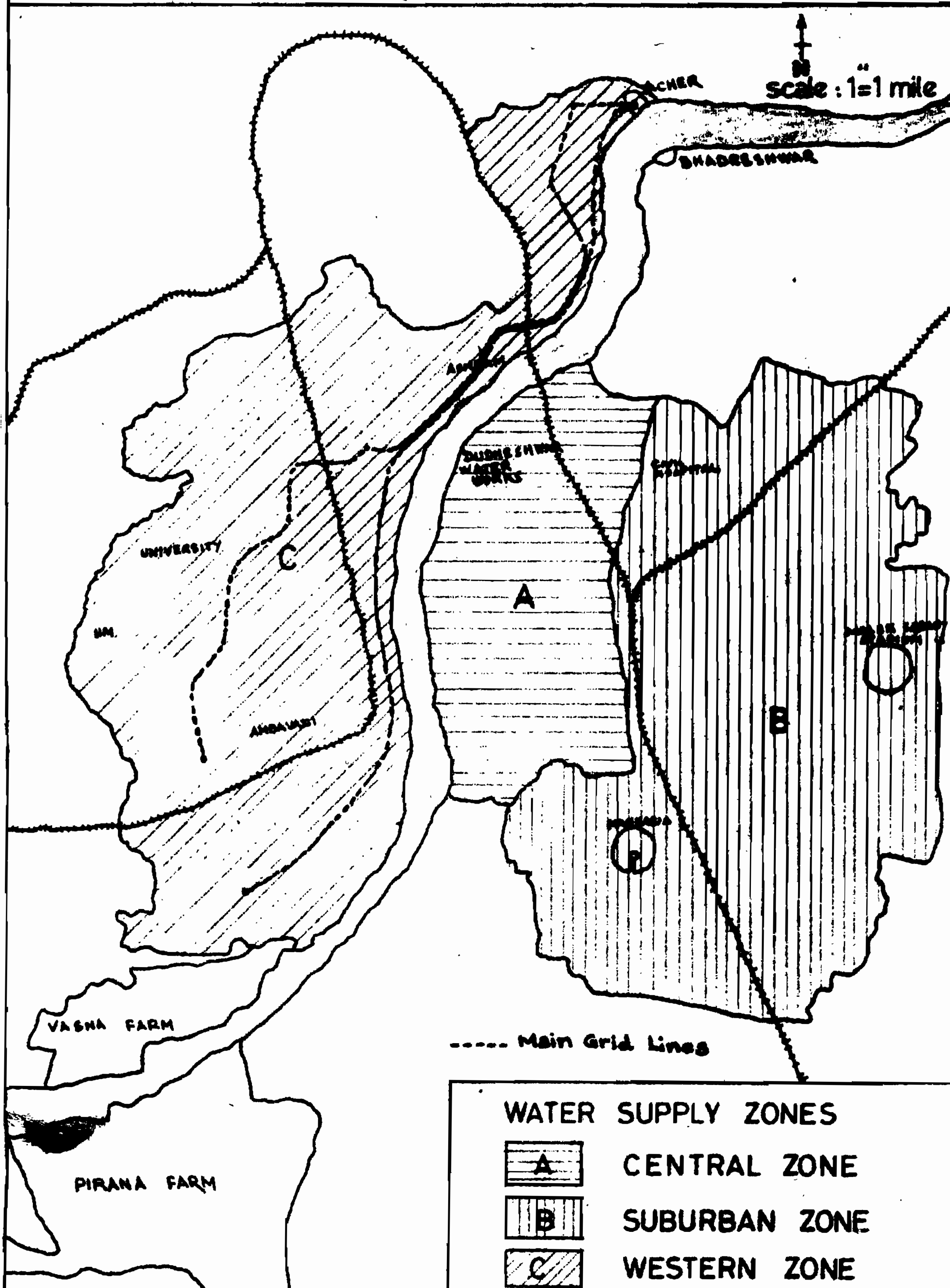
MAP OF AHMEDABAD CITY



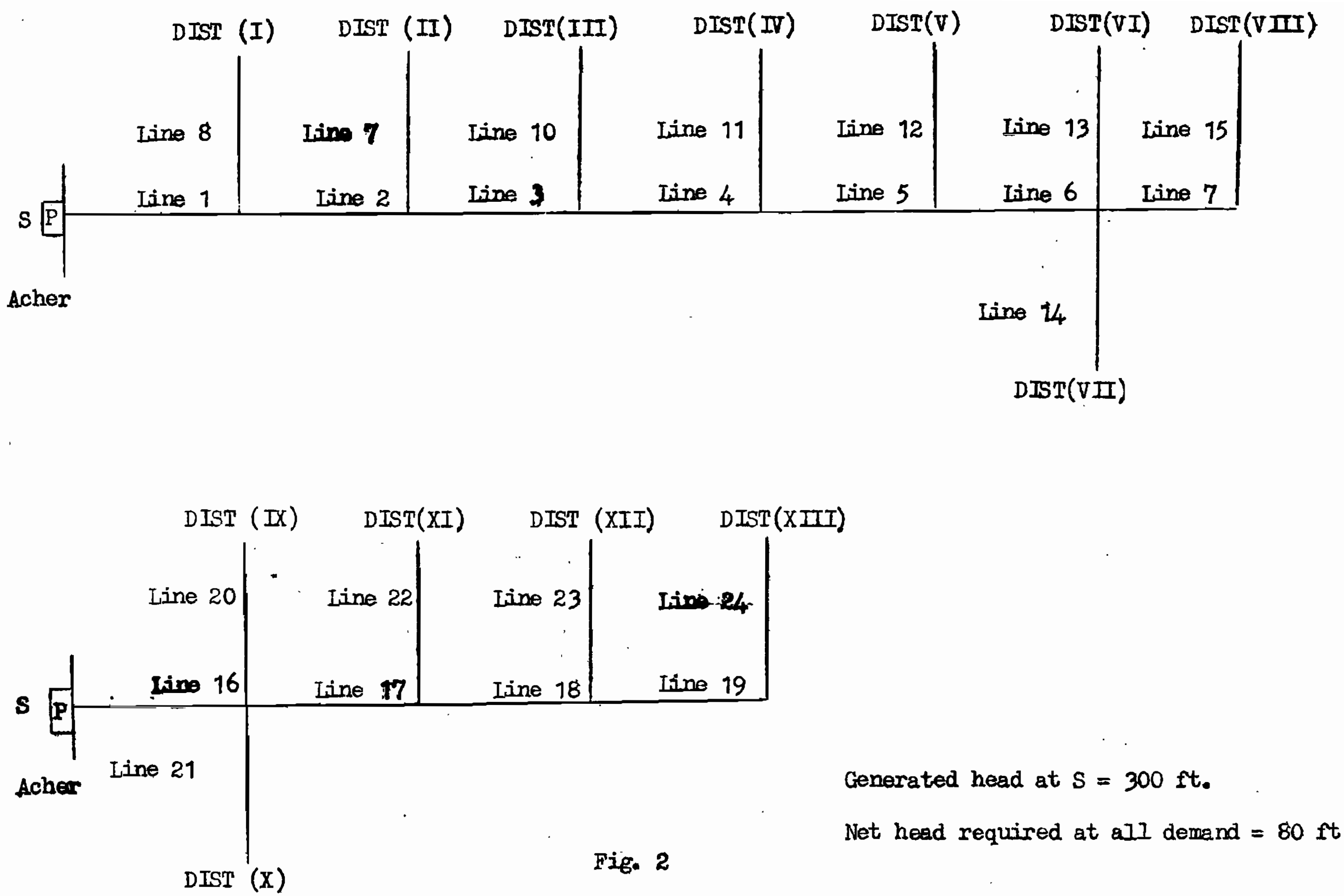


Fig. 2

Schematic diagram of Western Zone
Water Transmission System

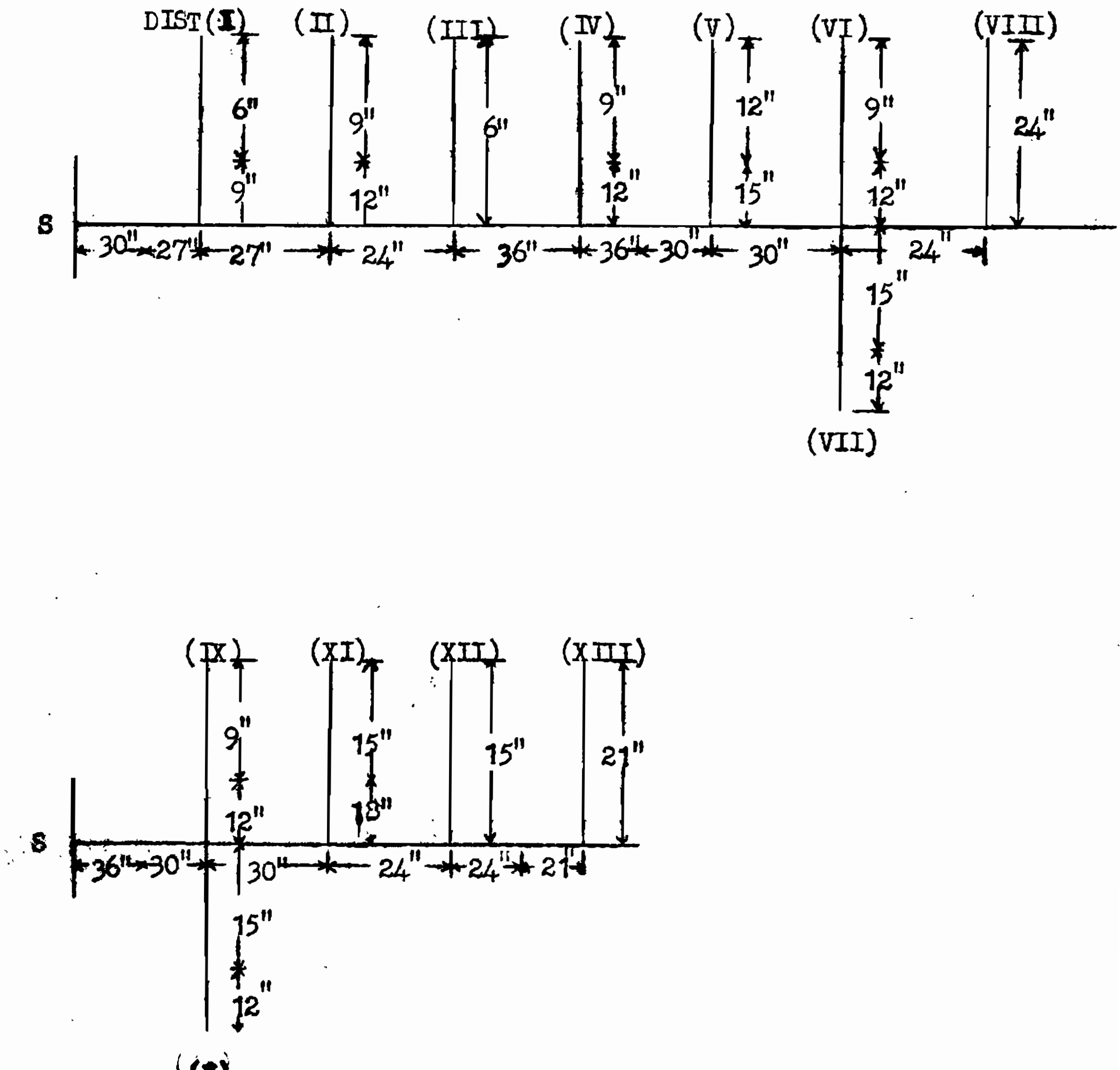


Fig. 3

Schematic Diagram of Optimal
Water Transmission System.