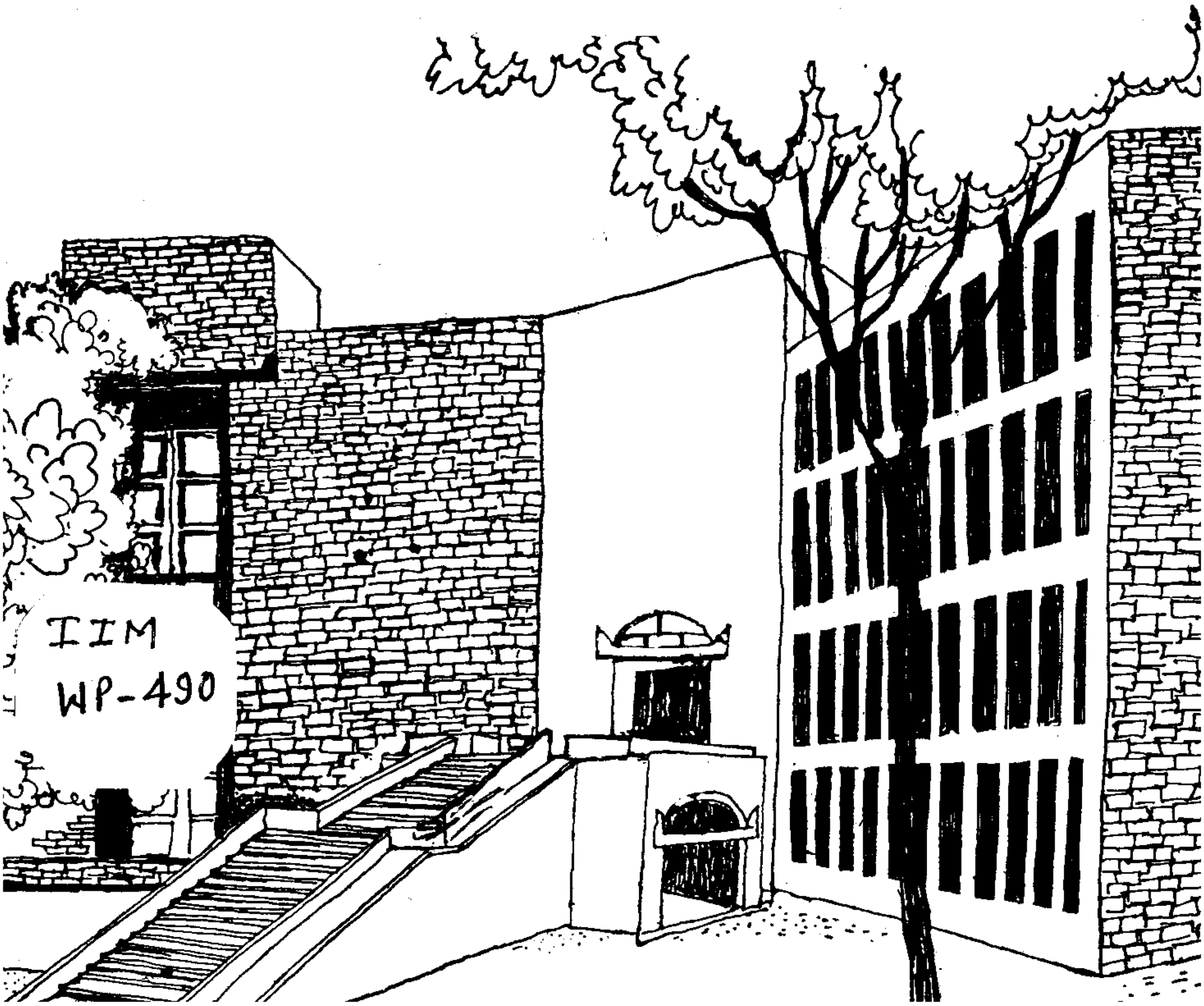
विद्याविनियोगाद्विकासः
IIM
AHMEDABAD

Working Paper



TWO PROBLEMS IN COGNITIVE ALGEBRA: IMPUTATIONS AND AVERAGING-VERSUS-MULTIPLYING

By

Ramadhar Singh

W P No. 490
January 1984

The main objective of the working paper series of the IIMA is to help faculty members to test out their research findings at the pre-publication stage

INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD-380015
INDIA

Two Problems in Cognitive Algebra :
Imputations and Averaging-versus-Multiplying

RAMADHAR SINGH
Organizational Behavior Area
Indian Institute of Management
Ahmedabad, India

Note. This chapter has been prepared for N.H. Anderson (Ed.), _Contributions to information integration theory_. Suggestions for revision will be greatly appreciated.

January 1984

TWO PROBLEMS IN COGNITIVE ALGEBRA

IMPUTATIONS AND AVERAGING VERSUS MULTIPLYING

Ramadhar Singh

Two problems in cognitive algebra are studied in this chapter. One is a problem of model diagnosis, distinguishing between two integration rules, averaging and multiplying, in causal attribution. This was the initial focus of the experiments reported here. Study of this diagnosis problem, however, led to a further problem concerning imputations, that is, inferences about missing information.

Imputations are important, but they have received only scattered attention. Most studies of information integration present specific pieces of information that control the judgment. The wide success of cognitive algebra testifies to the usefulness of this tactic.

In some situations, however, needed information may be missing. Subjects may then impute some value to the missing information to arrive at their judgments. Such imputations are important in cognitive analysis if, as may be expected, they play a substantial role in processing information.

Indeed, as this chapter will show, imputations are central in cognitive algebra. Conversely, cognitive algebra provides a unique and powerful approach to the study of imputations.

THEORETICAL BACKGROUND

IMPUTATIONS

The problem of imputations may be illustrated with the cognitive algebra of the equation

$$\text{Gift size} = \text{Generosity} \overset{*}{\times} \text{Income}, \qquad (1)$$

which is the focus of the later experiments. Given information about a person's generosity and income, subjects readily judge how much the person might give to a charitable cause. The hypothesized multiplying rule follows the rationale that generosity acts as a motivational factor, or action tendency, that operates on a capability potential. Application of the linear fan theorem showed that these judgments appear to obey the hypothesized multiplying rule (Graesser & Anderson, 1974).

But suppose that subjects receive information about only one stimulus variable, either generosity by itself or income by itself. This is logically insufficient for making a judgment. If subjects do make judgments about gift size, that suggests they have imputed some value to the missing information, for example, that it had an average value.

This rational argument does not, of course, control subjects' behavior. Experimental evidence is necessary to demonstrate imputations. And beyond demonstration lie problems of detailed analysis of how imputations depend on what information is given and on the subject's background knowledge.

The theoretical logic is straightforward, and the experimental studies reached a definitive conclusion. The path to the conclusion was somewhat complex, however, so it may be useful to summarize it here.

Two different integration rules can account for the results of the initial experiments: an averaging rule and a multiplying rule. The two rules lead to different conclusions about the imputations. The purpose of the experiments, therefore, was to distinguish between these two integration rules.

The results showed that the stimulus integration obeys the multiplying rule, not the averaging rule. If only one piece of stimulus information is given, subjects impute a value for the missing information and multiply it with the given information. Missing generosity information is imputed a single fixed value, regardless of the given income information. In contrast, the imputed value of missing income information varies directly with the given generosity information.

AVERAGING VERSUS MULTIPLYING

Evidence for Multiplying. The present research began with the two experiments summarized in Figure 1. Subjects received information about annual income and generosity of hypothetical persons and estimated the size of gift they might have sent to a family whose house had burned down, exactly as in the experiments of Graesser and Anderson (1974). Figure 1 plots mean judgment of gift size as a function of generosity, listed on the horizontal axis, and annual income, listed as curve parameter.

The four solid curves in each experiment show the judgments when both stimulus variables, income and generosity, were specified. These four curves form a linear fan pattern, evidence for the operation of a multiplying rule. These data replicate those of Graesser and Anderson and, at face value, support their interpretation in terms of the gift size equation of Eq. (1).

Evidence for Averaging. But another interpretation of the data is also possible--*in terms of an averaging model. With differential weighting, the averaging model can yield an approximate linear fan pattern (Anderson, 1971, p. 85). If lower values of income have greater importance or weight in the judgment of gift size, then the corresponding curves will have lower slope in Figure 1, in agreement with the obtained results. The possibility of this interpretation was noted by Anderson and Butzin (1974) in a study of the analogous equation, Performance = Motivation x Ability, but this question was not pursued further until the present experiments.

To test between the averaging and multiplying rules, the present experiments added another condition, represented by the dashed curves in Figure 1, which corresponds to judgments based on information only about generosity. In each experiment, the dashed, generosity-only curve crosses over one of the solid curves. This crossover violates the linear fan pattern and thereby seems to rule out the multiplying model.

Such crossover effects are, of course, well known as evidence for the averaging rule (e.g., Lampel & Anderson, 1968; Oden & Anderson, 1971). Although the logic of the crossover test is well known, it may be useful to summarize it here, for it is important in the later theoretical analyses. Rupees 12,000 is a moderate income; when this information is added to the lowest levels of generosity, their average is higher than the value of the generosity alone. At the left side of the graph, therefore, the solid curve lies above the dashed curve. In contrast, when the moderate income information is added to the highest level of generosity, their average is lower than the value of the generosity alone. At the right side of the graph, therefore, the solid curve lies below the dashed curve. A similar pattern appears in the right panel of Figure 1. At face value, these results infirm the Graesser and Anderson interpretation in terms of the multiplying rule. Instead, the results seem to favor the averaging rule.

Problems of Imputations. The foregoing interpretations of Figure 1 ignored the possibility that subjects might make imputations about the missing income information. This problem was pointed out by Norman Anderson (personal communication, August, 1977), and his suggestion seemed to have merit inasmuch as subjects logically cannot judge gift size without information about income. Accordingly, a more systematic analysis of this problem was undertaken.

There are two natural patterns of imputation. First, subjects may impute a single fixed value, presumably around the average of the specified values in the experiment. But then the generosity-only condition is essentially the same as the other conditions; the imputed income information operates just like the explicitly specified income information. In this case, the dashed curve should form part of the linear fan defined by the solid curves—for both rules. With this pattern of imputation, neither rule can explain the results.

The imputation may, however, depend on the value of the given information. An ungenerous person would be assumed to have lower income than a generous person. Similarly, a rich person would be assumed to be more generous than a poor person. In this case, the dashed curve should cross over the medium income curve—for both rules. Such crossover would result because, with this imputation rule, the imputed value of income is not constant, but increases as generosity increases on the horizontal axis.

With this second pattern of imputation, therefore, both rules can explain the results. If imputations are allowed, then this crossover test does not discriminate between the two rules. The following experiments were conducted to resolve this issue.

EXPERIMENTS 1*3

ANALYSIS WITH A TWO-OPERATION MODEL

A general way to attack the problem of imputations is to use a two-operation model, in which two pieces of information are given about generosity and one about income. One of the generosity cues can then be omitted without creating any problem of missing information.

This task leads to a two-operation, averaging*multiplying model. Since the two generosity cues are qualitatively similar, previous work on person cognition implies that they will be averaged (Anderson, 1981). In line with Eq. (1), generosity and income should multiply. This leads to the following averaging*multiplying rule

$$\text{Gift size} = (\text{Generosity}_1 + \text{Generosity}_2) \times \text{Income}. \qquad (2)$$

The factorial graph of the two generosity cues should exhibit the parallelism pattern, at least if the equal-weighting condition holds. Both income*generosity graphs, however, should exhibit the linear fan pattern.

METHOD

Experiments 1*3 were similar in purpose and design. Their first purpose was to test the averaging*multiplying model of Eq. (2). Their second purpose was to study the associated problem of imputations by using stimulus conditions with only one or two of the three pieces of information.

Stimuli and Designs. There were two main stimulus designs. The first was a 4 * 3 * 3, Income * Generosity$_1$ * Generosity$_2$ factorial that yielded descriptions of 36 stimulus persons. The four levels of income were 5, 12, 19, and 26 Rs. thousands, just as in the first initial experiment shown in the left panel of Figure 1. The three levels of the generosity factor differed across the three experiments. In Experiment 1, generosity was defined by contributions to the Flood Relief Fund for 1974 and 1976. The three contribution levels were none, moderate, and large. In Experiment 2, generosity was defined by the opinions of two friends. The three levels of Friend 1 were slightly generous, fairly generous, and quite generous; the three levels of Friend 2 were somewhat generous, pretty generous, and very generous. These levels were specified in terms of labels on an 8-step opinion scale that also included the two end steps, not at all generous and extremely generous. In Experiment 3, the ranges were increased by decreasing the lowest level for Friend 1 and increasing the highest level for Friend 2.

The second design was a Generosity$_1$ * Generosity$_2$ factorial. The levels of both factors were the same as in the main design of the respective experiment. In addition, each level of both generosity factors was given alone. Thus, there were 51 experimental stimulus persons altogether.

Nine practice examples were also constructed, including 4 with all three cues, 3 with just two generosity cues, and 2 with a single generosity cue. The levels of the practice descriptions were chosen more extreme than for the regular experimental stimuli, so they served also as end anchors (Anderson, 1982, Section 1.1.4).

Procedure. In Phase 1, subjects made judgments about generosity based on one or two generosity cues. A written sheet of instructions described the nature of the stimuli, the task, and the response measure. Five practice examples were given to make the task clear. Following practice, the deck of cards describing the 15 experimental stimulus persons was presented to the subjects, who read through them to gain familiarity with the persons to be judged. Each subject then shuffled the deck of 15 cards and rated generosity of the person described by each card in two successive replications. For each card, the subject entered the code number of the stimulus card on a response sheet, together with his rating of generosity. Ratings were made on a 0–20 scale printed at the top of the response sheet and anchored by the phrases *not at all generous* and *extremely generous*.

In Phase 2, subjects estimated how large a gift each stimulus person might have given to a family whose house had burned down. The subject read the typed sheet of instructions twice, listened to the description of the task by the experimenter, worked on 9 practice examples described earlier, and then read through all 51 experimental descriptions. The deck of 51 cards was then shuffled and the subject rated gift size of each stimulus person on a 1–21 scale. In Experiment 1, the 51 stimulus persons were rated twice in different shuffled orders. In Experiments 2 and 3, there were three such replications. Data from all replications were included in the analyses.

Subjects. There were 20, 12, and 12 subjects in Experiments 1–3, respectively. All were second-year engineering students at the Indian Institute of Technology, Kanpur, India, who were fulfilling a course requirement in introductory psychology. They were run two at a time in the psychology laboratory.

RESULTS

Averaging Rule for Judgments of Generosity. The first theoretical question concerns the integration rule for the two pieces of information about generosity in Phase 1 of the experiments. The factorial graphs for these data are shown in Figure 2. The three solid curves for each experiment show a pattern of near-parallelism, except for a single deviant point at the lower left for Experiment 1. This pattern supports a simple integration model, either adding or averaging.

The statistical analysis mirrored the visible patterns. The interaction term in the analysis of variance, which tests for deviations from parallelism, was nonsignificant in Experiments 2 and 3, $F(4, 44) = .96$ and .06, respectively. For Experiment 1, the interaction was statistically significant, $F(4, 76) = 5.49$, but the one-point discrepancy in the graph may be interpretable as an end-effect in the rating scale that did not reappear in the two subsequent experiments.

A critical test between the adding and averaging rules is provided by the dashed curve in each panel, which represents the judgments based on a single generosity cue. By the logic outlined in the introduction, this crossover infirms the adding rule and supports the averaging rule.

Further support for the foregoing analyses is available from the main design. The solid curves in Figure 3 present the factorial graphs for the two generosity cues, averaged over the income cue. These solid curves also exhibit near-parallelism. The dashed curves are based on the single generosity cue listed on the horizontal axis. In the same way as before, this crossover infirms the adding rule and supports the averaging rule for the integration of information about generosity.

Linear Fan Pattern for Judgments of Gift Size. The second theoretical question concerns the integration rule for information about income and generosity. Evidence on this rule comes from the two-factor graphs of Figure 4, each of which represents the income cue and one of the two generosity cues from the main design. All six graphs show a similar pattern of a diverging linear fan. This replicates the results already seen in Figure 1 even though two generosity cues are involved. Statistical evidence for the linear fan pattern is provided by the Linear x Linear components of the interaction, which were significant for all six graphs, whereas the

Linear x Quadratic components were small and generally nonsignificant.

A different view of this linear fan pattern is shown by the solid curves of Figure 5. This figure treats the main design as a two-way, Income x Generosity design with nine levels of generosity, corresponding to the nine pairs of generosity information. These nine generosity conditions are spaced on the horizontal axis according to their functional scale values, derived from the marginal means of the specified 4 x 9 design. This shows that the linear fan patterns of Figure 4 were not an artifact of averaging over one generosity factor.

The dashed curves in Figure 5 come from the $Generosity_1$ x $Generosity_2$ design, in which income was not specified. These dashed curves show clear crossovers, at least for Experiments 1 and 3, similar to those seen in Figure 1. This violates the linear fan pattern and at face value argues against the multiplying rule.

DISCUSSION

The first theoretical implication of the foregoing results is that generosity cues are averaged to arrive at judgments of generosity. Moreover, the parallelism means that the equal weighting condition was satisfied. As far as generosity is concerned, therefore, the possibility of differential weighted averaging can be ruled out.

The second theoretical implication concerns the linear fan pattern in the Generosity x Income graphs. The preferred interpretation is that this supports the two-operation model of Eq. (2): the two generosity cues are averaged and this average is multiplied into the income cue.

Unfortunately, the problem of imputations discussed in the introduction has not been entirely eliminated in these experiments. The crossovers of the dashed curves in Figure 5 violate the linear fan pattern and so might seem to infirm the multiplying rule and support the averaging rule. The multiplying rule could be saved, however, by assuming that subjects impute a value to the missing income information that is positively correlated with the given generosity information. This imputed income would act as a multiplier for the generosity-only curve that increases as generosity increases. Hence the slope of the generosity-only curve would increase, causing it to cross over the solid curves for which income is a given constant. Against this imputation rule, therefore, the crossovers in Figure 5 are ambiguous.

EXPERIMENT 4

TEST FOR IMPUTATIONS

The main purpose of Experiment 4 was to resolve the ambiguity just noted in the interpretation of Experiments 1-3. This was done by pairing the income information with either one or three pieces of information about generosity. If the multiplying rule holds, then the curves from the two-cue and four-cue designs will form a common linear fan. If the averaging rule holds, however, then the curves from the two-cue design will cross over the curves from the four-cue design.

A second purpose of Experiment 4 was to obtain further information about the operation of imputations. Accordingly, judgments of gift size were obtained from information about either income alone or generosity alone.

METHOD

<u>Stimuli</u> <u>and</u> <u>Designs</u>. There were 11 stimulus designs. The first was a four-way factorial with three generosity factors and one income factor. Each generosity factor was defined by the opinion of a different friend:

Friend 1 (very much below average in generosity, average in generosity, and very much above average in generosity); Friend 2 (not at all generous and very generous); and Friend 3 (low in generosity and high in generosity). The three levels of annual income were 5, 15, and 25 Rs. thousands.

The second design was a three-way factorial using the three generosity factors already listed. Designs 3-5 were two-way factorials using each of the three pairs of generosity factors. Designs 6-8 were also two-way factorials that used one of the generosity factors with the income factor. Designs 9-11 had only one factor, either the second or third generosity cue or the income cue, respectively.

Altogether, these designs yielded 92 stimulus persons. In addition, 16 practice examples were constructed. Four had 4 cues, two had 3 cues, six had 2 cues, and the remaining four had a single cue. The practice examples included information more extreme than the regular stimulus persons and so served as end anchors. The four 4-cue practice examples were also presented along with the main 92 experimental stimuli.

Procedure. Subjects were 14 second-year engineering students from the same population as the preceding experiments, who received Rupees 10 for their services. They were run individually over two successive evenings in sessions of approximately 2.5 and 2 hours. General procedure was the same as in the second phase of the preceding experiments. Each subject rated the 96 stimulus persons two times in different shuffled orders on each evening. Data from all four replications were analyzed.

RESULTS AND DISCUSSION

Multiplying versus Averaging. To establish the multiplying rule, two kinds of evidence are needed. First is the linear fan pattern illustrated in Figure 6. In the left panel, the three solid curves with filled circles are from the two-cue, Generosity$_1$ x Income design. The corresponding curves in the center and right panels are from the similar two-cue designs,

$\text{Generosity}_2 \overset{*}{x} \text{Income}$ and $\text{Generosity}_3 \overset{*}{x} \text{Income}$, respectively. All three sets of curves show the linear fan pattern.

The linear fan pattern is also found in the data from the full four-factor design. The two-factor graphs of these data are shown in each panel of Figure 6 by the solid curves with open circles. All three sets of these curves also show the linear fan pattern.

The critical discrimination between the multiplying and averaging rules comes from comparison of the filled-circle and open-circle curves in Figure 6. If the multiplying rule holds, then each curve should be a straight-line function of income with slope equal to the value of generosity. Hence the filled-circle and open-circle curves of Figure 6 should form a common linear fan--as indeed they do.

Further support for the multiplying rule comes from comparing slopes for the open-circle and filled-circle curves. The slopes of the open-circle curves, since they are averaged over the other two generosity factors, should lie between the slopes for the low and high generosity curves from the two-cue designs. This prediction also is well supported in all three panels of Figure 6.

The averaging rule, in contrast, requires that the filled-circle curves should cross over the open-circle curves in each panel. This prediction follows the logic of the crossover test already presented. Little or no sign of such crossover can be seen. The slight crossover in the left panel did not approach statistical significance. This test avoids the problem of missing information because some generosity information, either one or three pieces, is always present. These results demonstrate, therefore, that the averaging rule is not applicable and that the multiplying rule is.

For completeness, the statistical analyses are given in Table 1, which presents trend analyses for the six sets of data just considered. The multiplying model requires that the Linear $\overset{*}{x}$ Linear component of the interaction be significant and that the higher-order components be nonsignificant. The table shows that the Linear $\overset{*}{x}$ Linear components have very high <u>F</u> ratios, and all are highly significant. The higher-order components are generally small and nonsignificant, except for the case of the two-way design in the left panel of Figure 6. However, inspection of the graph shows no serious deviations from the linear fan pattern.

Also included at the bottom of Table 1 are the statistical tests for the two-factor and four-factor designs combined. These combined data should also form a linear fan, as already noted, and the statistical tests support this. In short, the statistical analyses agree completely with the visual inspection of Figure 6. Thus, the averaging rule can be eliminated, whereas the multiplying rule

$$\text{Gift size} = \text{Generosity} \overset{*}{x} \text{Income}$$

receives very strong support.

<u>Imputations</u>. Because the multiplying rule has been established, it is possible to show that subjects made imputations about missing information. Two rules of imputation must be considered, one for generosity, the other for income.

When only information about income is given, subjects impute a fixed value to generosity and multiply that into the given income information. The evidence for this conclusion is shown in Figure 6, in which the dashed curves represent judgments based on income alone. These dashed curves fall into the linear fan pattern defined by the solid curves in each panel. The natural interpretation is that subjects impute a fixed value to generosity, approximately average in value, and multiply it with income. This will yield a straight line with slope and elevation approximately the same as for the solid curves for given generosity information of average value, in line with the observed results.

If subjects made no imputation, then the slope and elevation of the dashed curves would be hard to understand. On the other hand, if subjects imputed a nonconstant value of generosity, one that depended on the given income information, then the dashed curves would have variable slope and cross over the solid curve.

A different imputation rule applies when only generosity information is given. In this case, subjects impute an income value that is a direct function of the given generosity information. One clue to this interpretation is shown in Figure 7, which plots the data from the four-way design in the form of a two-way, Generosity x Income design in the same way as indicated for Figure 5. Figures 5 and 7 both show the same pattern: the solid curves form a linear fan, but the dashed curves have steeper slope and violate the linear fan. The steeper slope can be explained if subjects assume that people who are more generous have greater income. In other words, they impute a value to the missing income information in direct relation to the given generosity information and multiply the imputed and given information to arrive at their judgments.

Much stronger evidence for this interpretation is given in Figure 8, which plots the three Income x Generosity graphs for both the two-cue and the four-cue designs. For the issue at hand, what is important is that the dashed curves for generosity-only information are steeper and cross over the open-circle solid curves from the four-factor design. By the reasoning already given, this supports the conclusion that the imputation about missing income information depends on the value of the given generosity information.

Averaging Rule for Generosity Information. When more than one piece of generosity information is given, these are averaged. This averaging rule was supported in Experiments 1-3, and the present experiment provides even stronger support.

One line of evidence for averaging is shown by the parallelism patterns in Figure 9. The solid, filled-circle curves in the three panels represent the three two-way designs with two generosity cues; the solid, open-circle curves represent the corresponding factorial graphs from the four-way design, averaged over the other two design factors. All six sets of curves exhibit near-parallelism, and none of the corresponding interaction terms in the analysis of variance approached significance.

Furthermore, the slopes of the curves agree with averaging theory. Theoretically, the slope represents the relative weight of the generosity cue listed on the horizontal axis. This cue has higher relative weight when it is one of two than when it is one of three generosity cues. Hence the filled-circle curves should have steeper slope than the open-circle curves, as in fact they do.

To complete the slope pattern, the dashed curve, which represents the response to a single generosity cue is also included. It has the steepest slope of all, as it theoretically should, and indeed shows the crossovers that eliminate the adding rule.

Additional evidence of the same kind is presented in Figure 10, which replots the two-cue generosity data of Figure 9, together with the corresponding data from the design with three generosity cues. Here again the two-cue data show the steeper slope. It is also worth noting that the three-cue curves of Figure 10 have essentially equal slope to the corresponding four-cue curves of Figure 9. This agrees with the two-operation model of Eq. (2), for the fourth cue is income, which is not averaged, but multiplied.

Configural Effect. One complication arose in the statistical analysis. The three-cue generosity interaction was statistically significant in both the three-cue and the four-cue designs, $F(2, 26) = 5.59$ and 11.60, respectively. These two interactions are profiled in Figure 11.

Inspection of Figure 11 shows a simple pattern for the three-way interaction. From left to right, the three panels exhibit divergence, parallelism, and convergence. These patterns have a simple explanation: when two friends have similar opinions about the person's generosity, the opinion of the third friend is discounted; when two friends differ in their opinions, the opinion of the third friend is given higher weight.

This interpretation rests on detailed reasoning that deserves attention as an illustration of the analytical power of cognitive algebra. Each 2 x 2 design represents the generosity information given by Friends 2 and 3, which has either high or low values. The two middle panels correspond to the average value of the generosity information given by Friend 1. These two panels exhibit parallelism, and are taken as the no-interaction reference. Hence those curves in the left and right panels that are parallel to those in the center panel also presumably show no interaction. Taking into account the vertical separation of the pairs of curves, the interaction may be localized at single points: the top right point in both left panels, and the bottom left point in both right panels. It thus follows that the opinion of Friend 1 is discounted when and only when Friends 2 and 3 agree, but Friend 1 disagrees with them. Substantively, not too much should be made of one incidental result. This analysis, however, does illustrate how integration theory can provide incisive information about interaction processes.

EXPERIMENT 5

VERIFICATION OF AVERAGING*MULTIPLYING RULE USING SET-SIZE EFFECT

This experiment was run chiefly as a verification check on the two-operation, averaging*multiplying model. It also provided information on imputations about missing generosity information. The main change was to base the theoretical interpretation on the set-size effect, in which adding more information of equal value causes a more polarized response (Anderson, 1965, 1967).

Within averaging theory, the set-size effect is handled in terms of the initial state, which is averaged in with the given stimulus information. In terms of the present averaging*multiplying model, the integration rule for multiple pieces of generosity information and one piece of income information is

$$\text{Gift size} = \left|\frac{kwG + I_0(1 - w)}{kw + (1 - w)}\right| \overset{*}{x} \text{Income}, \qquad (3)$$

where k is the number of pieces of generosity information of equal value, G; I_0 is the initial state; and w and 1 - w are the weights for one given piece of generosity information and the initial opinion, respectively. This model predicts crossovers when set size is varied, but a linear fan pattern in the factorial graph for Generosity $\overset{*}{x}$ Income.

IMPUTATIONS

METHOD

Stimuli and Designs. The first design was a Number of opinions $\overset{*}{x}$ Generosity $\overset{*}{x}$ Income factorial. The number of opinions about generosity was 1 or 5, all of equal value. The values of the opinions were not at all generous, okay, and very generous. The levels of income were the same four values used in Experiments 1&3.

The second design was an Order of presentation $\overset{*}{x}$ Number of opinions $\overset{*}{x}$ Income factorial. There were two orders of presentation of the generosity information: positive*negative and negative*positive. The number of opinions was 2 or 5. The levels of income were the same as in the first design. Sets with two opinions had one positive and one negative; sets with five opinions has two positive, one neutral, and two negative. Each set thus had a mid-scale or neutral value (Edwards & Ostrom, 1971; Singh & Byrne, 1971).

In addition, the four levels of income were presented alone to replicate the imputation rule for missing generosity information. The complete set of experimental stimuli included the foregoing 44 descriptions, plus two end anchors and four fillers.

Procedure. General procedure was the same as in Experiment 4. There were 12 subjects from the same population, each of whom received Rupees 5 for his services. Subjects were run individually, and each rated the 50 descriptions three times in different snuffled orders. Data from all three replications were included in the analysis.

RESULTS AND DISCUSSION

Averaging versus Multiplying. The first question is whether generosity and income information are averaged or multiplied. The three curves in the right panel of Figure 12 provide clear evidence against averaging and for multiplying. The two solid curves are from the second design, and they represent judgments based on the one piece of income information. listed on the horizontal, and either two or five pieces of information about generosity. The averaging model requires that the curve with more generosity information have shallower slope than the curve with less generosity information, because the slope is an index of the relative weight of the income information. Since the two curves are virtually identical the averaging hypothesis is inadequate.

The multiplying model, in contrast, predicts equal slope for the two solid curves because the mean value of the generosity information is the same for both, by virtue of the choice of stimulus values in the second design. Multiplication of this same value of generosity with the given income information will produce curves of equal slope. The same result that causes difficulty for the averaging model thus provides strong support for the multiplying rule, Gift size = Generosity x Income.

<u>Averaging-Multiplying Model</u>. According to the averaging-multiplying hypothesis, the information about generosity is averaged and then multiplied with the income information. Support for this prediction is shown in the middle panel of Figure 12, in which the six solid curves represent the data from the first design, with one curve for each combination of generosity information. These six solid curves form a linear fan, in agreement with the multiplying model.

The curves for five pieces of generosity information are more extreme than the corresponding curves for one piece of generosity information. This also agrees with the multiplying model. By virtue of the set-size effect, the effective value of generosity is more extreme when based on more information. Hence the curves labeled 5 should have more extreme slope and more extreme elevation than the corresponding curves labeled 1. The two lowest curves are no exception; <u>not at all generous</u> has a near-zero value so the 5-curve has lower value and lower slope than the corresponding 1-curve, by virtue of the action of the initial state in Eq. (3).

Statistical support for the graphical trends evident in the middle panel of Figure 12 comes from the trend analyses. The model requires a significant Linear $\overset{*}{x}$ Linear component and nonsignificant Linear $\overset{*}{x}$ Quadratic, Quadratic $\overset{*}{x}$ Linear, and Quadratic $\overset{*}{x}$ Quadratic components. The corresponding tests yielded $F(1, 11)$ = 29.58, .02, .29, and .02, respectively, in excellent agreement with the theory.

The preceding experiments showed that generosity information is averaged, and the present results agree with this. This may be seen in the left panel of Figure 12, which plots the data of the center panel averaged over income. The crossover of the two curves represents the standard set-size effect. For the highest and lowest values of generosity, the curve for five pieces of information is more extreme than the curve with one piece of information, as required by Eq. (3).

Imputations. Previous experiments had shown that missing generosity information is imputed an average value, and further support for this interpretation is present in Figure 12. In the right panel, the dashed, income-only curve is essentially identical to the two solid curves. Although there is a hint of steeper slope, the interaction did not approach significance, $F(6, 66) = 1.55$. The natural interpretation is that subjects imputed a fixed, average value to the missing generosity information and multiplied this with the given income information.

Similar evidence for imputations is provided by the dashed curve in the center panel. Although the dashed curve crosses over the 1-okay curve, the difference between them did not approach significance, either in the overall interaction or in its bilinear component. This suggests, therefore, that the imputation is equivalent to one piece of generosity information of okay value.

EXPERIMENT 6

TEST WITH SOURCE RELIABILITY

At the time the preceding experiments were being written up, a test of the analogous model, Performance = Motivation $\overset{*}{x}$ Ability, was published by Surber (1981a), who found evidence for averaging rather than multiplying. Since this seemed inconsistent with the preceding results, it was decided to replicate Surber's method and design under the conditions used in the preceding experiments.

According to Anderson's averaging model, which was employed by Surber, the judgment of gift size should be

$$\text{Gift size} = \frac{w_G G + w_I I + w_O I_O}{w_G + w_I + w_O}, \quad (4)$$

where G, I, and I_O are the scale values of generosity, income, and the initial state, and w_G, w_I, and w_O are their respective weights.

Reliability of information is assumed to affect the weight parameters (Anderson, 1971). If the value of w_G is increased by varying the reliability of the generosity information, as in Surber's experiment, then the relative weight and relative effect of the income information should be reduced. This property provides a strong test of the averaging model.

If generosity and income are multiplied, however, then

$$\text{Gift size} = w_G G \overset{*}{x} w_I I. \quad (5)$$

In this case, as Surber pointed out, an increase in w_G will increase the effect of the income information. The two models thus make sharply contradictory predictions, and these predictions were tested in the following experiment.

METHOD

Stimuli and Design. The design had four factors: Generosity; Reliability of generosity information; Income; and Reliability of income information. The three levels of generosity were not at all generous, okay, and very generous. The three levels of income were 5, 15, and 25 Rs. thousands.

Reliability of the information was manipulated by specifying the source of the information. For generosity, the three sources were a person of one hour acquaintance, a fast friend for five years, and the local flood relief officer. For income, the three sources were a casual visitor to the home of the stimulus person, a fast friend for five years, and the income tax file. These levels were selected in consultation with two pilot subjects who also helped in the preparation of detailed instructions.

In addition to the 81 person descriptions from the foregoing design, 9 filler descriptions were included. Four used extreme levels of generosity and income and served as end anchors. The remaining five had stimulus levels different from those in the main design. The initial practice included these nine descriptions together with six from the regular design.

Procedure. General procedure was the same as in the previous experiments. Written instructions helped establish the reliability manipulation by stating that the most credible source of generosity information would be the local flood relief officer, who maintained records of previous donations. When this was not available, a friend or acquaintance was approached for an opinion. Analogous instructions were used for reliability of the income information. It was emphasized that these information sources varied widely in credibility and this should be taken into account in making judgments of expected gift size.

Subjects were 28 first-year students enrolled in a course on personal and interpersonal dynamics at the Indian Institute of Management, Ahmedabad, who received Rupees 5 for the approximately 1.5 hours spent on the experimental task. After each subject completed the practice and indicated his understanding of the task, he received the complete deck of 90 person descriptions and rated them twice in different shuffled orders. Data from both replications were included in the analysis.

RESULTS AND DISCUSSION

Linear Fan Pattern. A complete overview of the data is given in Figure 13, which plots the mean judgments from all 81 cells of the complete design. Each panel shows the results for one Generosity * Income design, with generosity as curve parameter and income on the horizontal axis. Every panel shows the linear fan form demanded by the multiplying model.

These results provide strong support for the multiplying model, and they are supported by the trend analyses. Partition of the overall Generosity * Income effect into Linear * Linear, Linear * Quadratic, Quadratic * Linear, and Quadratic * Quadratic components yielded $F(1,27)$ = 75.40, .16, .04, and .00, respectively. The Linear * Linear component is highly significant and the linear*quadratic components are nonsignificant, just as predicted by the multiplying model.

The averaging model, however, might give an approximate account of the linear fan pattern if differential weighting were allowed for the income variable. The previous experiments show that generosity is equally weighted, so the model would have the semilinear form (Anderson, 1983), which predicts a set of straight lines of varied slope. This predicted pattern is not a linear fan, however, because the straight lines do not in principle intersect at a single point. In principle, therefore, one of the linear-quadratic components would be real, but in practice the power of the test could be low. Accordingly, some better way is needed to discriminate between the two models, as in the following subsection.

Reliability Manipulation: Multiplying versus Averaging. The effects of the reliability manipulation are straightforward: all support the hypothesis that generosity and income are multiplied; none support the hypothesis that generosity and income are averaged. The logic of these tests has already been indicated. If the multiplying rule holds, then increasing the reliability of the generosity information will increase the effect of the income information, and vice versa. But the opposite will happen if the averaging rule holds: increasing the reliability of the generosity information will decrease the effect of the income information, and vice versa.

The first test between the two rules is given in the top layer of Figure 14. Each panel plots the Generosity * Income graph for one level of reliability of generosity information: low, medium, and high, from left to right. As the reliability of the generosity information increases, from left to right, the three curves become steeper and more spread out. In other words, the effect of the income information increases as the reliability of the generosity information increases. This supports the multiplying rule but contradicts the averaging rule.

A similar result is seen in the middle layer of Figure 14. Each panel shows the Generosity * Income graph for one level of reliability of income information. As this reliability increases, from left to right, the curves become steeper and more spread out. In other words, the effect of the generosity information, which is represented by the steepness of the curves, increases as the reliability of the income information increases. By the same logic as before, this supports the multiplying rule and contradicts the averaging rule.

A third test between the two rules is shown in the bottom layer of Figure 14, which plots factorial graphs for generosity and reliability of generosity information. The multiplying and the averaging rules can both account for the crossover, but they make different predictions about its trend. The magnitude of this crossover should increase as the income information becomes more reliable, from left to right, according to the multiplying rule, whereas it should decrease according to the averaging rule. The observed crossover increases, which supports the multiplying rule and contradicts the averaging rule. This three-way interaction was statistically significant, as were those in the other two panels.

A different test between the two models, similar to that used by Surber, is shown in Figure 15. The two left panels show the factorial plots for the reliability and value of the two kinds of information. Both panels show the crossover interaction required by the multiplying model. Although this crossover seems less clear in the second panel, the trend analyses supported the model in both cases. The bilinear F's were 34.76 and 17.23 for the generosity and income graphs, respectively, whereas none of the linear quadratic components approached significance. The averaging model, however, also accounts for this crossover.

A discriminative test between the two models is provided by the two right graphs, which show the factorial plots for the reliability of one kind of information and the value of the other kind of information. The averaging model requires that generosity information have greater effect when income reliability is lower. Hence the curve labeled high should have lower slope than the curve labeled low and lie below it at the right end. Just the opposite is the case, as may be seen in the second panel from the right. A similar prediction holds for the rightmost panel, and here again the data disagree with the averaging model.

In contrast, the ordering of the three curves agrees with the multiplying model. One discrepancy from the multiplying model is that it requires a linear fan pattern in both right panels. Although this is weak in the panel second from right, it was highly significant, with a bilinear F of 10.52. In the right panel, however, it did not approach significance. With this exception, the data of Figures 13-15 provide remarkably good support for the hypothesis that generosity and income are integrated by a multiplying rule.

GENERAL DISCUSSION

The results of this experimental investigation can be summarized under the two issues considered in the introduction: cognitive algebra and imputations. The data provide striking evidence for cognitive algebra, but this interpretation requires that allowance be made for imputations about missing information. At the same time, cognitive algebra provides an incisive diagnostic for the imputation rules. Following discussion of these two issues, therefore, the approach developed in these experiments will be applied to other tasks.

COGNITIVE ALGEBRA AND IMPUTATIONS

Two-Operation Integration Rule. A major finding of the present series of experiments concerns the operation of the two-operation, averaging* multiplying rule. Given two pieces of information about generosity and one about income, judgments of gift size obeyed the rule

$$\text{Gift size} = (\text{Generosity}_1 + \text{Generosity}_2) \overset{*}{\times} \text{Income}.$$

Information about generosity was averaged, and this average was multiplied with the income information. This rule accounted for the results of six experiments in remarkable detail.

Evidence for the averaging operation was seen in the parallelism patterns and in the crossover tests. These analyses agree with much previous work on averaging processes for cues that have the same informational quality.

Evidence for the multiplying operation was seen in the linear fan patterns. This replicates the results obtained by Graesser and Anderson (1974), but with additional conditions that rule out alternative interpretations of their data. Indeed, the full theoretical analysis of the present data takes account of three alternative hypotheses.

Weighted Averaging Hypothesis. One alternative explanation of the linear fan pattern is that generosity and income, instead of being multiplied, are averaged with differential weighting. If weighting is greater for lower generosity or for lower income, as would seem reasonable with judgments of gift size, then the averaging model would produce an approximate linear fan.

The generosity variable, however, must be equally weighted because of the parallelism patterns for the designs with two generosity cues, as in Experiments 1–3. Similar designs might have been included to test for equal weighting of the income variable. However, the designs of the later experiments allowed a more direct test of the multiplying rule.

One specific result may be cited to illustrate the theoretical logic. Experiment 4 compared a two-cue and a four-cue design, with one piece of income information and either one or three pieces of generosity information. If generosity and income are averaged, the two-cue curves will be markedly steeper than the four-cue curves. But if generosity and income are multiplied, the two sets of curves will form a common linear fan. As Figure 6 showed, the results clearly supported the multiplying rule. Different experimental manipulations in Experiments 5 and 6 gave further support to the multiplying rule.

Imputations. A second theoretical complication concerned the possibility that subjects imputed values to missing information and integrated these imputed values. This problem of imputations appeared in interpreting the crossovers in Experiments 1–3. With qualitatively similar information, crossovers are ordinarily taken as evidence for averaging, as with the present designs that used one and two generosity cues.

The matter is different, however, with qualitatively different information, as with generosity and income. Since both cues are logically necessary to judge gift size, the imputation hypothesis must be taken seriously. Indeed, if missing income information is imputed a value positively related to the given generosity information, then the crossovers observed in Experiments 1-3 can be explained by both the averaging and the multiplying rules.

This matter was also resolved by Experiment 4, by virtue of the joint two-cue and four-cue designs mentioned in the preceding subsection. Because the curves from these two designs formed a common linear fan, the averaging rule can be rejected. Furthermore, the multiplying rule provided an excellent quantitative account of all the results.

The existence and nature of the imputations in the present task were made manifest through cognitive algebra. Establishment of the two-operation, averaging-multiplying rule of Eq. (2) provided the basis for diagnosis of the imputations. These diagnoses were straightforward, but it is notable that two different imputation rules were operative, depending on which kind of information was missing.

The imputation rule for generosity was simple. When generosity was not specified, it was imputed an average value. This constant imputed value was multiplied with the given income information.

The imputation rule for income was also simple. When income was not specified, it was imputed a value directly related to the value of the given generosity information. The evidence for these two rules was summarized in Experiments 4-5 and will not be repeated here. It deserves emphasis, however, that the diagnosis of these imputation rules was bound up with the cognitive algebra of the task.

The asymmetry in the imputation rules for the two kinds of information reflects the subjects' implicit theory of personality. The results imply that subjects did not consider that earning more makes a person more generous. They did, however, expect a more generous person to have greater income. This asymmetry in the imputation rules may reflect a two-fold meaning of generous. One meaning refers to a motivational trait, the other to a behavioral description. To say that a person is generous connotes not only a disposition to future action, but also past actions of generosity--for which income is prerequisite.

Response Scale Validity. Finally, there is an alternative interpretation of the linear fan pattern that may need brief comment. In this interpretation, the linear fan is an artifact of a biased, or nonlinear, response scale. Instead of the ratings being a true linear (equal interval) scale, the true scale might be a logarithmic function of the ratings. The true pattern would not be a linear fan, therefore, but one of parallelism, and the multiplying rule would be incorrect.

The data on the two-operation model avoid this difficulty because they simultaneously satisfied the parallelism pattern and the linear fan pattern, depending on the integration stage. Both patterns support and buttress each other. This follows the two-operation logic discussed by Graesser and Anderson (1974; see also Anderson, 1981, 1982).

Inconsistency in Cognitive Algebra. In their study of the gift size equation, Graesser and Anderson (1974) found an apparent mathematical inconsistency. They used three conditions, in which subjects received information about two of the variables and made judgments about the third. They claimed that one judgment obeyed a multiplying rule,

$$\text{Gift size} = \text{Generosity} \overset{*}{\times} \text{Income},$$

but that the other two judgments obeyed subtracting rules,

$$\text{Income} = \text{Gift size} - \text{Generosity},$$

and

$$\text{Generosity} = \text{Gift size} - \text{Income}.$$

The evidence was the linear fan pattern for the first equation and the parallelism for the second and third equations.

Mathematically, of course, the second and third equations should obey a dividing rule, not the subtracting rule, if they are to be consistent with the first equation. Accordingly, Graesser and Anderson concluded that cognitive algebra is not a mirror of mathematical algebra. A similar conclusion was reached by Anderson and Butzin (1974) in their study of the analogous equation, Performance = Motivation $\overset{*}{\times}$ Ability (see Anderson, 1981, Section 1.5.6).

Neither experiment, however, ruled out the possibility that the linear fan pattern might have been produced by averaging with differential weighting. Anderson and Butzin mentioned this possibility but did not pursue it. Their results, therefore, allow the possibility that all three judgments are consistent manifestations of the same underlying process of averaging.

The present experiments clear up this ambiguity, for they clearly demonstrate that generosity and income are integrated by the multiplying rule. This shows that the inconsistency in cognitive algebra claimed by Graesser and Anderson is correct.

IMPUTATIONS

FURTHER WORK

As the present experiments have made clear, rule diagnosis can be complicated when imputations are present. Previous evidence indicates that imputations are situation sensitive, but that they may be expected when the information variables are qualitatively different, as with generosity and income. In this case, the methods used in previous work may be ambiguous about the operative rule. This applies to tests between multiplying and differential weighted averaging as well as to tests between adding and equal weight averaging. Four such cases will be considered here and it will be shown how the ambiguities can be removed using the design and approach developed in this research program.

Consumer Products. Yamagishi and Hill (1981) obtained judgments of desirability of consumer products described by price alone, by quality alone, and by price and quality together. Imputations about missing information are expected in this task because price and quality appear to be distinct cognitive units, both necessary for the judgment. Since price and quality are positively correlated, missing information about one should be imputed a value proportional to the other. Product desirability, however, depends directly on quality but inversely on price. Hence, if imputations are present, the price-only curve should have reduced slope, showing a crossover pattern opposite to that usually obtained in tests between averaging and adding. This prediction holds for both averaging and adding rules.

Indeed, their price-only curve had a near-zero slope. This can be accounted for by an adding rule with imputations (their path analytic model). It cannot be accounted for by an averaging model without imputations. Using this double standard, Yamagishi and Hill rejected the averaging rule and claimed support for the adding rule.

Yamagishi and Hill were wrong to disallow imputations in averaging theory. The paper by Singh et al. (1979), which they cited in their Footnote 1, explicitly discusses evidence for inferences about missing information within the averaging model. Once imputations are allowed, the design of Yamagishi and Hill does not distinguish between adding and averaging.

An unambiguous and powerful test between adding and averaging in their task can be obtained using the design that was developed in Experiments 4 and 5. The essential idea is to use three pieces of information about one of the variables, say, product quality, with price specified by one piece of information. The three quality cues would be combined individually and jointly with the price cue to yield three two-cue designs and one four-cue design. The adding rule implies that the common plot for the two-cue and four-cue designs will exhibit parallelism. The averaging rule, by contrast, implies a crossover.

The present approach also provides information about the imputation rule itself. Applied to the data reported by Yamagishi and Hill, this analysis points to asymmetric imputation rules for price and quality, analogous to the present asymmetry for income and generosity. First, their price-only curve was nearly flat; product attractiveness failed to increase as price decreased. This implies an imputation of low (high) quality when only low (high) price is given, as was also assumed by Yamagishi and Hill. Averaging then accounts for the flat curve because the attractiveness values of the given price and the imputed quality are opposite in sign. Their quality-only curve, on the other hand, was almost the same as the curve for average quality, with both having steep slope as a function of price. This disagreed with their model, which implies unequal slopes. In the averaging model, in contrast, equal slopes would mean that the missing price information was imputed a constant value. Further work is needed to pin down the cognitive algebra of this price-quality task.

Prediction of Performance. Intuitively, the cognitive equation studied here, Gift size = Generosity x Income, seems to have the same psychological structure as the equation, Performance = Motivation x Ability. Income serves as an ability factor for making gifts; generosity may be viewed as a form of motivation. This was the reasoning in the original investigations (Anderson & Butzin, 1974; Graesser & Anderson, 1974), both of which found similar results in support of the multiplying rule. The present work confirms and extends the multiplying rule for the Gift size equation.

For the Performance equation, however, subsequent work has not always yielded the linear fan pattern, but more often parallelism or approximate parallelism (Gupta & Singh, 1981; Singh et al. 1979; Surber, 1981a). The conventional distinguishing tests with single cues of motivation or ability yielded crossover interactions that argue for averaging and against adding or multiplying. The present results, however, suggest that subjects will make imputations when they receive single cues of motivation or ability. These cues are qualitatively different, so a judgment of performance cannot logically be made with only one (see Singh et al., 1979). Hence the crossovers are not unambiguously diagnostic of the underlying cognitive algebra. To diagnose the rule underlying these fan and parallelism patterns, it is necessary to avoid the problems connected with imputations.

Anderson (1983, pp. 75-76) has verified the multiplying interpretation for the fan pattern under conditions that eliminated alternative interpretations involving differential weight averaging or imputations. Further applications of the logic and method of the present experiments 4-6 have clearly disclosed the operation of imputations in predictions of exam performance (Singh & Bhargava, 1982a) and life performance (Singh & Bhargava, 1982b). In these studies, the fan pattern was always due to the multiplying rule, whereas the parallelism pattern was due to the adding rule with adults but the averaging rule with children. When problems associated with imputations about missing information are eliminated, therefore, predictions of performance obey adding, averaging, or multiplying rules, contingent on the age and culture of the subjects as well as on the nature of the task (Bhargava, 1985; Srivastava, 1985).

Evidence for the averaging model in predictions of exam performance has also been found in one American study. Surber (1981a) manipulated reliability of information just as in the present Experiment 6 and found a converging fan pattern in the factorial plot for Motivation x Ability. More important, the effect of the reliability manipulation was in accord with the averaging rule. In this study, the possibility of imputations can be ruled out, so the evidence for the averaging rule seems unequivocal.

Two other studies by Surber (1980, 1981b), however, used the conventional distinguishing test based on single cues about motivation alone or ability alone, which leaves open the likely possibility of imputations. The fan pattern was present in some conditions in both studies, so the crossovers are ambiguous about averaging. Present methods could be applied to resolve this ambiguity. This would shed light on the plausibility of the several hypotheses of cultural difference (Singh, 1981), nature of task (Bhargava, 1985; Singh & Bhargava, 1982b; Srivastava, 1985), and difficulty of task (Surber, 1981a,b) that have been offered to account for the emergence of fan and parallelism patterns in predictions of performance.

Deserved Punishment. In Leon's (1980) developmental study, children made judgments of deserved punishment given information about the intent behind a harmful action and the amount of damage caused by the action. The factorial plot for intent and damage exhibited parallelism, and the intent-only curve did not cross over but formed part of the parallelism pattern. If only this data pattern is considered, it argues for an adding rule, Punishment = Intent + Damage.

However, imputations were evident in Leon's data. The relative elevation of the intent-only curve corresponded to a moderate, nonzero value of damage. This value must have been imputed since no damage was specified. Hence the lack of crossover does not distinguish between adding and averaging. Leon preferred the averaging interpretation, which seems reasonable in view of the overwhelming support for averaging in many tasks, but which may still require justification.

This problem may be studied using the design of Experiments 4-5. Information about intent would come from three independent sources, for example, from three playmates of the child who caused the damage. The critical test comes from the common plot of the four-cue and two-cue data. If the adding rule holds, then both sets of curves should exhibit parallelism. If Leon's averaging interpretation is correct, on the other hand, then the two-cue curve should cross over the corresponding four-cue curves.

Attractiveness of Dates. In Lampel and Anderson (1968), female subjects judged attractiveness of dates described by two personality traits and a photograph in a $\text{Trait}_1 \times \text{Trait}_2 \times$ Photograph design. These judgments showed near-parallelism in the $\text{Trait}_1 \times \text{Trait}_2$ factorial plot, whereas both Trait $\times$ Photograph plots had the linear fan form. If only this data pattern is considered, the data would follow the averaging*multiplying rule

$$\text{Attractiveness} = (\text{Trait}_1 + \text{Trait}_2) \overset{*}{x} \text{Photograph}.$$

This multiplying interpretation was considered but rejected by Lampel and Anderson. One ground for rejection was that the attractive and unattractive photographs presumably had positive and negative value, respectively. Hence the multiplying rule would predict crossovers in each Trait * Photograph plot, which was not so. An additional ground for rejection was that judgments based on the photograph alone crossed over the trait*photograph curves. On these grounds, the trait*photograph integration was interpreted as averaging, but with differential weighting for the photograph. The pattern of data indicated that less attractive photographs received greater weight, a negativity effect.

This interpretation might need reconsideration if imputations are present in this task. Following the stereotype that "what is beautiful is good" (Berscheid & Walster, 1974), it is possible that imputations about personality traits were made when only the photograph was presented. This question could be studied using the design already described.

CONCLUDING COMMENTS

In their paper on cognitive algebra of gift size, Graesser and Anderson (1974) state

> To establish an algebraic model is only a first step in the analysis of the judgment process. The model is only a surface form, and more than one underlying mechanism of integration can produce the same data pattern [p. 697].

There can hardly be any disagreement with Graesser and Anderson. In fact, the present research started with the goal to find out whether the multiplying rule or the differential-weight averaging rule would give the better account of the linear fan pattern in predictions of gift size and of performance (Anderson & Butzin, 1974). The present results show that model diagnosis can be complicated not only by "mechanisms of integration," but also by the way in which given and imputed information are processed. The operation of imputations in cognitive algebra and the resulting ambiguity in the conventional distinguishing test between rules bear upon this.

In our everyday life, we make inferences about many social variables, including motivation, ability, integrity, and sincerity. Information available for making these inferences is seldom complete. Imputations about missing information are inherent in all day-to-day attributions; they cannot simply be ignored merely because they are not known. The present series of experiments shows that imputations about missing information can be studied precisely through cognitive algebra, and that methods of the theory of information integration can provide penetrating analysis of processes underlying social cognition.

This chapter is based on a research project, Prediction and Postdiction in Social Judgments, funded by the Indian Institute of Management, Ahmedabad, India. The author thanks S. K. Aithal, Ajit K. Dalal, Meenakshi Gupta, Udai Pareek, Y. N. Pradhan, and N. K. Sharma for their cooperation in data collecting, Shivganesh Bhargava, Raj K. Kataria, Kamini Pandya, Rohini Phanse, and C. Shyam Prasad for their assistance in data analysis, and Ajay Shah and R. Usha for their assistance in preparing this report. Special thanks to Norman Anderson for his invaluable guidance in the preparation of this chapter and also for emotional support he provided during this time.

REFERENCES

Anderson, N. H. Averaging versus adding as a stimulus-combination rule in impression formation. Journal of Experimental Psychology, 1965, 70, 394-400.

Anderson, N. H. Averaging model analysis of set-size effect in impression formation. Journal of Experimental Psychology, 1967, 75, 158-165. (b)

Anderson, N. H. Integration theory and attitude change. Psychological Review, 1971, 78, 171-206.

Anderson, N. H. Cognitive algebra. In L. Berkowitz (Ed.), Advances in experimental social psychology (Vol. 7). New York: Academic Press, 1974. (b)

Anderson, N. H. Foundations of information integration theory. New York: Academic Press, 1981.

Anderson, N. H. Methods of information integration theory. New York: Academic Press, 1982.

Anderson, N. H. Schemas in Person Cognition (Tech. Rep. CHIP 118). La Jolla: Center for Human Information Processing, University of California, San Diego, September 1983.

Anderson, N. H., & Butzin, C. A. Performance = Motivation x Ability: An integration-theoretical analysis. Journal of Personality and Social Psychology, 1974, 30, 598-604.

Berscheid, E., & Walster, E. Physical attractiveness. In L. Berkowitz (Ed.), Advances in experimental social psychology (Vol. 7). New York: Academic Press, 1974.

Bhargava, S. Developmental trends in prediction of life performance. Unpublished doctoral dissertation, Gujarat University, Ahmedabad, India, 1983.

Edwards, J. D., & Ostrom, T. M. Cognitive structure of neutral attitudes. Journal of Experimental Social Psychology, 1971, 7, 36-47.

Graesser, C. C., & Anderson, N. H. Cognitive algebra of the equation: Gift size = Generosity x Income. Journal of Experimental Psychology, 1974, 103, 692-699.

Gupta, M., & Singh, R. An integration-theoretical analysis of cultural and developmental differences in attribution of performance. Developmental Psychology, 1981, 17, 816-825.

Kun, A., Parsons, J. E., & Ruble, D. N. Development of integration processes using ability and effort information to predict outcome. *Developmental Psychology*, 1974, *10*, 721‡732.

Lampel, A. K., & Anderson, N. H. Combining visual and verbal information in an impression-formation task. *Journal of Personality and Social Psychology*, 1968, *9*, 1‡6.

Leon, M. Integration of intent and consequence information in children's moral judgments. In F. Wilkening, J. Becker, & T. Trabasso (Eds.), *Information integration by children*. Hillsdale, New Jersey: Erlbaum, 1980.

Oden, G. C., & Anderson, N. H. Differential weighting in integration theory. *Journal of Experimental Psychology*, 1971, *89*, 152‡161.

Singh, R., & Bhargava, S. Cognitive algebra of exam performance: Tests of hypotheses of cultural difference, task difficulty, and imputations. Unpublished experiments. Ahmedabad, India: Indian Institute of Management, 1982. (a)

Singh, R., & Bhargava, S. Life Performance = Motivation ‡ Ability: An integration-theoretical analysis. Unpublished experiments. Ahmedabad, India: Indian Institute of Management, 1982. (b)

Singh, R. & Byrne, D. Cognitive certainty and affective neutrality as a function of stimulus homogeneity‡heterogeneity. *Psychonomic Science*, 1971, *25*, 207‡208.

Singh, R., Gupta, M., & Dalal, A. K. Cultural difference in attribution of performance: An integration-theoretical analysis. *Journal of Personality and Social Psychology*, 1979, *37*, 1342‡1351.

Srivastava, P. *An information integration analysis of developmental trends in social perception*. Unpublished doctoral dissertation, Gujarat University, Ahmedabad, India, 1983.

Surber, C. F. The development of reversible operations in judgments of ability, effort, and performance. *Child Development*, 1980, *51*, 1018‡1029.

Surber, C. F. Effects of information reliability in predicting task performance using ability and effort. *Journal of Personality and Social Psychology*, 1981, *40*, 977‡989. (a)

Surber, C. F. Necessary versus sufficient schemata: Attribution for achievement in difficult and easy tasks. *Journal of Experimental Social*

Psychology, 1981, *17*, 569-586. (b)

Yamagishi, T., & Hill, C. T. Adding versus averaging models revisited: A test of a path-analytic integration model. *Journal of Personality and Social Psychology*, 1981, *41*, 13-25.

FIGURE LEGENDS

Figure 1. Mean judgments of gift size as a function of income and generosity of stimulus persons. The five curves of the left panel are from the first initial experiment (n = 27); the five curves of the right panel are from the second initial experiment (n = 15). The solid curves are from a 4 x 4, Income x Generosity design identical to that used by Graesser and Anderson (1974); dashed curves are based on information about generosity only.

Figure 2. Mean judgment of generosity as a function of one or more pieces of information about generosity of the stimulus person. Data from Experiments 1 through 3 are shown from left to right.

Figure 3. Mean judgment of gift size as a function of two pieces of information about generosity of stimulus person. Data averaged over income factor of the Income x Generosity$_1$ x Generosity$_2$ design of Experiments 1, 2, and 3. Near-parallelism of the solid curves and crossover by the dashed curves show that two pieces of information about generosity were integrated by an averaging rule with equal weighting.

Figure 4. Two-way, Income x Generosity factorial graphs from Income x Generosity$_1$ x Generosity$_2$ design of Experiments 1 through 3. All six panels show the linear fan pattern predicted by the multiplying model.

Figure 5. Mean judgment of gift size as a function of income and generosity of stimulus persons. Data from Income x Generosity$_1$ x Generosity$_2$ design of Experiments 1, 2, and 3. The nine levels of the 3 x 3 Generosity$_1$ x Generosity$_2$ component of the three-factor design are spaced on the horizontal axis according to their functional measurement values. The dashed curves are based on two-cue, Generosity$_1$ x Generosity$_2$ design of the three respective experiments. Crossover by the dashed curves replicates findings of the two initial experiments.

Figure 6. Factorial plots of Generosity$_1$ x Income, Generosity$_2$ x Income, and Generosity$_3$ x Income effects from the main four-cue design (solid curves with open circles) and from corresponding two-cue designs (solid curves with filled circles). The dashed curve represents judgments based on information about income only. Data from Experiment 4.

Figure 7. Mean judgment of gift size as a function of income and generosity of stimulus persons. Data from the main four-cue design of Experiment 4. The levels of the three generosity cues are spaced on the horizontal axis in accord with their functional values (functional values were nearly equal for three pairs of the stimulus levels, so these three pairs were averaged to yield 9 rather than 12 functional values). The dashed curve is based on judgments from the design with three generosity cues, with income unspecified.

Figure 8. Factorial plots of Income $\overset{*}{x}$ Generosity$_1$, Income $\overset{*}{x}$ Generosity$_2$, and Income x Generosity$_3$ effects from the main four-cue design (solid curves with open circles) and from corresponding two-cue designs (solid curves with filled circles). The dashed curves are based on just one generosity cue as listed on the horizontal axis.

Figure 9. Factorial plots of three Generosity $\overset{*}{x}$ Generosity designs, Experiment 4. Solid curves with filled circles represent the three two-cue generosity designs (Designs 3-5); solid curves with open circles represent the same factorial plots for the main four-cue design. Dashed curves represent the one-cue Generosity designs with levels listed on the horizontal axis.

Figure 10. Factorial plots of Generosity $\overset{*}{x}$ Generosity designs, Experiment 4. Solid curves with filled circles represent the three two-cue generosity designs replotted from Figure 9. Solid curves with open circles represent the same factorial plots for Design 2 with all three generosity cues.

Figure 11. Factorial plot of three-way, Generosity$_1$ $\overset{*}{x}$ Generosity$_2$ $\overset{*}{x}$ Generosity$_3$ interaction effect from the three-cue design (upper panel) and from the main four-cue design (lower panel). Data from Experiment 4.

Figure 12. Factorial plots from the main design of Experiment 5.

Figure 13. Factorial plot of Generosity $\overset{*}{x}$ Income effect under each of the nine conditions of Reliability of Generosity information x Reliability of Income information. Data from Experiment 6.

Figure 14. Three-way interactions from Experiment 6. Factorial graphs of Reliability of generosity information $\overset{*}{x}$ Income x Generosity (top panel), Reliability of Income information $\overset{*}{x}$ Income $\overset{*}{x}$ Generosity (middle panel), and Reliability of income information x Reliability of generosity information $\overset{*}{x}$ Generosity (bottom panel).

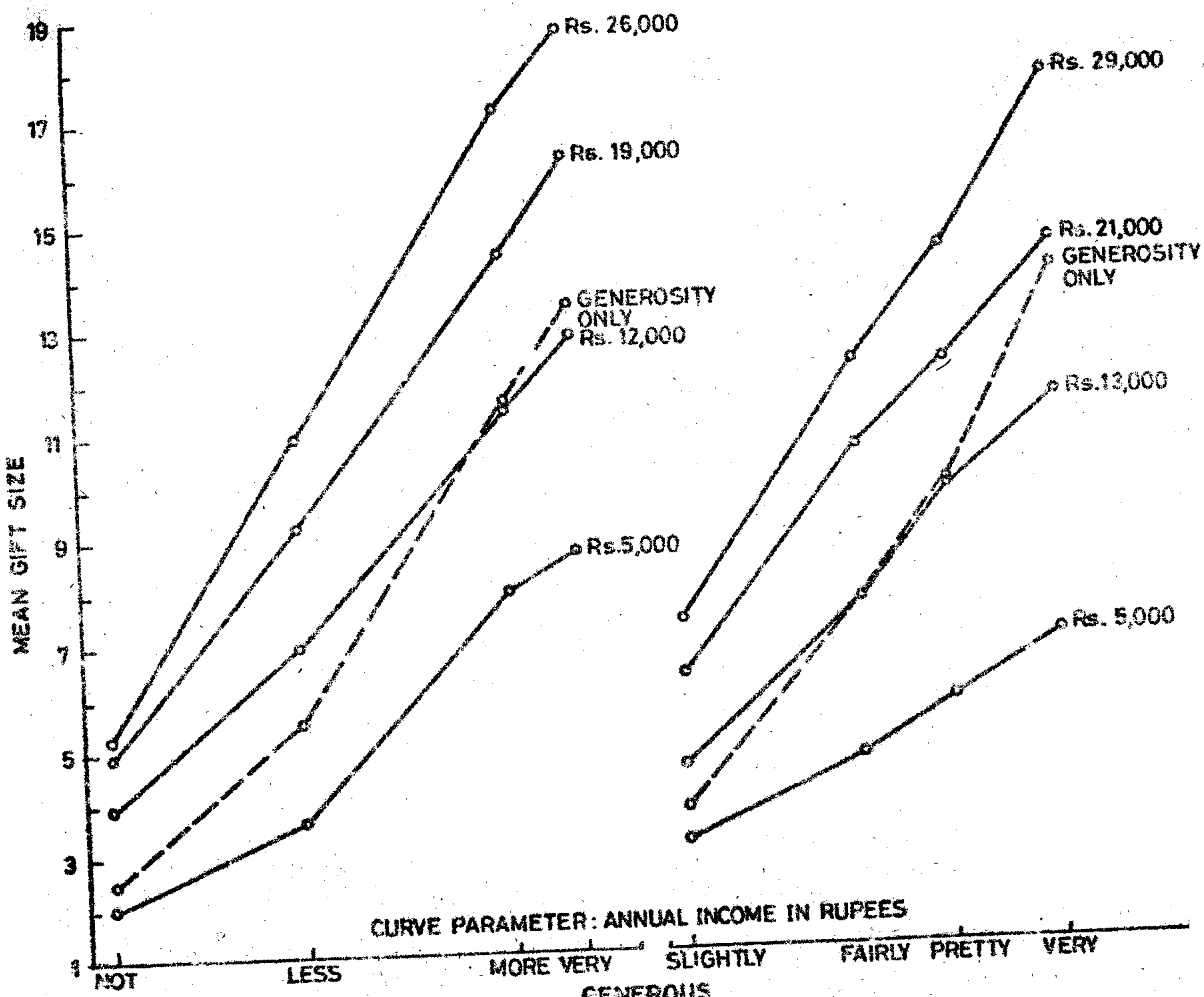
Figure 15. Profiles of four two-factor interactions, Experiment 6.

TABLE 1

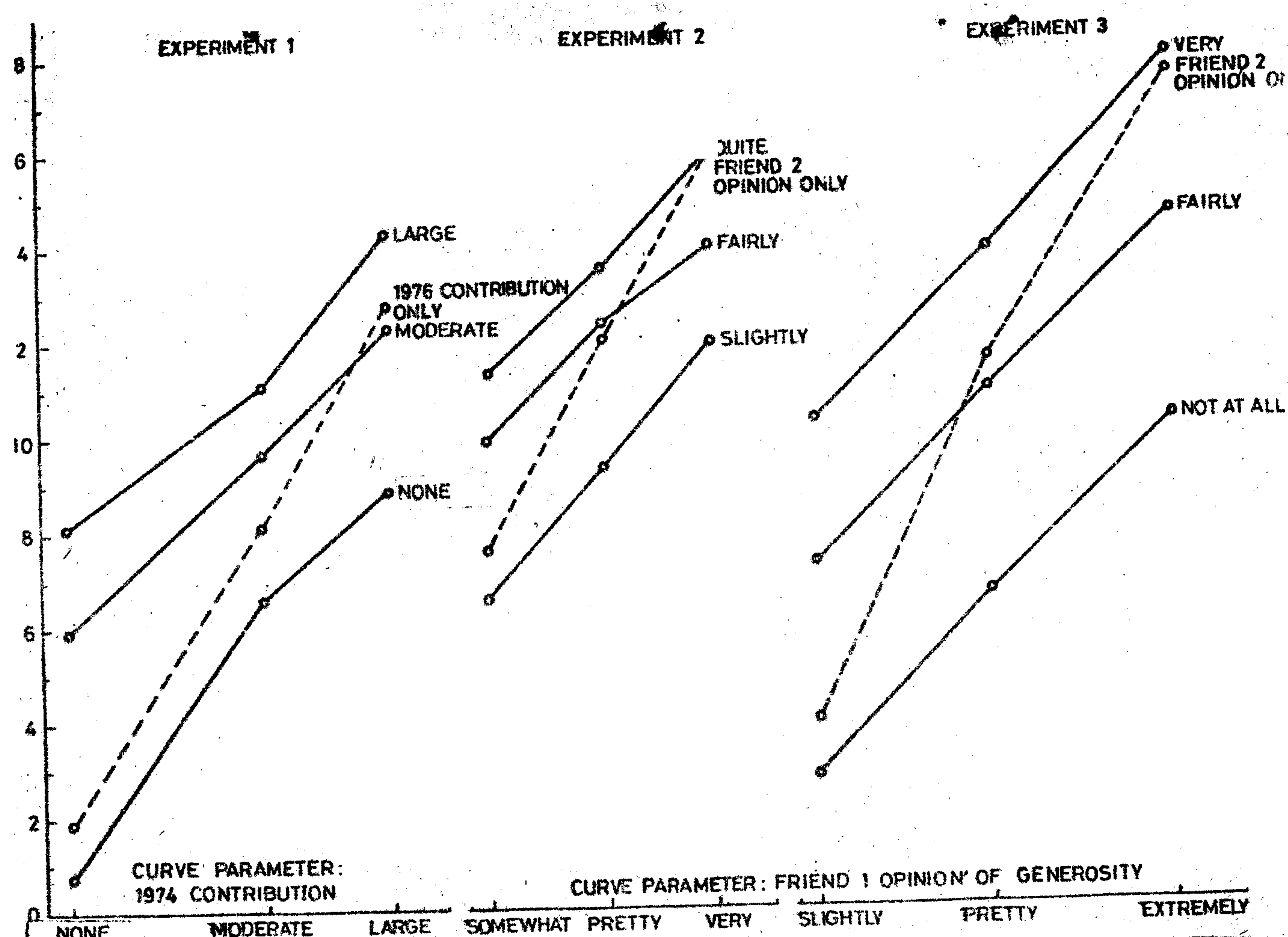
POLYLIN ANALYSIS OF GENEROSITY AND INCOME INTERACTION
EXPERIMENT 4

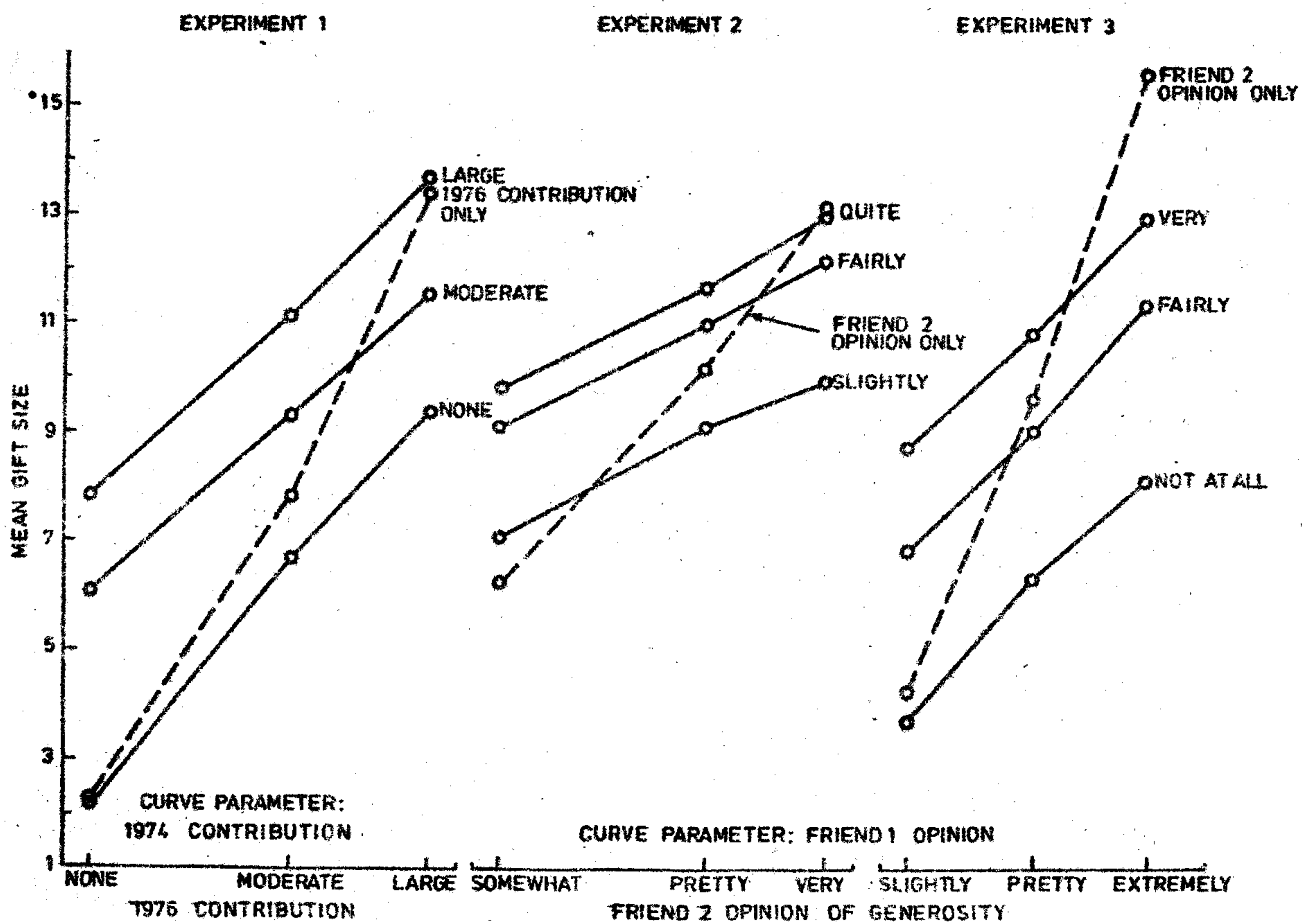
Design	POLYLIN Components			
	L × L·	L × Q	Q × L	Q × Q
Two-Cue design				
Generosity-1 × Income	37.40	.80	9.89	4.98
Generosity-2 × Income	51.11	.64	-	-
Generosity-3 × Income	47.27	.60	-	-
Four-cue design				
Generosity-1 × Income	33.93	2.46	.01	.01
Generosity-2 × Income	30.68	.63	-	-
Generosity-3 × Income	36.28	1.18	-	-
Combined designs				
Generosity-1 × Income	42.27	.78	.71	5.22
Generosity-2 × Income	49.13	.53	.76	.13
Generosity-3 × Income	34.88	1.01	1.45	.00

NOTE. Letters L and Q refer to linear and quadratic components, respectively. Each F ratio has 1 and 13 *df*. Critical F's are 4.67 and 9.07 at the .05 and .01 levels, respectively.

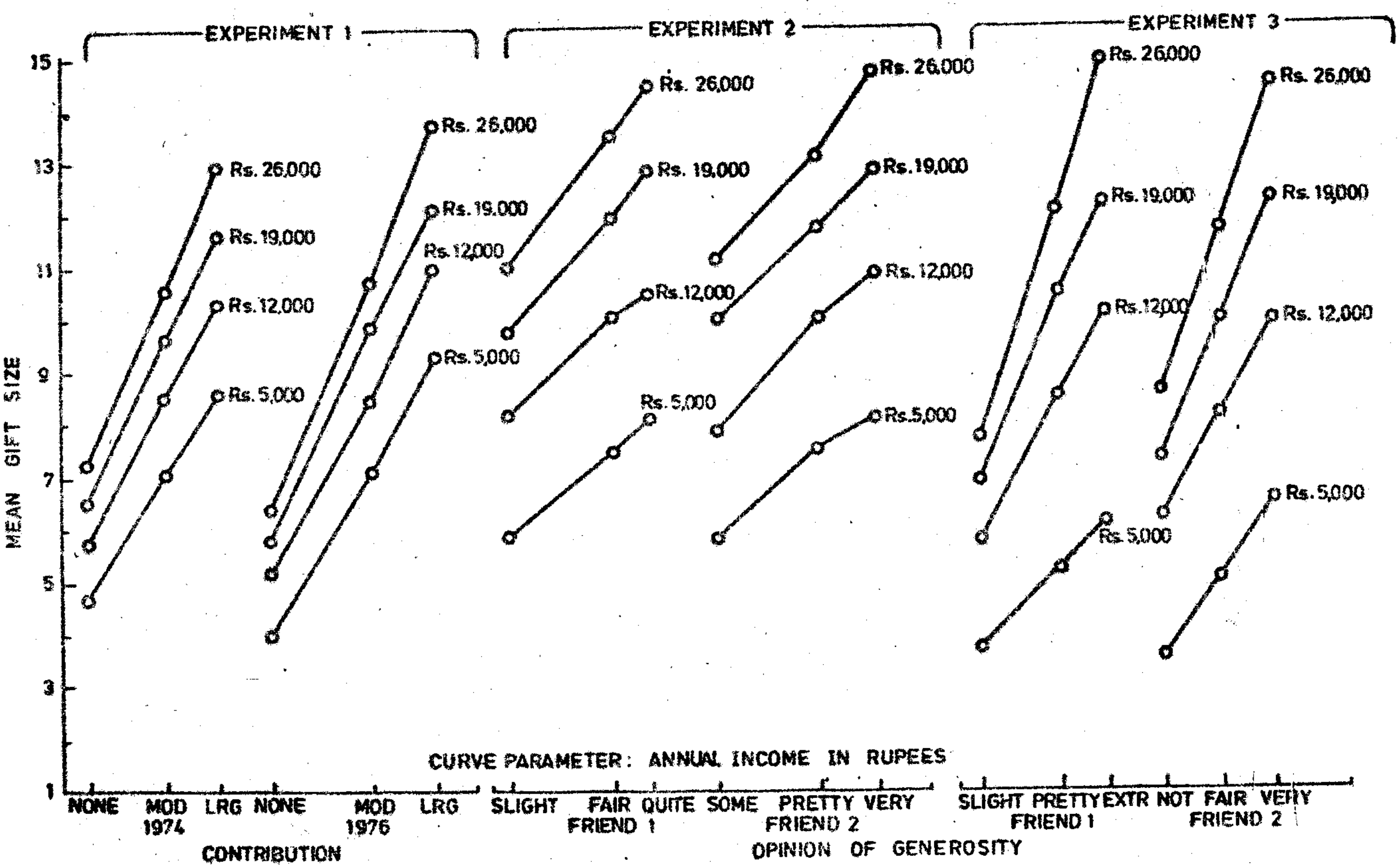


MEAN GIFT SIZE
19
17
15
13
11
9
7
5
3
1
NOT
LESS
MORE
VERY
SLIGHTLY
FAIRLY
PRETTY
VERY
Rs. 26,000
Rs. 19,000
GENEROSITY ONLY
Rs. 12,000
Rs.5,000
Rs. 29,000
Rs. 21,000
GENEROSITY ONLY
Rs.13,000
Rs. 5,000
CURVE PARAMETER: ANNUAL INCOME IN RUPEES

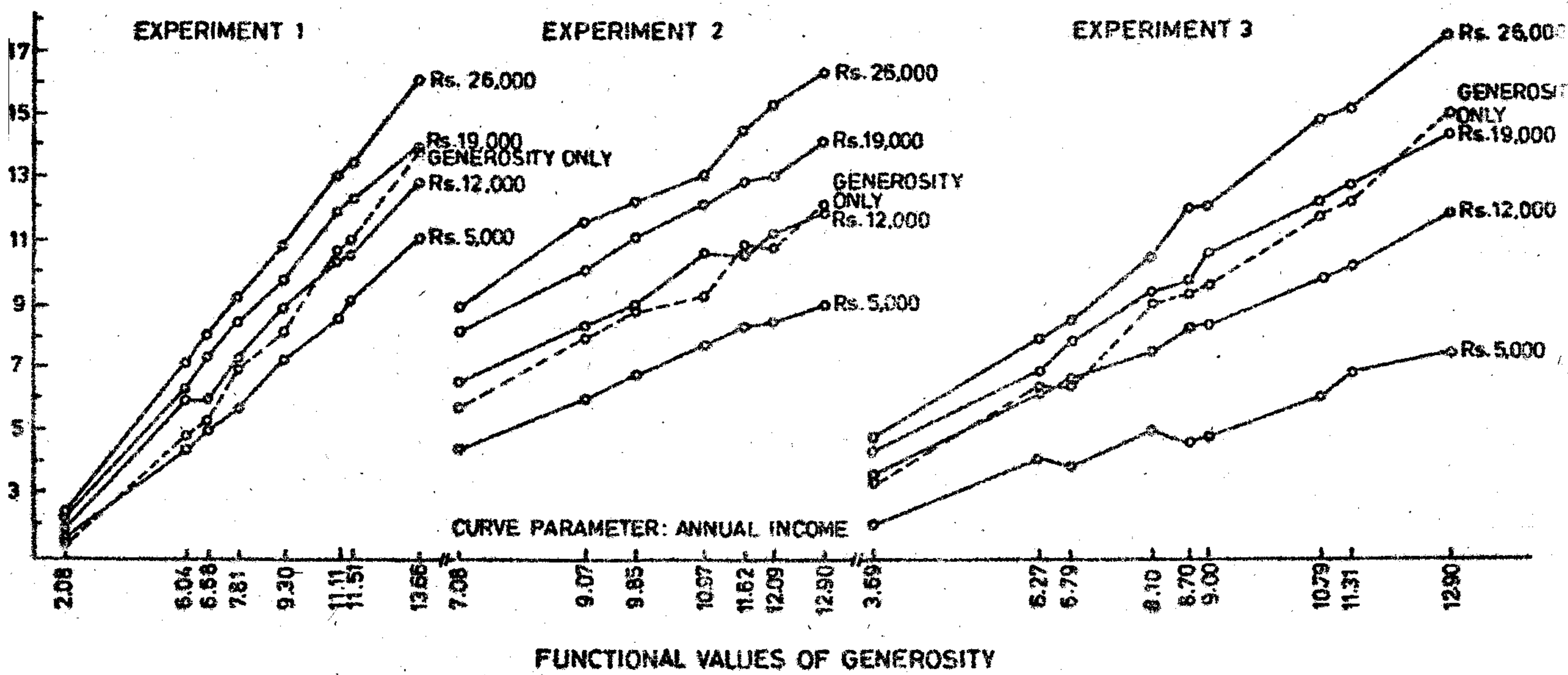

EXPERIMENT 1
EXPERIMENT 2
EXPERIMENT 3
8
6
4
2
10
8
6
4
2
0
LARGE
1976 CONTRIBUTION
ONLY
MODERATE
NONE
QUITE
FRIEND 2
OPINION ONLY
FAIRLY
SLIGHTLY
VERY
FRIEND 2
OPINION
FAIRLY
NOT AT ALL
CURVE PARAMETER:
1974 CONTRIBUTION
CURVE PARAMETER: FRIEND 1 OPINION OF GENEROSITY
NONE
MODERATE
LARGE
SOMEWHAT
PRETTY
VERY
SLIGHTLY
PRETTY
EXTREMELY

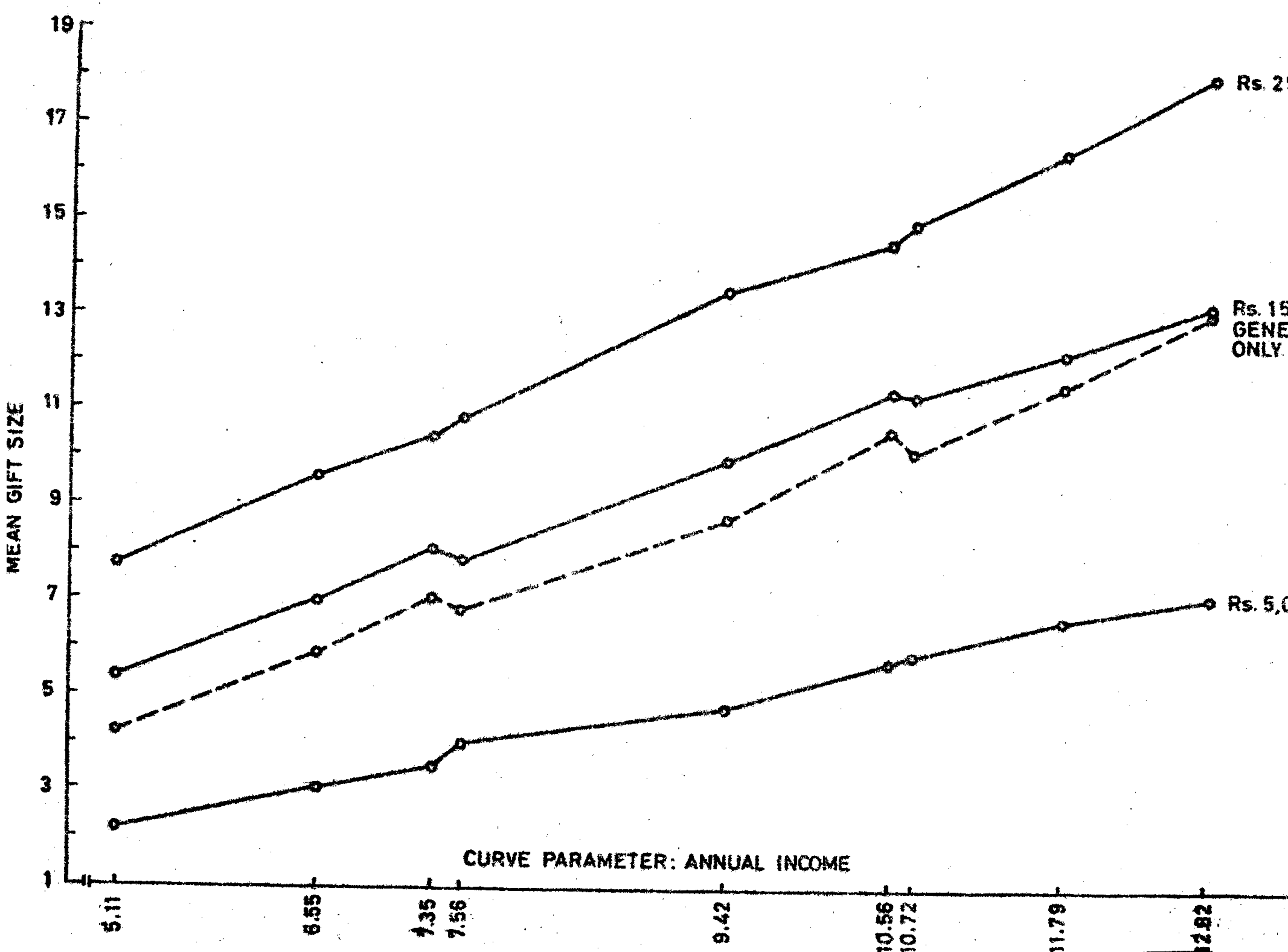


EXPERIMENT 1
EXPERIMENT 2
EXPERIMENT 3
MEAN GIFT SIZE
15
13
11
9
7
5
3
1
LARGE
1976 CONTRIBUTION ONLY
MODERATE
NONE
QUITE
FAIRLY
FRIEND 2 OPINION ONLY
SLIGHTLY
FRIEND 2 OPINION ONLY
VERY
FAIRLY
NOT AT ALL
CURVE PARAMETER: 1974 CONTRIBUTION
CURVE PARAMETER: FRIEND 1 OPINION
NONE
MODERATE
LARGE
SOMEWHAT
PRETTY
VERY
SLIGHTLY
PRETTY
EXTREMELY
1976 CONTRIBUTION
FRIEND 2 OPINION OF GENEROSITY

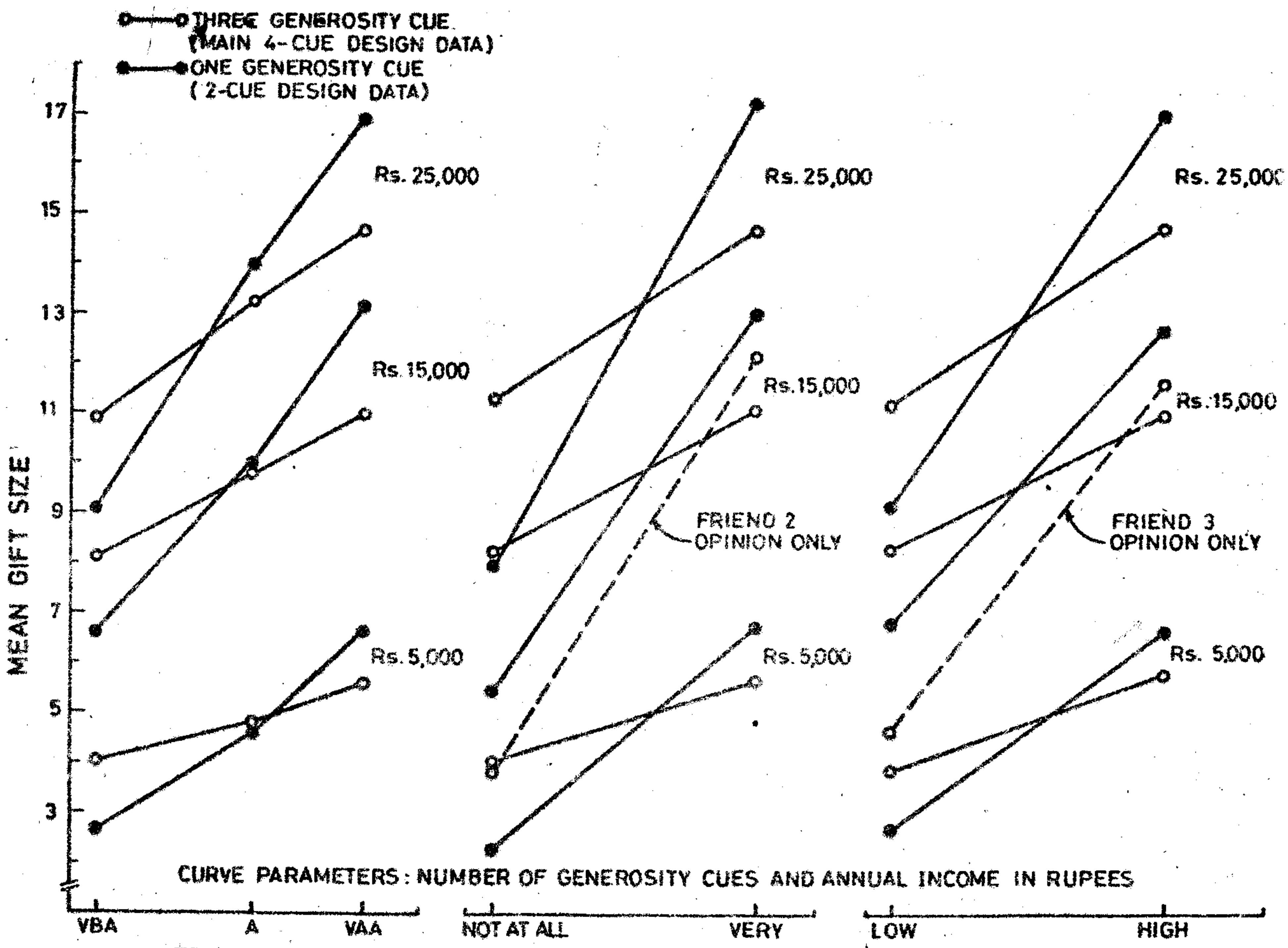


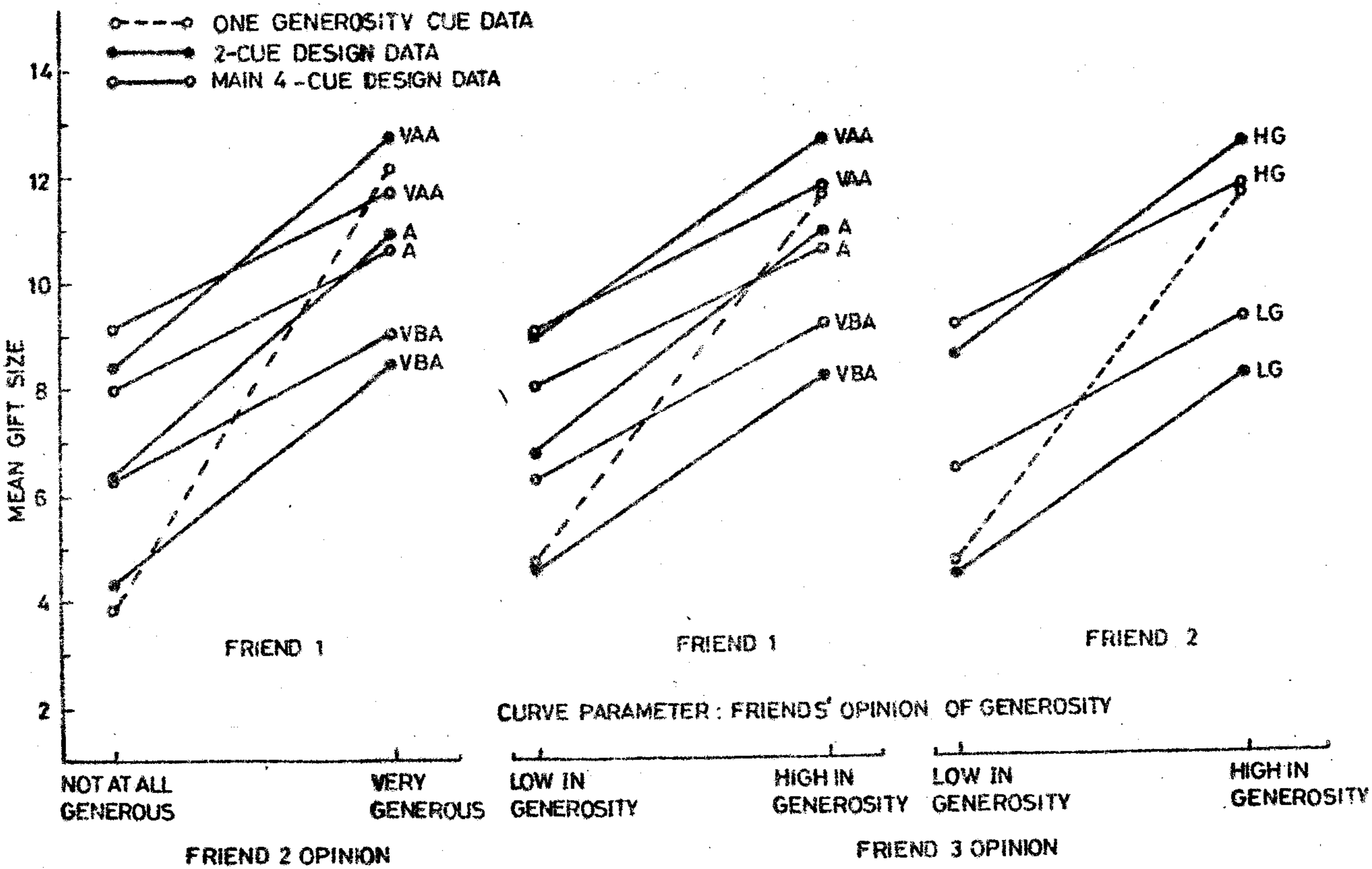
EXPERIMENT 1
EXPERIMENT 2
EXPERIMENT 3
MEAN GIFT SIZE
15
13
11
9
7
5
3
1
Rs. 26,000
Rs. 19,000
Rs. 12,000
Rs. 5,000
CURVE PARAMETER: ANNUAL INCOME IN RUPEES
NONE MOD LRG NONE MOD LRG
1974
1976
CONTRIBUTION
SLIGHT FAIR QUITE SOME PRETTY VERY
FRIEND 1
FRIEND 2
SLIGHT PRETTY EXTR NOT FAIR VERY
FRIEND 1
FRIEND 2
OPINION OF GENEROSITY

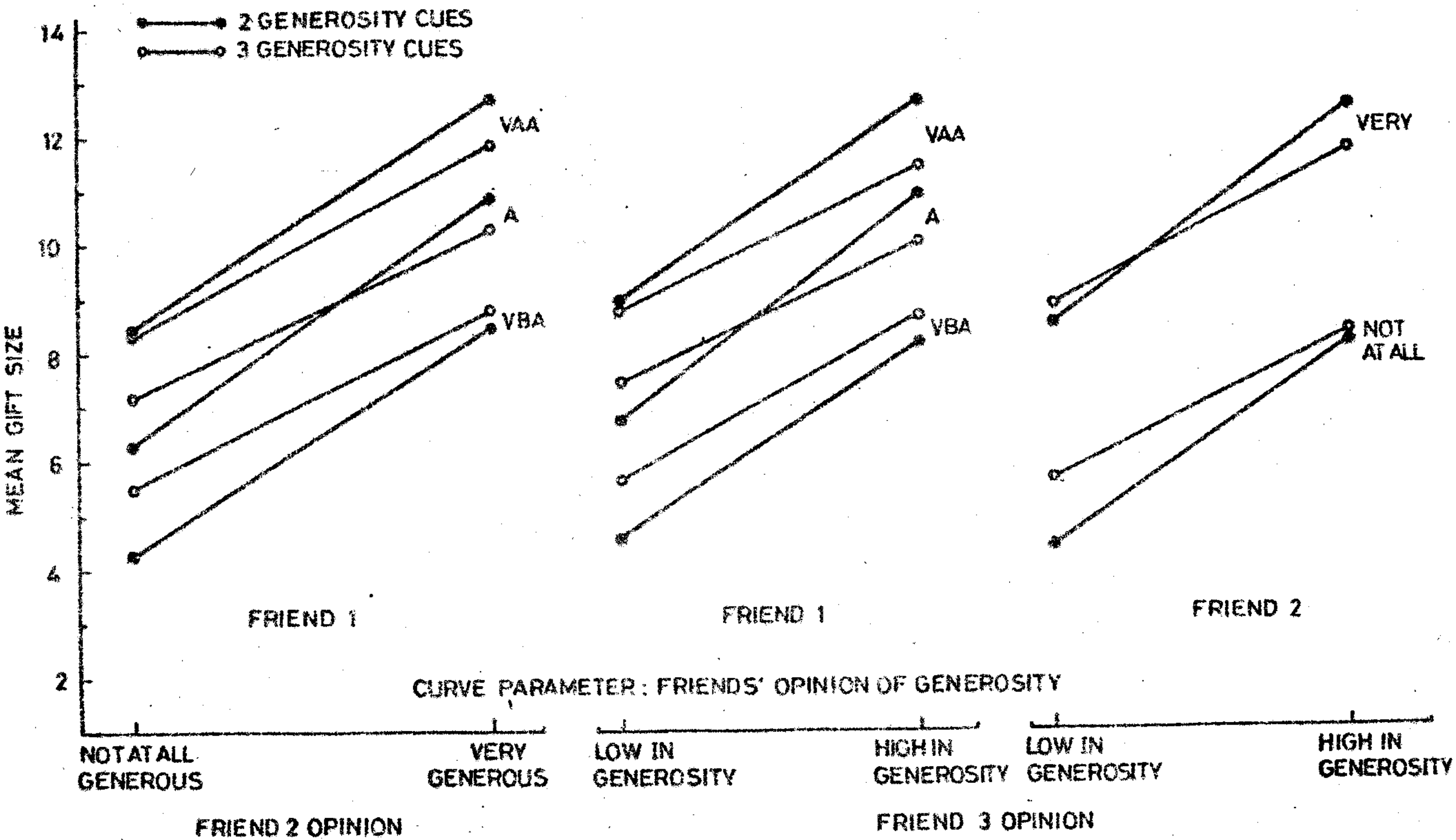

EXPERIMENT 1
EXPERIMENT 2
EXPERIMENT 3
Rs. 26,000
Rs.19,000
GENEROSITY ONLY
Rs.12,000
Rs. 5,000
Rs. 26,000
Rs.19,000
GENEROSITY ONLY
Rs. 12,000
Rs. 5,000
Rs.19,000
Rs.12,000
Rs. 5,000
17
15
13
11
9
7
5
3
2.08
6.04
6.58
7.81
9.30
11.11
11.51
13.66
7.08
9.07
9.85
10.57
11.62
12.09
12.90
3.69
6.27
6.79
8.10
8.70
9.00
10.79
11.31
12.90
CURVE PARAMETER: ANNUAL INCOME
FUNCTIONAL VALUES OF GENEROSITY



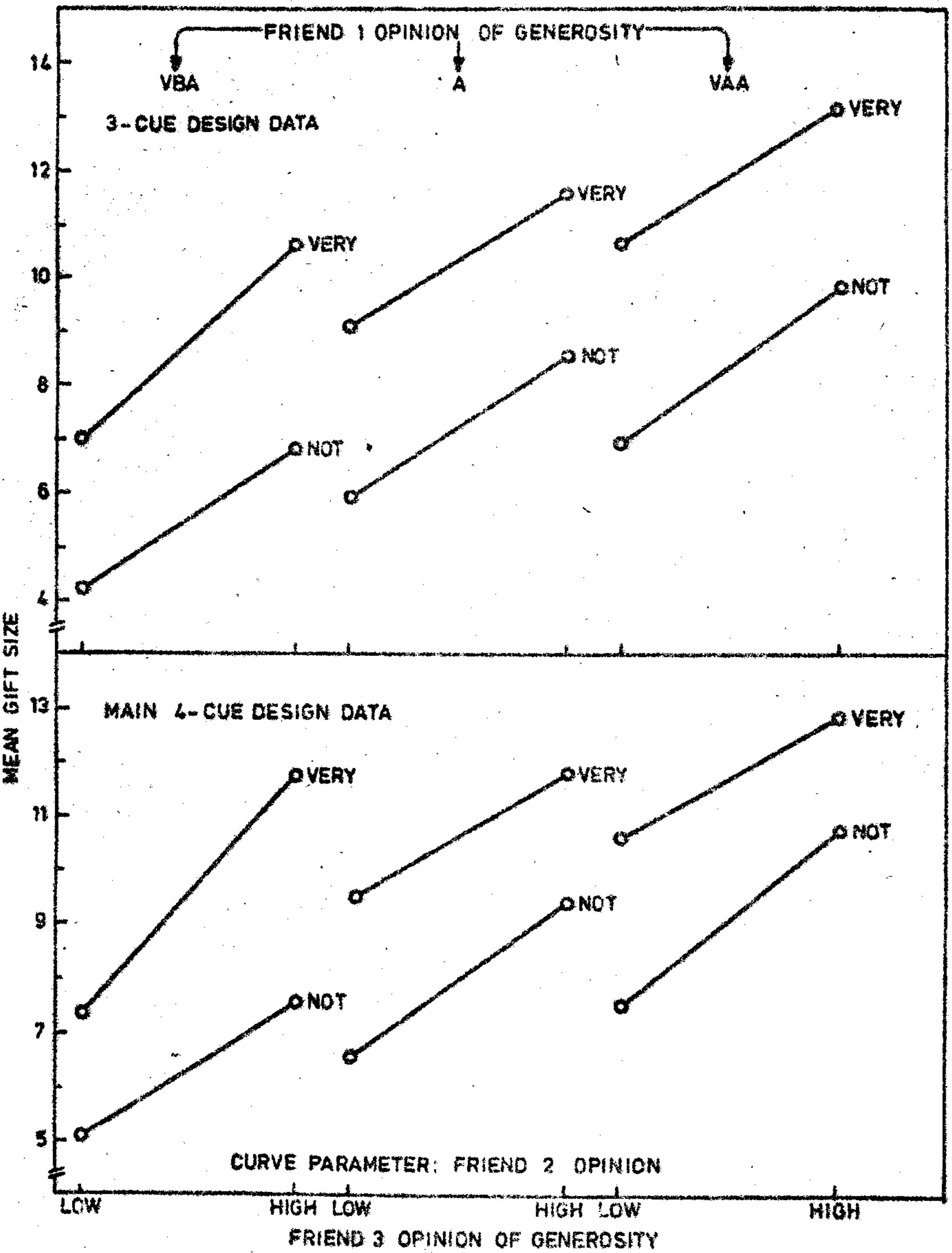
MEAN GIFT SIZE
19
17
15
13
11
9
7
5
3
1
Rs. 2
Rs. 15
GENE
ONLY
Rs. 5,0
CURVE PARAMETER: ANNUAL INCOME
5.11
6.55
7.35
7.56
9.42
10.56
10.72
11.79
12.82


THREE GENEROSITY CUE
(MAIN 4-CUE DESIGN DATA)
ONE GENEROSITY CUE
(2-CUE DESIGN DATA)
MEAN GIFT SIZE
17
15
13
11
9
7
5
3
Rs. 25,000
Rs. 15,000
Rs. 5,000
Rs. 25,000
Rs. 15,000
FRIEND 2
OPINION ONLY
Rs. 5,000
Rs. 25,000
Rs. 15,000
FRIEND 3
OPINION ONLY
Rs. 5,000
CURVE PARAMETERS: NUMBER OF GENEROSITY CUES AND ANNUAL INCOME IN RUPEES
VBA
A
VAA
FRIEND 1 OPINION
NOT AT ALL
VERY
FRIEND 2 OPINION
LOW
HIGH
FRIEND 3 OPINION
GENEROSITY

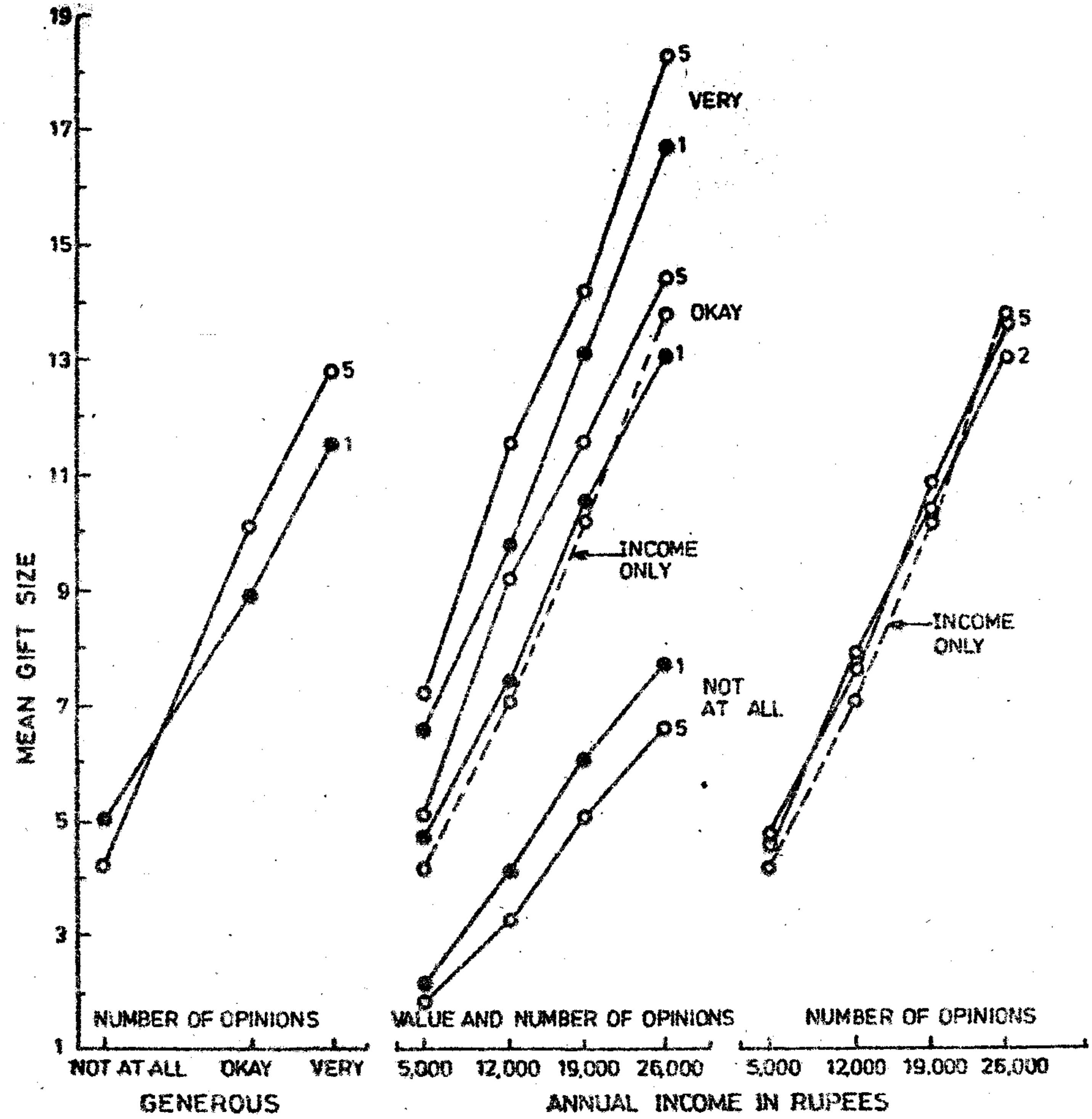

ONE GENEROSITY CUE DATA
2-CUE DESIGN DATA
MAIN 4-CUE DESIGN DATA
MEAN GIFT SIZE
14
12
10
8
6
4
2
VAA
VAA
A
A
VBA
VBA
HG
HG
LG
LG
FRIEND 1
FRIEND 1
FRIEND 2
CURVE PARAMETER : FRIENDS' OPINION OF GENEROSITY
NOT AT ALL GENEROUS
VERY GENEROUS
LOW IN GENEROSITY
HIGH IN GENEROSITY
LOW IN GENEROSITY
HIGH IN GENEROSITY
FRIEND 2 OPINION
FRIEND 3 OPINION



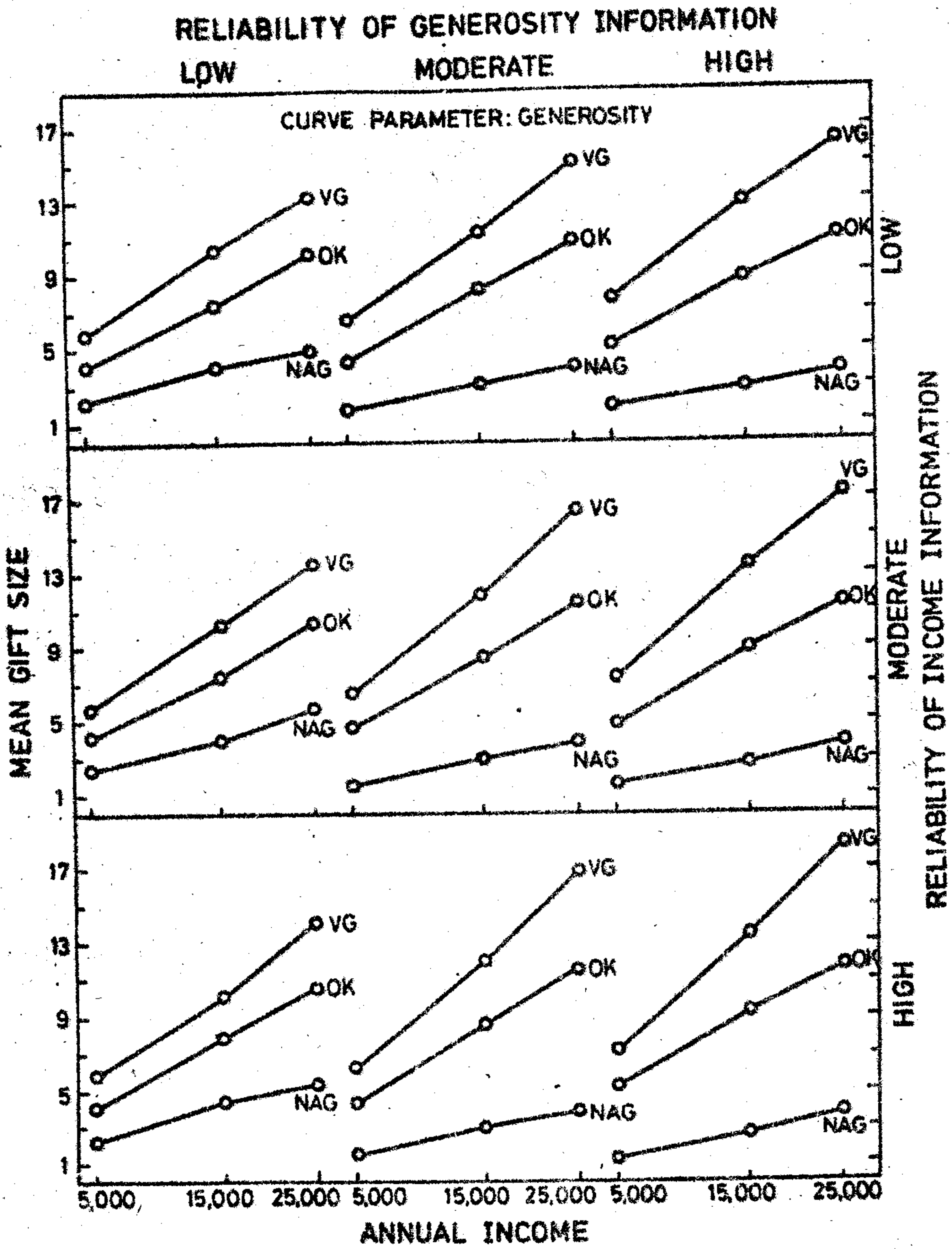
2 GENEROSITY CUES
3 GENEROSITY CUES
MEAN GIFT SIZE
14
12
10
8
6
4
2
VAA
A
VBA
VAA
A
VBA
VERY
NOT AT ALL
FRIEND 1
FRIEND 1
FRIEND 2
CURVE PARAMETER: FRIENDS' OPINION OF GENEROSITY
NOT AT ALL GENEROUS
VERY GENEROUS
LOW IN GENEROSITY
HIGH IN GENEROSITY
LOW IN GENEROSITY
HIGH IN GENEROSITY
FRIEND 2 OPINION
FRIEND 3 OPINION

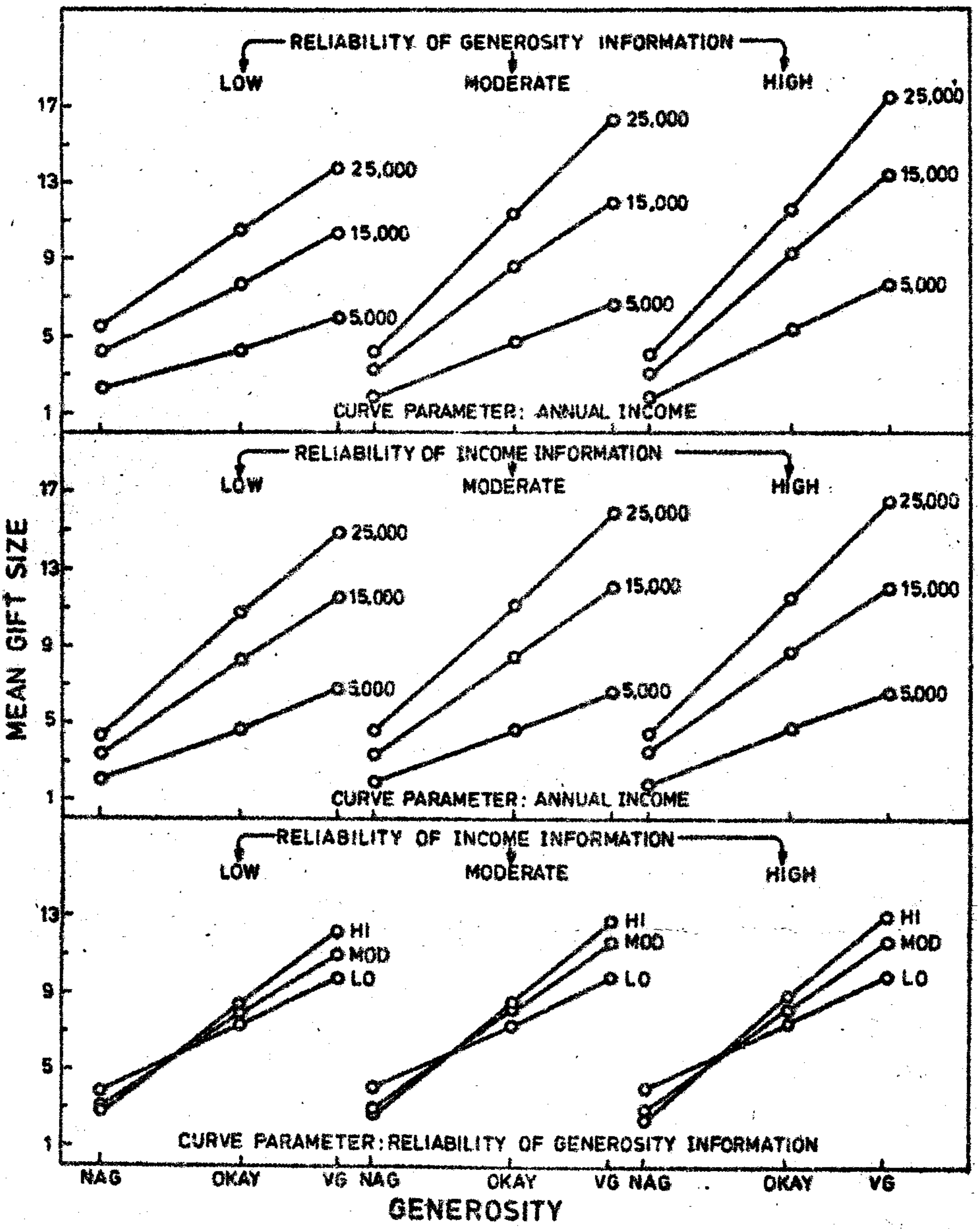


FRIEND 1 OPINION OF GENEROSITY
VBA
A
VAA
3-CUE DESIGN DATA
MAIN 4-CUE DESIGN DATA
VERY
NOT
MEAN GIFT SIZE
14
12
10
8
6
4
13
11
9
7
5
CURVE PARAMETER: FRIEND 2 OPINION
LOW
HIGH LOW
HIGH LOW
HIGH
FRIEND 3 OPINION OF GENEROSITY

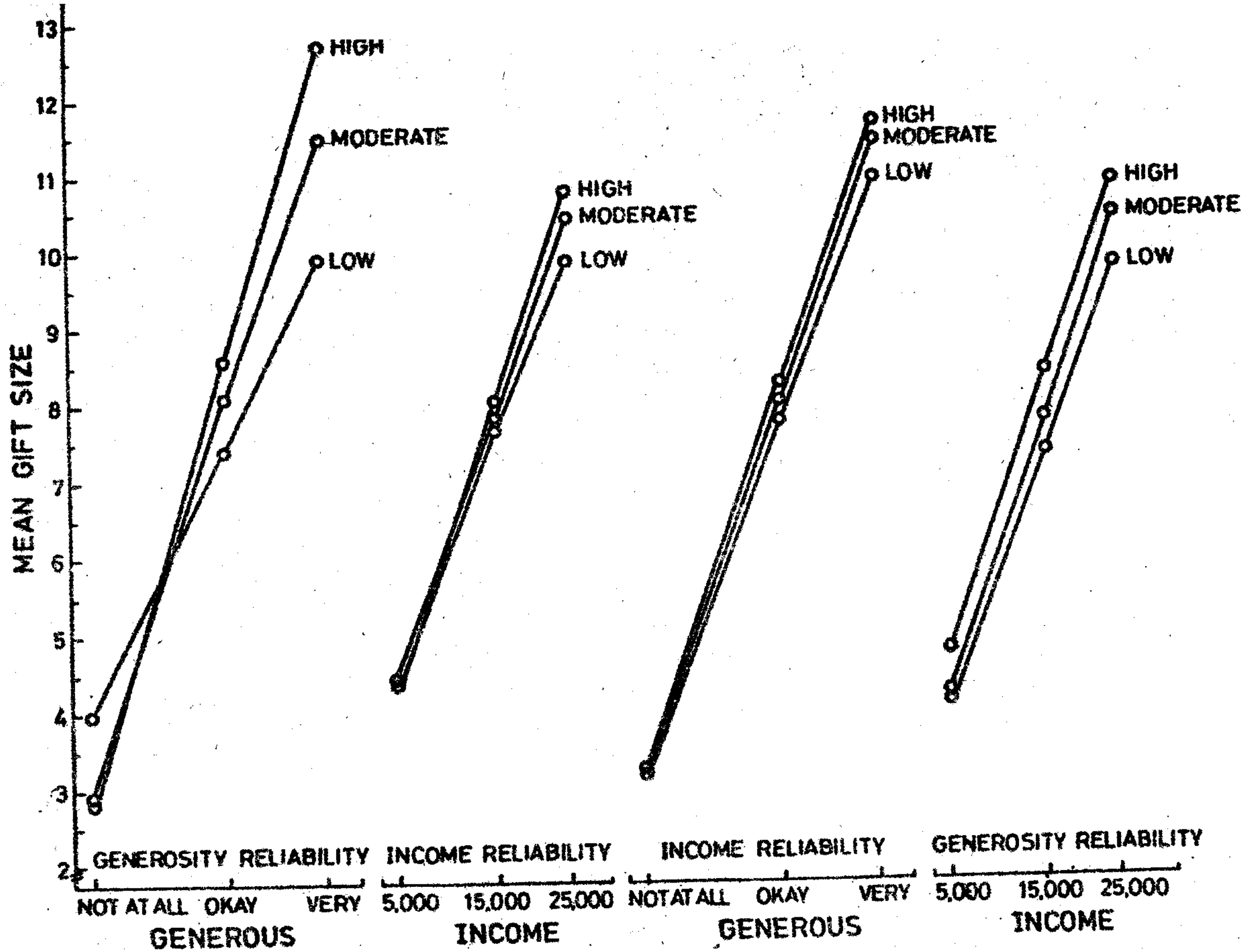


MEAN GIFT SIZE
19
17
15
13
11
9
7
5
3
1
5
1
NUMBER OF OPINIONS
NOT AT ALL
OKAY
VERY
GENEROUS
5
VERY
1
5
OKAY
1
INCOME ONLY
1
NOT AT ALL
5
VALUE AND NUMBER OF OPINIONS
5,000
12,000
19,000
26,000
5
2
INCOME ONLY
NUMBER OF OPINIONS
5,000
12,000
19,000
26,000
ANNUAL INCOME IN RUPEES


RELIABILITY OF GENEROSITY INFORMATION
LOW
MODERATE
HIGH
CURVE PARAMETER: GENEROSITY
VG
OK
NAG
MEAN GIFT SIZE
RELIABILITY OF INCOME INFORMATION
LOW
MODERATE
HIGH
1
5
9
13
17
5,000
15,000
25,000
ANNUAL INCOME



RELIABILITY OF GENEROSITY INFORMATION
LOW
MODERATE
HIGH
25,000
15,000
5,000
CURVE PARAMETER: ANNUAL INCOME
RELIABILITY OF INCOME INFORMATION
LOW
MODERATE
HIGH
CURVE PARAMETER: ANNUAL INCOME
RELIABILITY OF INCOME INFORMATION
HI
MOD
LO
CURVE PARAMETER: RELIABILITY OF GENEROSITY INFORMATION
MEAN GIFT SIZE
1
5
9
13
17
NAG
OKAY
VG
GENEROSITY


MEAN GIFT SIZE
13
12
11
10
9
8
7
6
5
4
3
2
HIGH
MODERATE
LOW
GENEROSITY RELIABILITY
NOT AT ALL
OKAY
VERY
GENEROUS
INCOME RELIABILITY
5,000
15,000
25,000
INCOME
INCOME RELIABILITY
NOT AT ALL
OKAY
VERY
GENEROUS
GENEROSITY RELIABILITY
5,000
15,000
25,000
INCOME