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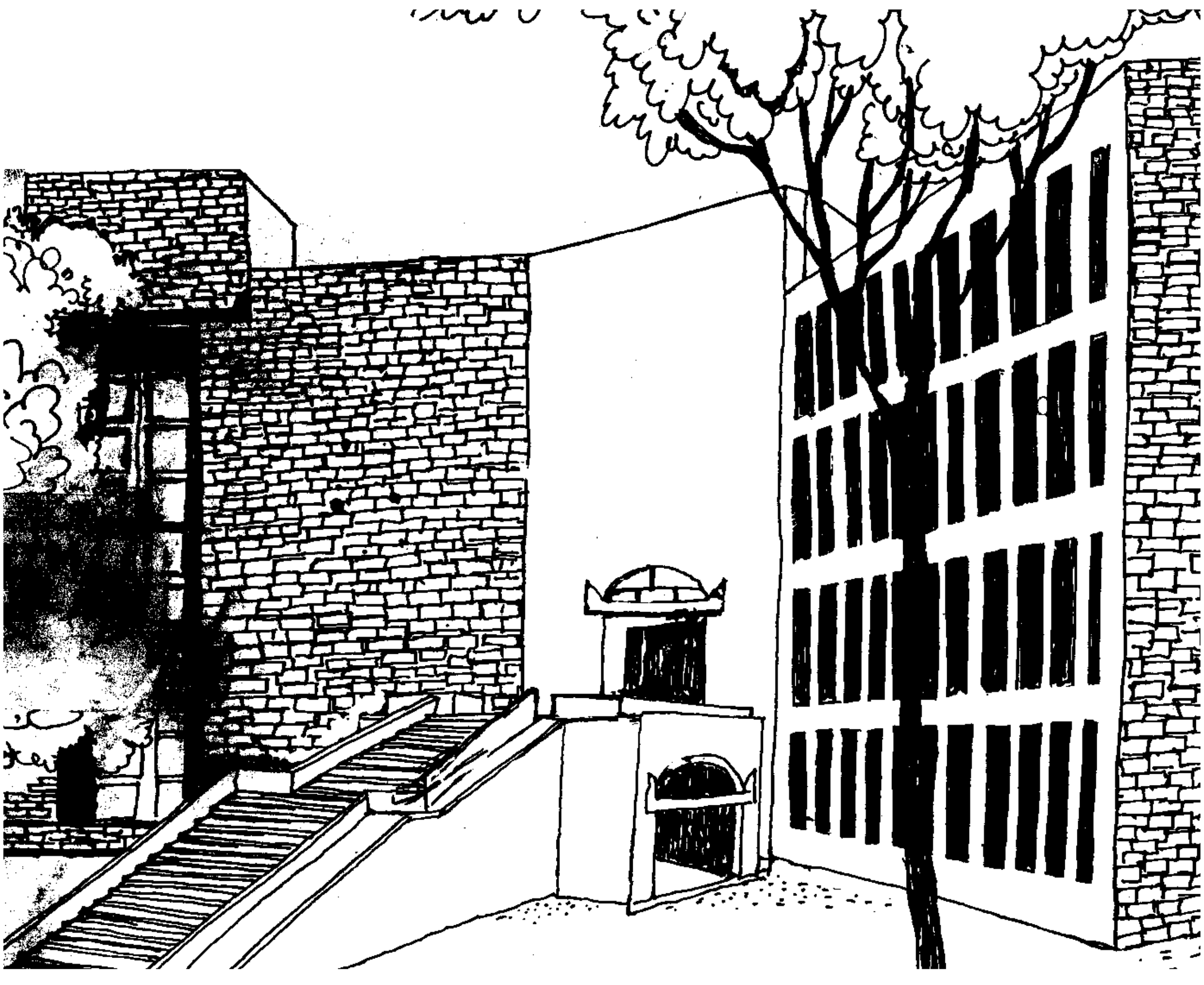

PRODUCTION FUNCTION AND OPTIMUM INPUT MIX IN FISH FARMING IN INDIA

By

G.S. Gupta

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Production Function and Optimum Input Mix in Fish Farming in India*

G.S. Gupta**
Indian Institute of Management, Ahmedabad

Introduction

Fisheries has been an important occupation in India and its significance is increasing over time. Over 50 per cent of the population eats fish in the country and the demand for this commodity has been projected to outstrip its supply at the current market conditions. Both marine and inland fisheries have to be developed to meet the ever growing demand for fish. However, fish farming, also called pisci-culture, has a special role to play in this respect. This is because it is this part of fisheries alone where human efforts play a predominant role both in terms of control on fish-species, to which Indian consumers are quite sensitive, and timing of harvest, which is very important in view of the meagre and prohibitive storage facilities and serious price fluctuations. Besides, culture fisheries provide flexible employment to otherwise sizeable under-employed or disguised employed people in the country.

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* The paper is based on the data collected for the project "All India Inland Fish Marketing Study", sponsored by the Ministry of Agriculture and Cooperation, Government of India.

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Inspite of the special role which culture fisheries plays in the economy, its contribution to employment and national income is far from satisfactory. As yet, a lot of fresh water area (estimated at 10 lakh hectares out of a total of 16 lakh hectares) remain outside this venture and the average yield rate stands at about 898 kg per hectare in contrast to some achievements of upto 6,000 kg. per hectare.[1] This is basically due to the wrong technique (input-mix) of cultivation. The present paper attempts to study the input-output relationship in fish farming through the estimation of production function and the derivation of the optimum input combination. A comparison of the actual and the optimum input-mix would indicate the extent of inefficiency in the production system and suggest the ways to improve upon the productivity and the profitability of fish farming in the country.

Production Function

The production function in fish farming was hypothesized as follows:

$$Q = f\,(P,\ L,\ S,\ SF,\ IF,\ OF) \qquad \ldots (1)$$

$$f_i > 0, \quad \text{for} \quad i = \overline{1,\ 6}$$

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[1] The pond size in hectare is defined in terms of the effective storage level (ESL), which comes to about 40% of the full storage level (FSL) and 140% of the dead storage level (DSL). The yield rate figure given here is the simple average of the yield rate obtained on 649 fish ponds surveyed by the author for the project "All India Inland Fish Marketing Study". For detail, see Fish Marketing in India : Fresh Water Culture Fisheries (Vol.3), Indian Institute of Management, Ahmedabad, 1983.

where

Q = Total production (kg)	SF = Supplementary feed (kg)
P = Pond area (Hectares)	IF = Inorganic fertilizer (kg)
L = Labour (Mandays)	OF = Organic fertilizer (kg)
S = Seed (Number of fingerlings)	f_i = Partial derivative of function 'f' with respect to the ith independent variable.

The functional form for the production function was not predecided. The Cobb-Douglas, linear and quadratic forms were tried, and the best one was selected on the basis of both economic theory and econometric principles. The data for the estimation came from a cross-section of the selected 45 districts, covering 12 states spread all over the country. These data were collected through a survey of the selected 649 ponds, about 15 ponds from each of the 45 districts. The data pertain to 1981-82. The Ordinary Least Squares' estimation results for the five alternative specifications are provided in Table 1.

It appears from the results that the Cobb-Douglas form best fits the data. This is true both on the basis of the received economic theory and the regression statistics, including t-values, R^2(coefficient of determination) $\bar{R}^2$(R^2 adjusted for degrees of freedom) and F-values. No tests for multi-collinearity and autocorrelation were conducted, for these are unlikely to be the serious problems in view of the use of cross-section data. The linear form has the wrongly signed coefficient for inorganic fertilizer and it is unacceptable, for it assumes perfect substitutability among various factors of production, which may not be true even in the operating region. Each of the three quadratic forms reported in the table yields negative marginal productivity for one or the other inputs and contains mostly insignificant coefficients. In the Cobb-Douglas form, the coefficient of organic fertilizer assumes a wrong but highly insignificant coefficient. All other inputs barring seed are the significant (at 5 per cent level and beyond) explanatory variables for fish production; seed is significant at about 11 per cent level by the usual one-tail t-test. The regression equation explains about 95% of the total variation in fish production and it is highly significant by the F-test. The returns to scale parameter (the sum of the coefficients of all input variables in the Cobb-Douglas form) comes to 1.048, which is very close to unity. Thus, there is an evidence of constant returns to scale in fish farming in the country.

Due to the wrong sign of the coefficient of organic fertilizer and the finding of near constant returns to scale, the Cobb-Douglas form of the production function was re-estimated under the constraint of constant returns to scale. For this purpose, all that was needed was to run the double-log regression on the variables redefined in terms of the per unit (hectare) of pond area instead of their absolute values.[2] The estimation

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2 This can be proved mathematically as follows:

$$Q = A\, P^{\beta_1}\, L^{\beta_2}\, S^{\beta_3}\, SF^{\beta_4}\, IF^{\beta_5}\, OF^{\beta_6} \quad \ldots\ldots\ldots \quad \text{(i)}$$

$$\text{constraint: } \beta_1 + \beta_2 + \ldots\ldots + \beta_6 = 1 \quad \ldots\ldots\ldots \quad \text{(ii)}$$

Substituting (ii), i.e. $\quad = 1 - \beta_2 - \beta_3 - \beta_4 - \beta_5 - \beta_6$ in (i) and dividing both the sides by P gives

$$\frac{Q}{P} = A \left(\frac{L}{P}\right)^{\beta_2} \left(\frac{S}{P}\right)^{\beta_3} \left(\frac{SF}{P}\right)^{\beta_4} \left(\frac{IF}{P}\right)^{\beta_5} \left(\frac{OF}{P}\right)^{\beta_6}$$

(β 's are parameters)

results are given column 1 of Table 2. Since in this form, the regression is on the yield (production per hectare of pond area), linear and quadratic forms were also estimated for yield as dependent variable to get a comparative view of various functional forms. The results on these are also included in Table 2.

Once again, the Cobb-Douglas form is found to best fit the data. All the variables now assume correct signs. The labour variable is significant at 1 per cent level, inorganic fertilizer at 5 per cent level, seed and supplementary feed at 10 per cent level, and the coefficient of . organic fertilizer is highly insignificant. Since the organic fertilizer is easily and cheaply (often freely) available, the results are not surprising. The five explanatory variables explain 71 per cent of the variation in yield, which is quite good; the F-value is significant even at 1 per cent level.

The elasticity of production (yield) with respect to labour, seed, supplementary feed, inorganic fertilizer and organic fertilizer comes to 0.324, 0.158, 0.044 0.053 and 0.001, respectively. The elasticity of total production with respect to pond area stands at 0.420 (1- .324 - .158 - .044 - .053 - .001). Thus fish production is the most sensitive to pond area and the least to organic fertilizer. The production is, ofcourse, subject to constant returns to scale, which under the assumption of constant input prices implies that the average cost is invariant over fish production.

Optimum Input-Mix

The production technology in fish farming is best described by the equation contained in column 1 of Table 2. Given that and the input and output prices, the optimum input combinations can be derived through the optimization technique. Three kinds of optimum production techniques are relevant here:

(a) Maximum yield input combination
(b) Minimum cost for a given yield input combination
(c) Maximum profit input combination

Since the selected production function is the Cobb-Douglas type, the yield is a non-decreasing function of each input and so is unbounded. Again, for this functional form, which implies a linear expansion path, though the input combinations for minimum cost and maximum profit might differ, the proportion of various inputs would be the same under both the optimizations. The cost minimizing input combination requires that the ratios of marginal physical products (MPP) to price be the same for each input. The profit maximising combination necessitates that the marginal revenue product be equal to price for each factor of production.[3]

Before deriving the optimum input combinations, it will be useful to see if the actual input-mix is the optimum on either consideration. The relevant information for these checks is provided in Table 3. In this table,

[3]There is also a sufficient condition for each optimization issue here, which, as usual, is assumed to hold good in this paper.

all the variables are in their total magnitudes (i.e. not per hectare) and the mean values refer to arithmatic (simple) means. The mean MPPs are derived by multiplying the respective constant elasticities by the ratio of total mean fish production to the mean value of the corresponding input variables. For example, mean MPP for pond area ($\overline{MPP}_p$) was obtained as

$$\overline{MPP}_p = (0.420) \frac{21747.5}{32.7}$$

$$= 271.53 \text{ kg.}$$

Column 6 in Table 3 provides a check on the least-cost input combination. The various numbers in this column are not equal and hence the current input mix is not the optimum on this count. Further, differences are rather large, varying between 0.0014 and 1.3558 and hence the mix is also far from the least cost input combination. In particular, the numbers reveal that while there is an excessive use of organic fertilizer, the use of inorganic fertilizer is very deficient. The marginal contribution of each rupee spent on organic fertilizer stands at 0.0014 kg. of fish, that on inorganic fertilizer stands at 1.3558 kg. of fish. The contribution of supplementary feed assumes the second rank (0.4862 kg) in this respect. Column 7, which is the inverse of column 6, gives the cost of fish on the basis of the marginal contribution of various inputs. Organic fertilizer leads to an estimated cost of Rs.714.29 per kg. of fish, inorganic fertilizer puts this figure at 74 paise per kg. only.

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Since the current input-mix is not the least-cost optimum, it, ofcourse, cannot be the profit maximizing one.[4] The infomation for the exact derivation is contained in Table 3, column 9. It will be seen that while the last rupee spent on organic fertilizer contributes only one paisa that on inorganic fertilizer yields Rs.10.45; the contribution of that on supplementary feed, seed, labour and pond stands at Rs.3.74, Rs.2.27, Rs.1.52 and Rs.1.06, respectively. Thus, of all the inputs, only organic fertilizer has the negative net contribution. The findings imply that the application of organic fertilizer to fish ponds must be cut rather drastically, while that of inorganic fertilizer, in particular, must be enhanced to a large extent.

To arrive at the concrete recommendations, it is imperative to derive the optimum input combinations. The least-cost combinations for the current yield rate (simple mean yield : $\frac{Q}{P}$ = 828 kg./hectare) would occur where

$$\frac{MPP_L}{P_L} = \frac{MPP_S}{P_S} = \frac{MPP_{SF}}{P_{SF}} = \frac{MPP_{IF}}{P_{IF}} = \frac{MPP_{OF}}{P_{OF}} \qquad \ldots \qquad (2)$$

(P_L = Price of input labour, and so on)

$$\text{and } \log\left(\frac{Q}{P}\right) = 3.34 + 0.324 \log\left(\frac{L}{P}\right) + 0.158 \log\left(\frac{S}{P}\right) + 0.044 \log\left(\frac{SF}{P}\right) + 0.053 \log\left(\frac{IF}{P}\right) + 0.001 \log\left(\frac{OF}{P}\right) \qquad \ldots \qquad (3)$$

The equation system (2) contains 4 equations, which along with the equation (3) can be solved for 5 variables, yielding the input combination as in column 4 of Table 4 below. Similarly, the maximum profit production process could be

4 An input combination, which is the profit maximizing, is necessarily the least-cost combination. See any good text in microeconomics, e.g. Fischer and Dornbusch (1983), : Economics, McGraw Hill.

derived by solving the following 5 equations for the 5 input variables:

$$MPP_L = \frac{P_L}{P_Q}, \quad MPP_S = \frac{P_S}{P_Q}, \quad MPP_{SF} = \frac{P_{SF}}{P_Q}$$

$$MPP_{IF} = \frac{P_{IF}}{P_Q} \text{ and } MPP_{OF} = \frac{P_{OF}}{P_Q}$$

(P_Q = price of output (fish))

The so obtained input-mix is given in Table 4, column 5. Also, included in this table are the associated production cost, yield and profit for the various input combinations.[5]

The findings in Table 4 reveal that

(a) As seen above, the current input-mix is sub-optimal.

(b) To achieve the least-cost input combination at the current yield rate, the application of inorganic fertilizer and the stockings of fingerlings must be enhanced while the use of all the remaining three inputs must be curtailed.

(c) The use of the least-cost input combination would lead to a reduction in the production cost/hectare by about 33% and to about a three-fold increase in the profit/hectare.

(d) The achievement of maximum profit/hectare calls for per hectare use of about 821 mandays of labour, stocking of 29,706 fingerlings and the application of 1485 kg. of supplementary feed, 980 kg. of inorganic fertilizer and 69 kg. of organic fertilizer. Thus, for this, the application of all inputs but organic fertilizer must increase from their present levels, though in different proportions.

(e) The practice of the maximum profit input mix would bring about a three-fold increase in the yield rate (from 828 kg to 2515 kg.) and more than a six-fold increase in the profit/hectare (from Rs.939 to Rs.6,192).

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5 The production cost per hectare for an input combination was obtained through the use of cost equation $C = P_P + (\frac{L}{P}) P_L + \ldots\ldots + (\frac{OF}{P}) P_{OF}$. The yield was obtained through the estimated yield equation (3). The profit (Π) associated with an input combination was obtained as $\Pi = R - C$, where R stands for total revenue per hectare, which equals $(\frac{Q}{P}) \times P_Q$.

Conclusion

The study reveals that the fish farming is subject to constant returns to scale. Since there is enough potential demand for fish at the current market price and there seems no paucity of inputs needed for pisci-culture at their current prices, there is a good scope for intensifying its cultivation. To this, one may hastily add that there appears an apparent shortage of fingerlings but the same is being resolved through vigorous efforts on this front.[6]

The production technology is far from the optimum. In particular, organic fertilizer is excessively applied while inorganic fertilizer is scarcely used. The application of the optimum technique of production has the potentiality of reducing the production cost by about 33% and raising the profit rate by 3 times at the current yield rate of 828 kg./ha. If the farming is attempted to achieve the maximum possible profit/hectare, the yield rate would multiply by 3 times and the profit rate by over six times. Needless to say, these findings are subject to the cross-district data for 45 districts in 12 states, each district data being a simple average of about 15 fish ponds for the year 1981-82.

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6. For detail, see Fish Marketing in India : Fish Seed Production and Marketing (Vol.II), Indian Institute of Management, Ahmedabad, 1983.

Table 1 : Production Function Estimates – I

Dependent variable : Q

Independent Variable	Functional Form									
	Cobb-Douglas		Linear		Quadratic					
	(1)		(2)		(3)		(4)		(5)	
Constant	3.41	(3.57)	-597.3	(.22)	-2634.8	(.34)	2576.3	(.67)	2561.8	(.79)
P	.499	(4.10)	122.30	(1.42)	- 43.84	(.12)	147.1	(1.31)	147.2	(1.67)
L	.335	(3.77)	2.421	(6.04)	1.33	(.33)	2.777	(2.69)	2.719	(5.0)
S	.146	(1.28)	.062	(2.28)	.021	(.48)	.021	(.49)	.021	(.56)
SF	.039	(1.76)	.099	(.21)	-2.18	(1.67)	-1.939	(1.58)	-2.00	(1.79)
IF	.034	(1.87)	-.484	(.45)	-.723	(.27)	-.379	(.16)	-.551	(.41)
OF	-.005	(.29)	.014	(.63)	.041	(.54)	.031	(.42)	.039	(.82)
PxL					.007	(.27)	-.23 D-3	(.01)		
SxSF					.11 D-4	(1.76)	.10 D-4	(1.67	.10 D-4	(1.89)
SxIF					.36 D-5	(.11)	-.59 D-5	(.19)		
SxOF					-.19 D-6	(.90)	-.16 D-6	(.80)	-.18 D-6	(1.08)
SF x IF					.35 D-3	(.71)	.25 D-3	(.56)	.28 D-3	(.69)
IFxOF					.21 D-5	(.09)	.44 D-5	(.19)		
P^2					1948.2	(.41)				
L^2					1112.4	(.23)				
Statistics										
R^2	.945		.774		.802		.797		.796	
$\bar{R}^2$	.936		.738		.709		.721		.745	
F (D.F.)	107.9	(6,38)	21.6	(6,38)	8.7	(14,30)	10.5	(12, 32)	15.3	(9,35)

Notes: (a) Numbers in parentheses are t-values.
(b) D denotes the place of decimal point, e.g. 0.11 D-4 means 0.000011

Table 2 : Pѳoduction Function Estimates II

Dependent Variable : Q/P

Independent Variable	Functional Form				
	Cobb-Douglas	Linear	Quadratic		
	(1)	(2)	(3)	(4)	(5)
Constant	3.34 (3.66)	314.8 (2.97)	-208.1 (.79)	161.4 (1.16)	-209.8 (.82)
L/P	.324 (4.15)	1.453 (4.28)	-.395 (.29)	1.733 (4.44)	-.448 (.36)
S/P	.158 (1.37)	.037 (1.31)	.056(1.44)	.072 (1.85)	.056 (1.54)
SF/P	.044 (1.32)	.040 (.25)	.236 (.43)	.469 (.86)	.112 (.65)
IF/P	.053 (1.85)	.649 (1.29)	4.09 (1.52)	6.06 (2.45)	4.22 (1.64)
OF/P	.001 (.02)	.002 (.17)	-.035 (.87)	-.049 (1.24)	-.026 (.99)
$\frac{S}{P} \times \frac{SF}{P}$			-.24 D-4 (.16)	-.74 D-4 (.47)	
$\frac{S}{P} \times \frac{IF}{P}$			-.86 D-3 (1.59)	-.001 (2.56)	-.86 D-3 (1.68)
$\frac{S}{P} \times \frac{OF}{P}$			.81 D-5 (.85)	.12 D-4(1.24)	.67 D-5 (.94)
$\frac{SF}{P} \times \frac{IF}{P}$			-.002 (.62)	-.004 (1.31)	-.002 (.66)
$\frac{IF}{P} \times \frac{OF}{P}$			.13 D-4 (.29)	.15 (.13)	
$(L/P)^2$			63.7 (1.63)		64.6 (1.74)
Statistics					
R^2	.710	.592	.694	.669	.693
$\bar{R}^2$	.673	.540	.592	.572	.614
F(D.F.)	19.1 (5,39)	11.3 (5,39)	6.8 (11,33)	6.9 (10,34)	8.8 (9,35)

Notes: (a) Numbers in parentheses are t-values.
(b) D denotes the place of decimal point, e.g. -.24 D-4 means -0.000024

Table 3: <u>Optimum Input Combination Check</u>

Production Function : $\log Q = 3.34 + .420 \log P + .324 \log L + .158 \log S + .044 \log SF + .053 \log IF + .001 \log OF$

Input	Unit of meas.	Mean[1]	Price (Average) (Rs./Unit)	Least - Cost Check: MPP (Mean) (kg/unit)	Least - Cost Check: MPP/Price (Mean) (kg/Rup.)	Least - Cost Check: Price/MPP (Mean) (Rs./kg.)	Max. Profit Check[2]: MRP = MVP[3] (Rs./unit)	Max. Profit Check[2]: MRP/Price
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
P	Hectares	[illegible]2.7	1969.3	271.53	.1379	7.25	2093.5	1.06
L	Mandays	46[illegible]7.7	7.67	1.51	.1969	5.08	11.64	1.52
S	Number	1128[illegible]9.7	.1033	.0304	.2943	3.40	.2341	2.27
SF	Kg.	3411.7	.5769	.2805	.4862	2.06	2.16	3.74
IF	kg.	820.2	1.04	1.41	1.3558	.74	10.87	10.45
OF	kg.	[illegible]6316.3	.2794	.0004	.0014	714.29	.0031	.01

1. Mean Q = 21,747.5 Kg.
2. Average fish price (P_Q) = Rs.7.71/kg.
3. $MRP = MVP = MPP \times P_Q$

Table-4 : Actual and Optimum Input Combinations and Associated Costs, Yields and Profits

(Rs. hectare)

Inputs (1)	Unit of measurement (2)	INPUT COMBINATION: Actual (3)	Least-cost (4)	Maximum Profit (5)
L	Mandays	217.4	120.3	820.6
S	No. of fingerlings	3,807.7	4,375.0	29,705.7
SF	Kg.	260.1	218.8	1,485.3
IF	Kg.	60.6	144.4	980.3
OF	Kg.	4,299.1	10.1	68.6
Prod.cost*	Rs.	5,445	3,623	13,197
Yield	Kg.	828	[illegible]	2,515
Profit	Rs.	[illegible]	2,761	6,192

*Includes pond cost of Rs.1,969.3/ha.