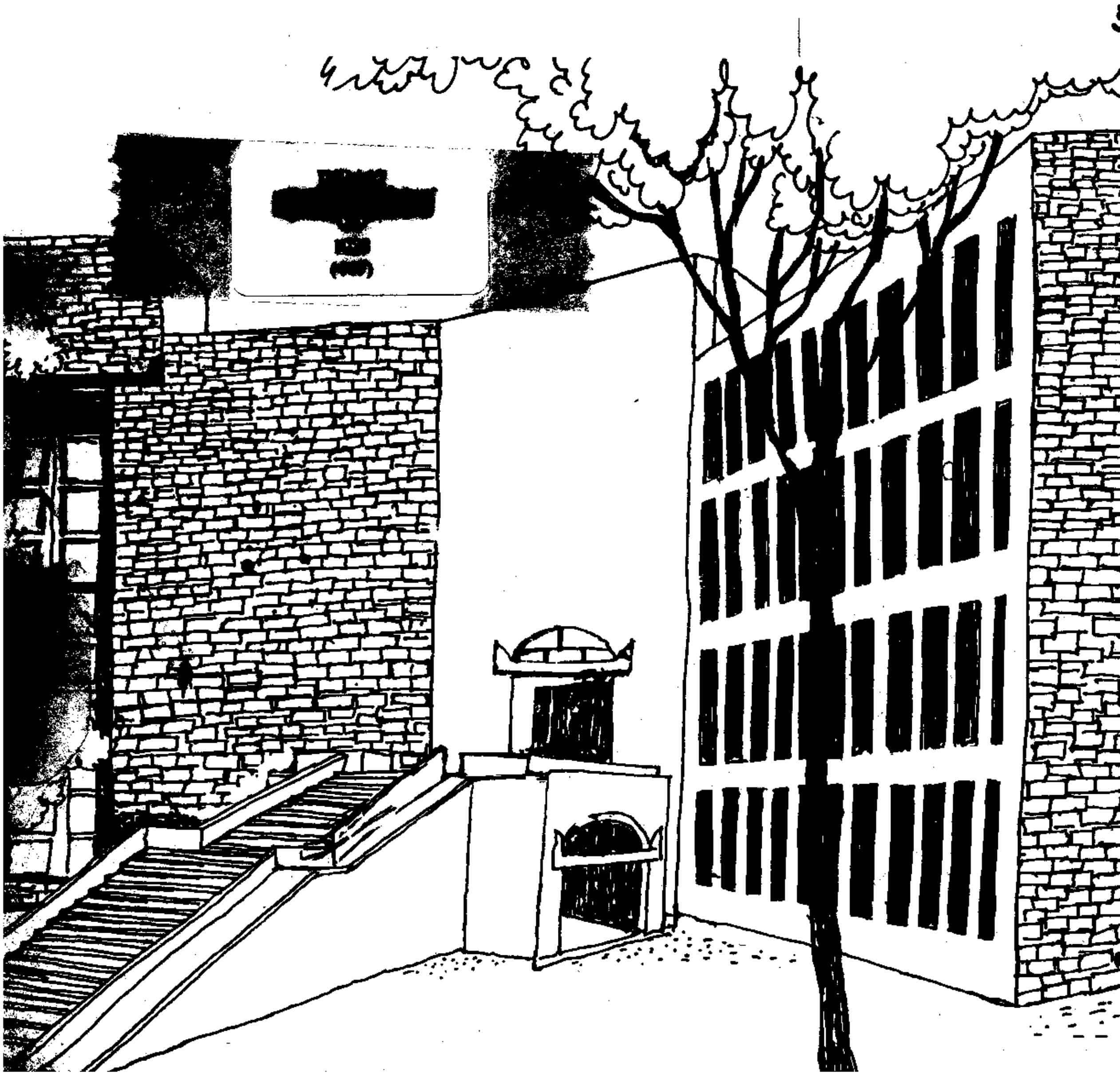
विद्याविनियोगाद्विकासः
IIM
AHMEDABAD

W. P.: 447

Working Paper



ANALYSIS OF TWO-UNIT PARALLEL REDUNDANT SYSTEM--A REVIEW

By

N. Ravichandran

W P No. 447
January 1983

The main objective of the working paper series of the IIMA is to help faculty members to test out their research findings at the pre-publication stage.

INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD-380015
INDIA

Analysis of two-unit parallel redundant system - A review*

N. Ravichandran*

Abstract

Eversince Gaver (1963) made an attempt to analyse a two unit parallel redundant system, several attempts have been made by many applied probabilists, engineers to analyse two-unit parallel redundant systems. While various authors have analysed systems with different assumptions on the failure and repair time distribution of the units, the solution to the case where both the failure and repair time distributions are arbitrary does not seem to be easy. In this article a systematic review of the methods that are available to solve the above mentioned system is made. Also, it explains why this particular case is not solvable by the methods like Semi Markov Process and regenerative process etc. Finally, it provides expressions for the measures like reliability, mean time to system failure, and availability for the most general soluable case.

Production & Quantitative Methods Area, Indian Institute of Management, Ahmedabad 380 015.

Introduction:

There are two basic configurations in the design of redundant repairable systems. One is the series structure and the other is the parallel structure. A series structure is characterised by the property that the overall system fails as and when a unit in the system fails. A parallel structure is characterised by the property that the overall system fails only when all the units in the system fail. In view of their importance, mathematical modelling of series and parallel structures in the context of redundant systems has been the objective of study by many probabilists, engineers etc. as can be evidenced by a large number of technical papers that have been published by various national and international journals. For a bibliography of the work done so far, see [9] .

The object of this contribution is to unify the attempts that have been made by many authors in analysing redundant systems with parallel configuration. This paper, starts with a simple and clear description of a redundant system with parallel structure, identifies the key factors in the system, defines various measures of interest in the analysis of such kind of systems, integrates the methods that are available/or made in analysing these systems and outlines the difficulties in analysing the most general problem in terms of the input parameters.

A parallel structure in the context of redundant systems can be completely specified by the number of units in the system, the failure and repair (if any) patterns of the units, the number of repair facilities that are available in the system as a whole, and the order in which the repair is entertained. Normally, the failure and repair durations of the units are taken to be random variables with specified probability distributions and this enables the analyser to identify the time dependent behavior of a specific system under consideration with a stochastic process. The complexity and the behaviour of the induced stochastic process is a function of the assumptions on the random variables in the system. Depending on the actual distributions of the random variables representing failure and repair patterns, Markov Process, [MP] , Semi Markov Process [SMP] , Regenerative Process[RP] and Stochastic Point Process[SP] were used by many individuals to study these redundant systems. To be precise the next section describes the system we are analysing in this article.

2. Problem Formulation:

(a) Description of the system

1. The system has two-units connected in parallel.
2. Any unit performs the system function satisfactorily.
3. When both the units are down the system is down.

4. There is a single repair facility to repair the failed units.

5. The repair policy is FIFO.

6. Each unit is 'new' after repair.

7. Switch is perfect with instantaneous switch over.

8. At time $t = o$, both the units are up.

9. The units are statistically independent.

(b) Quantities of interest

We describe in this sub-section some of the operating characteristics of the above described system which are attempted in any analysis of such systems.

1. Reliability, $R(t)$: Probability that the system is operable in $(0, t)$. If X denotes the random variable representing the time to system failure with a given initial condition then

$$R(t) = Pr\{ X > t\} .$$

2. Mean time to system failure, MTSF : $E[X] = \int_0^\infty R(t)\, dt$.

3. Availability, $A(t)$: Probability that the system is available at time t, with some specified initial condition.

4. Stationary availability, β : $\beta = \lim_{t \to \infty} A(t)$.

5. Interval reliability, $R(t, \tau)$: Probability that the system is up at time t and is operable in $(t, t+\tau)$

6. Stationary Interval Reliability: $R(\tau)$: $R(\tau)=\lim_{t\to\infty} R(t,\tau)$

Remark:

Reliability is an interval function and availability is a point function. Also $A(t) = R(t,0)$ and $R(\tau)=R(0,\tau)$.

(c) Transition diagram:

To study the behaviour of the system a clear cut description of the state of the system at any time is needed. We define the state of the system as, $(x(t),y(t))$ an ordered pair where $x(t)$ - represents the state of unit 1 and $y(t)$ represents the state of unit 2. We use the following symbols for $x(t)$ and $y(t)$.

0 - operable ; r - repairable ; qr - queueing for repair Thus a description of the form (0, r) will imply that unit 1 is operating and unit 2 is under repair and because of the parallel configuration, the system as a whole is functioning.

Figure 1 depicts the one step transition diagram of the system.

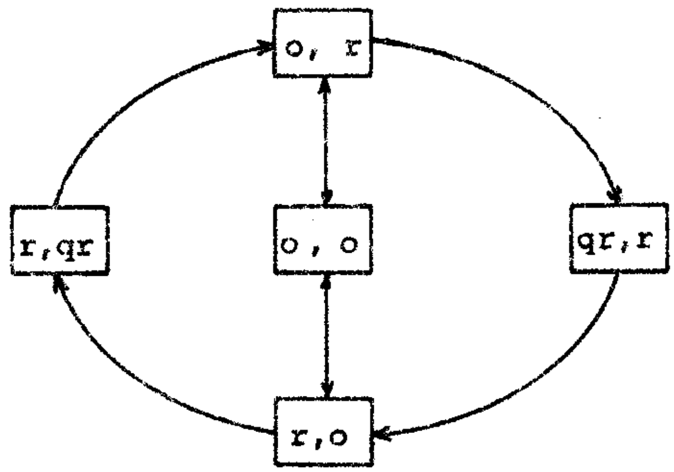


Figure 1 : One-step transition diagram

The above description of the state space of the system allows us to identify the behaviour of the system with a stochastic process as follows: Let X(t) represent the state of the system at any time t, then at any specified time point $X(t) \in \{(0,r) ; (r,0) ; (0,0) ; (qr,r) ; (r, qr)\}$, and hence a random variable with discrete value. Thus, $\{X(t), t \geq 0\}$ is a general stochastic point process [13].

(d) Notation

We use the following notation in our analysis.

$f_i(t)$ $[g_i(t)]$ - pdf of life (repair) time of unit i, $i = 1, 2$

$f^*(s)$ - Laplace transformation of f(t);

- often denoted as f(s).

* Convolution symbol

Eg: $f(t) * g(t) = \int_0^t f(u)\, g(t-u)\, du.$

$f^{(n)}(t)$ - n - fold convolution of f(t) in the interval (0,t) with itself.

Eg. $f^{(2)}(t) = f(t)*f(t)$

$F(t) = \int_0^t f(u)du$; $\bar{F}(t) = 1-F(t)$

3. Previous work:

This section describes briefly some of the important works in the analysis of two-unit parallel redundant system made in the literature so far.

Author(s)	Assumptions on distributions		Quantities obtained	Method of analysis
	Failure	Repair		
1.D.P.Gaver (1963)	Exp.	General	MTSF, β	SVT*
2.Kodama and Deguchi(1974)	Erlang	General	MTSF	SVT

3. D.G.Linton (1976)	Erlang General	General Erlang	Unit 1 Unit 2	MTSF	SVT
4. M.F.Neuts and K.S.Meier (1980)	Phase type	Phase type		MTSF, β	MP
5. R.Subramanian and N.Ravichandran (1979)	General Erlang	General General	Unit 1 Unit 2	A(t), MTSF β	RSP
6. R.Subramanian and N.Ravichandran (1980)	General Exp.	General General	Unit 1 Unit 2	R(t, τ)	RSP
7. N.Ravichandran (1981)	Erlang replaced by phase in (5)			R(t), MTSF A(t), β	RSP
8. M. Oheshi and T. Nishida (1980)	General	General		R(t), MTSF (attempted)	SVT
9. R.Subramanian and N.Ravichandran (1980)	General	General		R(t), A(t)	SVT

* Supplementary Variable Technique

Other related works on parallel redundant systems can be had from the bibliography of Osaki and Nakagawa (1976). It is however interesting to note that in all the works only the Laplace transforms of the reliability and availability of the system have been obtained except 4 and 6. Thus the most general system that has been attempted so far is the system considered in 8 and 9. In the next few sections of this paper we unify the treatment made by many of the contributions with special reference to the attributes of the stochastic process $\{X(t), t \geq 0\}$ considered in the earlier section.

4. Markovian Models:

Consider the simple case of constant failure and repair rates for the units. More precisely let $f_i(t)=\lambda_i e^{-\lambda_i t}$ and $g_i(t) = \mu_i e^{-\mu_i t}$. Designate the various possible states of the process as follows:

0 - (0,0); 1 - (0,r) ; 2 - (r,0); 3 - (r,qr); 4 - (qr,r).

Thus $\{X(t), t \geq 0\}$ as defined earlier is a stochastic process on $\{0, 1, 2, 3, 4\}$. Define $P_{ij}(t) = \Pr\{X(t) = j | X(0) = i\}$, i, j = 0, 1, 2, 3, 4 also set $P_j(t)=P_{0j}(t)$. Then by using the fact that X(t) is a Markov process, we obtain the following differential equations satisfied by $P_j(t)$'s.

$$\dot{P}_0(t) = (\lambda_1 + \lambda_2) P_0(t) + \mu_2 P_1(t) + \mu_1 P_2(t)$$

$$\dot{P}_1(t) = (\lambda_1 + \mu_2) P_1(t) + \lambda_2 P_0(t) + \mu_1 P_3(t)$$

$$\dot{P}_2(t) = -(\lambda_2 + \mu_1) P_2(t) + \lambda_1 P_0(t) + \mu_2 P_4(t)$$

$$\dot{P}_3(t) = -\mu_1 P_3(t) + \lambda_2 P_2(t)$$

$$\dot{P}_4(t) = -\mu_2 P_4(t) + \lambda_1 P_4(t)$$

with $P_i(0) = \delta_i 0 = 1$ if $i = 0$

$= 0$ otherwise

The solution of the above system of equations can be in principle, obtained by finding the spectral representation of the co-efficient matrix, or by using any of the numerical methods that are available to integrate these kind of simple differential equations. Of special interest is the stationary distribution of X(t); which we present here corresponding to the case $\lambda_i = \lambda$ and $\mu_i = \mu$.

Let $\pi_i = \lim_{t \to \infty} P_i(t)$, we have after resetting

$0 - (0,0)$; $\{ \begin{matrix} (0,r) \\ (r,0) \end{matrix} \} - 1$ and $\{ \begin{matrix} (r,qr) \\ (qr,r) \end{matrix} \} - 2$

$$\pi_1 = \frac{2\lambda}{\mu} \pi_o$$

$$\pi_2 = \frac{\lambda}{\mu} \left(\frac{2\lambda}{\mu} \right) \pi_o$$

$$\pi_0 = \left[1 + \frac{2\lambda}{\mu} \left(1 + \frac{\lambda}{\mu}\right) \right]^{-1}$$

The stationary distribution of $R(t)$, corresponding to distinct λ_i's and μ_i's are available in Neuts and Meiser [8] as a special case.

It is clear that the availability of the system conditioned by $X(0) = 0$ at the time origin denoted as $A_0(t)$ is given by

$$A_0(t) = P_0(t) + P_1(t) \quad \text{and}$$

$$\beta = \pi_0 + \pi_1(t)$$

The reliability of the system can be studied by exactly a similar procedure, by restricting the movements to those states which corresponds to a system failure. For this purpose it is useful to define,

$q_j(t) = Pr\{X(t) = j, \text{ No system down in}(0,t) | X(0)=0\} j \varepsilon \{1,2,3\}$

The $q_j(t)$'s satisfy the Chapman - Kolmogorov equations and the MTSF is given by

$$\text{MTSF} = \frac{(\lambda_2 + \mu_1)}{\lambda_2\mu_1 + \lambda_2(\lambda_1 + \lambda_2)}$$

Setting $\lambda_i = \lambda$ and $\mu_i = \mu$

$$\text{MTSF} = \frac{\lambda+\mu}{\lambda\ (\mu+{}^2\lambda)}$$

The measures interval reliability and stationary interval reliability can be obtained by using the following relations.

$$R(t,\tau) = \sum_{i \in \text{up states}} P_i(t)\, R_i(\tau)$$

$$SIR = \lim_{t \to \infty} R(t,\tau) = \sum_{i \in \text{up states}} \pi_i R_i(\tau)$$

It is possible to extend the analysis to the ease where the failure and repair durations are Erlang distributions. In this case as it is well known, it is possible to suitably redefine the state space of the stochastic process $\{X(t),\ t \geq 0\}$ so that $X(t)$ is Markovian. Then the entire procedure outlined earlier works and all the operating characteristics can be obtained.

It is also possible to do the same kind of analysis, even when the failure and repair durations are far more general, viz. of phase type, introduced by Neuts [7]. The introduction of phase type distribution is as follows: An Erlang distribution can be conceived as a Markov chain with only forward movement whenever there is a transition. Thus by Markov chain theory this Markov chain stays in a state for a random time whose distribution is negative exponential. With this understanding Erlang distribution can be viewed as the distribution of the random variable representing the time till absorption in to a state in a continuous time parameter Markov chain. If we relax the assumption of forward movement of the underlying Markov chain and consider the random variable representing the time till absorption in a particular state we get, what is known as a phase type random variable. For, further properties and definitions see [7]. The advantage of this class of distributions is that it has all the operational properties of Erlang, and hence the resulting process is Markov, which makes a detailed analysis of the system feasible as demonstrated in [8].

To summarise, what has been said in this section, the following few steps can be followed to obtain the operating characteristics of a system when the failure and repair time durations are all random variable of either exponential or Erlangian or phase type. The following procedure does not change even when the number of units in the system is large say $i(>2)$.

1. Describe the state space of the stochastic process and identify the possible transitionsamong various states.

2. Check whether the process is Markovian. If not make a suitable modification in the state description of the process so that the resulting process is always Markovian.

3. Write down the generator for this Markov process.

4. Using the forward or backward Kolmogorov equations the set of differential equations necessary to determine the distribution of X(t) for any t can be written. Choose an appropriate numerical method to get the actual result. Now, the availability of the system can be calculated.

5. The stationary availability of the system can be obtained by solving a system of simultaneous equations resulting from step 4. An efficient procedure to solve them can be developed as has been demonstrated by Neuts [8].

6. Block certain states which correspond to system down and go through steps 4 and 5 to obtain the reliability and MTSF.

5. Non-Markovian Models:

This section will describe certain models which does not result a Markovian stochastic process, still amenable for solution. Of course, the complexity of the stochastic process $\{X(t), t\geq 0\}$ will depend on how much of generality we would like to have on the failure and repair time distributions. We now consider the system where

$$f_i(t) = \lambda e^{-\lambda t} \quad \text{and } g_i(t) \text{ is arbitrary, } i = 1, 2$$

To analyse the system we start by defining the following events.

E_0 - Both the units are up

E_1 - One unit just down and the other unit up

Let $A_i(t)$ - denote the availability of the system conditioned by an E_i event, $i = 0, 1$.

The quantities $A_i(t)$ satisfy the following equations.

$$A_0(t) = e^{-2\lambda t} + 2\lambda e^{-2\lambda t} * A_1(t) \quad \text{and}$$

$$A_1(t) = \bar{G}(t)\, e^{-2\lambda t} + [g(t)\, e^{-\lambda t}] * A_0(t) + [g(t)\, (1-e^{-\lambda t})] * A_1(t)$$

On solving

$$A_o^*(s) = \frac{M + 2\lambda \bar{G}\ (s+\lambda)}{(s + 2\lambda)\ M-2\lambda g(s+\lambda)}$$

where $M = 1-g(s) + g(s+\lambda)$

and $\beta = \lim_{s \to o} s\ A_o(s) = \frac{g(\lambda) + 2\lambda \bar{G}(\lambda)}{2\lambda E[g] + g(\lambda)}$

Remarks:

1. The above result was obtained by Gaver (1963) by a rather involved analysis by using the supplementary variable technique.
2. On setting $g(\lambda) = \frac{\mu}{\lambda + \mu}$, we get the results obtained in the previous section.

It is worth while to observe more closely the stochastic process$\{X(t)\ t \geq 0\}$. It is interesting to note that a physical meaning can be attributed to $X(t)$ for any time t, Actually $X(t)$ represents the number of failed units in the system at any time t and $X(t)$ takes values on$\{0, 1, 2\}$. The next question we would like to answer is about the characteristics of $\{X(t), t \geq 0\}$. Obviously $X(t)$ is not a Markov process because of the assumption of non-constant repair rates of the units. Let us examine whether it is a Semi Markov process : For this purpose consider

$$E_i = \lim_{\Delta \to 0} \{X(t) = i = X(t-\Delta),$$

The events E_i represent the entry to state i, and the process $\{X(t)\ t \geq 0\}$ will be SMP if all such entries viz. the E_i events are regenerative [13]. Unfortunately, E_1 is non-regenerative and hence this process is not a SMP However, if one is interested in the restricted behaviour of X(t) for the reliability analysis then $\{X(t)\ t \geq 0\}$ is a SMP.

Some useful results:

Before proceeding with the operating characteristics of some more complicated systems, we digress in this section a little to study the behaviour of one of the units when the other unit is continuously operating. Let f(.) (g(.)) be the pdf of the failure and repair time durations of this unit. It is clear that, in a failure time interval of one unit, the behaviour of the other unit which we are interested in studying, is an alternating renewal process [3] and the quantities of interest are the following functions: Let S(t) denote the state of the unit at any time t. Then $\{S(t),\ t \geq 0\}$ is an alternating renewal process.

$$P_{ij}(t) = Pr\{S(t) = j \mid S(0) = i \neq S(0-)\}\ i, j = 0, r$$

We have

$$P_{oo}(t) = \bar{F}(t) + h(t)*\bar{F}(t)$$

$P_{or}(t) = 1 - P_{oo}(t)$

$P_{ro}(t) = g(t) * P_{oo}(t)$

$P_{rr}(t) = \bar{G}(t) + g(t) * P_{oo}(t)$

where

$$h(t) = \sum_{n=1}^{\infty} [f(t) * g(t)]^{(n)}$$

Often what is need in the analysis of the systems we are discussing in this paper, is the time spent in a particular state at an arbitrary time point. These situations are taken care of by means of the functions defined below.

$P_{ir}(t, x)dx = Pr\{S(t) = r,$ time spent in the state during its last visit to this state lies in $(x, x + dx) \mid S(0)=i= S(0-)\}$

and these functions are given by

$P_{or}(t, x) = f(t-x)\ \bar{G}(x) + [h\ (t-x) * f(t-x)]\ \bar{G}(x)$

$P_{rr}(t, x) = \bar{G}(t)\delta\ (t-x) + h\ (t-x)\ \bar{G}(x)$

These results are extensively used in the analysis of various kinds of redundant systems, see [11].

7. A General System:

Consider the system with the following specific form of the failure and repair pdf's.

$f_1(t)$ general, $f_2(t) = \lambda e^{-\lambda t}, \lambda \geq 0$; $g_i(t) = \mu_i e^{-\mu_i t}$, We indicate how the operating characteristics of this system can be obtained by using the following regenerative events.

Event symbol	Unit 1	Unit 2
E_{oo}	Just up	Up
E_{ro}	Just down	Up
E_{or}	Just up	Just down
$E_{qr,\ r}$	Queueing for repair	Under repair

The availability equations conditioned by the above events are given below:

$$A_{oo}(t) = \bar{F}_1(t) + [f_1(t)\ P_{oo}(t)] * A_{ro}(t)$$
$$+ [f_1(t)\ P_{or}(t)] * A_{qrr}(t)$$

The derivation of the above equation is as follows:

The system could be available at time t by either of the following two mutually exclusive and exhaustive cases.

(i) There is no failure in $(0,\ t)$.

(ii) There is a failure in $(u,\ u + du)$, $u < t$ and when unit 1 fails the other unit is either available or is under repair.

By similar arguments

$$A_{qrr}(t) = g_2(t) * A_{ro}(t)$$

$$A_{ro}(t) = \bar{G}_1(t)\, e^{-\lambda t} + [g_1(t)\, e^{-\lambda t}] * A_{oo}(b)$$

$$+ [g_1(t)\, (1-e^{-\lambda t})] * A_{or}(t)$$

$$A_{or}(t) = \bar{F}_1(t) + [f_1(t)\, P_{ro}(t)] * A_{ro}(t) + [f_1(1) P_{or}(t)] * A_{qrr}(t)$$

The solution of these equations in terms of the Laplace transformation of the availability is postponed till the end of this section, as is the case with the determination of stationary availability.

The reliability of the system is characterised by the following set of equations and is obtained by supressing the transitions to failure inducing (for the system) states. Thus the set of equations are given by

$$R_{oo}(t) = \bar{F}_1(t) + [f_1(t)\, P_{oo}(t)] * R_{ro}(t)$$

$$R_{ro}(t) = \bar{G}_1(t)\, e^{-\lambda t} + [g_1(t)\, e^{-\lambda t}] * R_{oo}(t)$$

On solving

$$R'_{oo}(t) = \alpha(t) + \sum_{n=1}^{\infty} f^{(n)}(t) * \alpha(t)$$

where $\alpha(t) = \bar{F}_1(t) + [f_1(t)\ P_{oo}(t)] * \bar{G}_1(t)\ e^{-\lambda t}$

and $f(t) = [f_1(t)\ P_{oo}(t)] * (g_1(t)\ e^{-\lambda t})$

$$\text{also MTSF} = \frac{E[F] + \bar{G}_1(\lambda)\int_0^\infty f_1(t)\ P_{oo}(t)\ dt}{1 - g_1(\lambda)\int_0^\infty f_1(t)\ P_{oo}(t)\ dt}$$

$$\text{with } \bar{G}_1(\lambda) = \int_0^\infty e^{-\lambda t}\bar{G}_1(t)\ dt = \frac{1}{\lambda + \mu_1}$$

$$g_1(\lambda) = \frac{\mu_1}{\lambda + \mu_1}$$

Note:

1. The expression for MTSF obtained here is true even when $g_1(.)$ is non-exponential and with this modification the system considered here becomes a special case of Linton (1976).

2. However, when one tries to obtain the availability measure for the model, with non-constant repair rates, the equations mentioned earlier need some changes and these are indicated below:
   For example $A_{oo}(t)$ will get changed to

$$A_{oo}(t) = \bar{F}_1(t) + [f_1(t)\ P_{oo}(t)] * A_{ro}(t) + \left[\int_0^t f_1(u)\ du \int_0^u P_{or}(u,x)\ \frac{g_2(t-u+x)dx}{\bar{G}_2(x)}\right] * A_{ro}(t)$$

Similar modifications in the other equations are necessary because of the non-exponential repair time duration of unit 2.

Thus to consolidate, when we want to consider system with more general failure and repair distributions, the availability and reliability are not obtained as simple expressions, rather, we get a system of equations governing these measures.

To be more precise the equations governing the availability and reliability of the system are given by:

$$A_i(t) = \alpha_i(t) + \sum_{j=1}^{n} v_{ij}(t) * A_j(t), \qquad i = 1, 2 \ldots n$$

and

$$R_i(t) = \nu_i(t) + \sum_{j=1}^{n_1} u_{ij}(t) * R_j(t), \qquad i = 1, 2 \ldots n_1$$

where $\alpha_i(t)$ and $\nu_i(t)$ are constant terms and $\sum_{j=1}^{n} v_{ij}^{*}(0) = 1$ and $\sum_{j} v_{ij}^{*}(0) \neq 1$.

The Laplace transform of availability and reliability can be obtained by transforming the above system of integral equations into a set of simultaneous equations and solving them. Also, the stationary availability β, can be proved to be independent of the initial condition and is obtained from the relation.

$$\beta = \frac{\sum_{i=1}^{n} \alpha_i \pi_{ik}}{\sum_{i=1}^{n} \pi_{ik} (\Sigma \dot{v}_{ij})}$$

where $\alpha_i = \alpha_i^*(0)$

$$\dot{v}_{ij} = [\frac{d}{ds} v_{ij}^*(s)]_{s=o}$$

and π_{ij} is the cofector of the (i,j)th element of the co-efficient matrix C obtained after seeking s = 0 in the system of equations governing the availability of the system. The resulting system of equations can be transformed to the form $C\underline{A} = \underline{\alpha}$ where C is an n x n matrix, $\underline{A}$ is n x 1 and $\underline{\alpha}$ is n x 1.

The MTSF is obtained by simply solving the resulting simultaneous equations obtained after setting s= 0, in the transformed form of the integral equations governing the reliability of the system.

Next, we observe that the above frame work is applicable to all those systems where exponential distribution is replaced by Erlang or phase type as demonstrated in [14] and [12] respectively. Interestingly the above method is based on the regenerative stochastic process approach and is comparitively easy than the analysis made by Linton (1976) for a less general system.

8. A general methodology:

By now we have discussed several models arising out of different assumptions on the failure and repair time durations of the units. At this point it is worthwhile to explore the question why some of these models are solvable and why some of them are not? Eventhough, a clear and characteristic answer to the above question will depend on the actual assumptions on several other details of the model one may choose to assume, the following broad conclusion may be arrived at.

Let $\{X(t), t \geq 0\}$ be a convenient stochastic process describing the dynamic behaviour of the system. For example, X(t) - may represent the number of units that are failed at any time t - or X(t) may correspond to the state space as defined in the earlier section. Also one may define several events associated with each specific definition of the stochastic process X (t). To illustrate the various events that can be defined for a specific process $X(t)$, consider: E_i - Entry to state i.

In general the E_i's thus defined form a stochastic point process [13]. Often the process can be studied by exploiting some of the special events E_i. To be more exact, if there exists an event E_j such that whenever E_j occurs the process is conditionally independent, or equivalently with respect to the events E_j's the process is a regenerative stochastic process, [1, 2], then the following general analysis is possible.

Let $f_j(t)$ - denote the pdf of the time interval between successive E_j's. Assume, that it is possible to determine $f_j(t)$ in terms of the failure and repair time durations. Then the analysis of any general quantity of interest is as follows: Obtain the quantity of interest within a cycle (is defined as the time interval between two successive E_j events), and use renewal theoretic arguments [3] to obtain the quantity of interest for any time point t.

We elobrate on this for the case of reliability and interval reliability. Let $R_j(t)$ and $R_j(t,\tau)$ be these quantities respectively conditioned by an E_j event.

Then, we have

$$R_j(t) = R_c(t) + \sum_{n=1}^{\infty} f_j^{(n)}(t) * R_c(t)$$

where $R_c(t)$ = Pr { System up in $(0,t)$ No E_j event in$(0,t)$ | E_j at $t=0$ }

$$R_j(t, \tau) = R_c(t+\tau) + \sum_{n=1}^{\infty} \int_0^t f_j^{(n)}(u) \, R_j(t+\tau-u) \, du$$

The above method of analysis has been demonstrated for the case of exponential failure for one unit and general failure for the other unit, with arbitrary repair rates in [15].

It may appear that the above approach is practically applicable to all systems since for any system with single repair facility and with some moderate other assumptions, the system recovery points are always regenerative. However, in practice this is not true, and that is the reason till today there is not a compact solution for the general case. The real problem is to be obtain the distribution of the random variable representing the cycle length. This does not seem to be feasible in the general case.

Recently, a general approach to solve this problem, by essentially using supplementary variables is given in [16]. However, the approach suggested in [16], is not suitable for any numerical solution. Recently Ohashi and Nishide [10] have attempted to solve the general problem for the case of system reliability. However, their analysis is also not complete in the sense for the end results they need some specific forms of f(.) and g(.).

References:

[1] E. Cinlar (1975): Markov Renewal Theory, A Survey: Management Science, Vol. 21, pp 729-752.

[2] J.W. Cohen (1969) : The Single Server Queue, North Holland Publishing Co., Amsterdam.

[3] D.R. Cox (1962) : Renewal Theory, Methuen, London.

[4] D.P. Gaver (1963) Time to failure and availability of parallel systems with repair, IEEE Tran. Rel. Vol. R-12, pp. 30-38.

[5] M. Kodama and H. Deguchi (1974) : Reliability considerations for a 2-unit redundant system with Erlang failure and repair distributions, IEEE Tran. Rel. Vol. R-23, pp 75-81.

[6] D.G. Linton (1976) : Some advancements in the analysis of two-unit parallel redundant system, Micro. Rel. pp 39-46.

[7] M.F. Neuts (1975) Probability distribution of phase type in Liber Amicorum Professor Emeritus H. Florin Department of Mathematics, University of Louvain, Belgium pp 173-206.

[8] M.F. Neuts and K.S. Meier (1980) : On the use of phase type distributions in reliability modelling of systems with a small number of components, Technical Report No. 62 B, University of Delaware, Newark.