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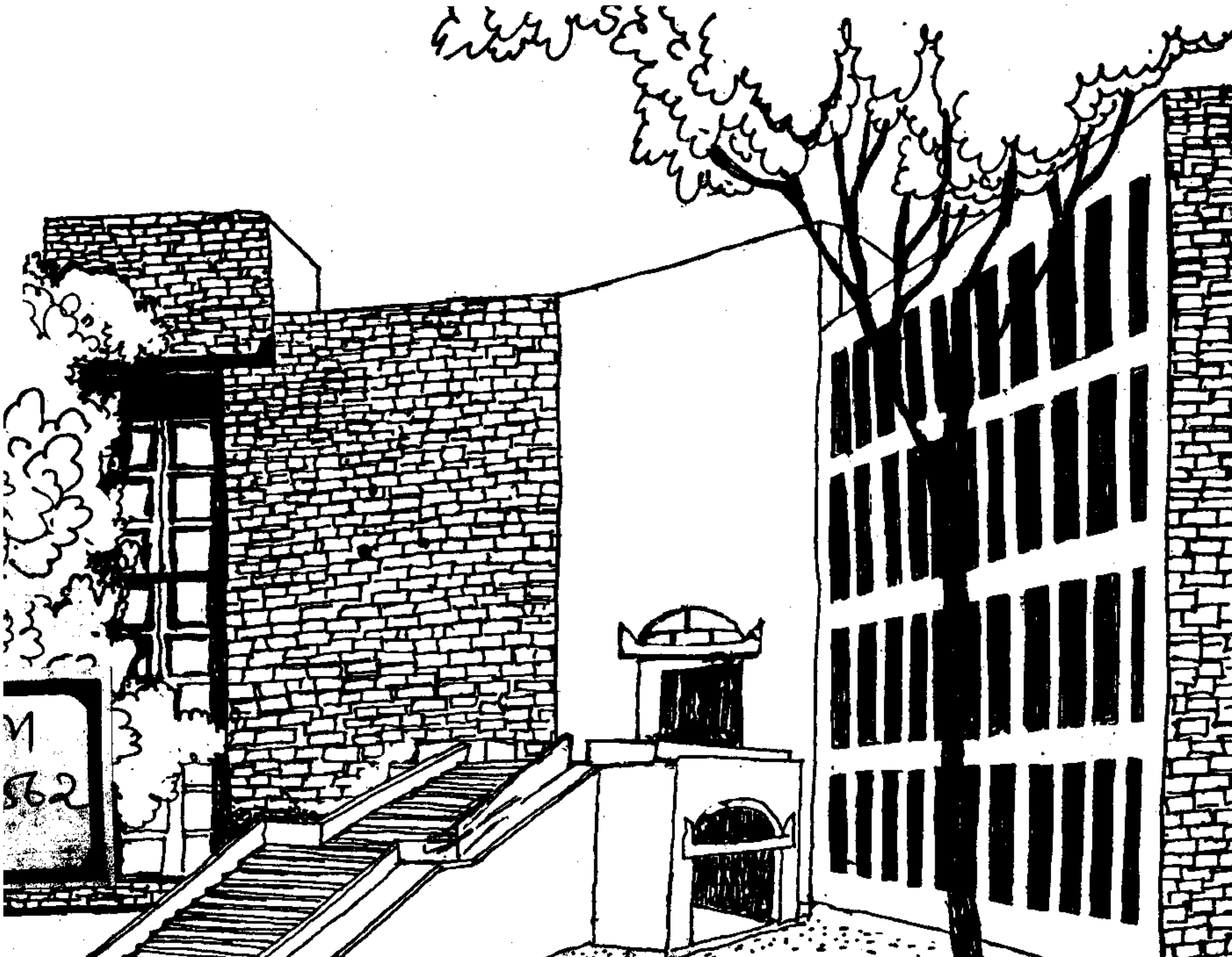

SOME NOTES ON EQUAL YIELD INCOME AND EXPENDITURE TAXES

By

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A.Das-Gupta, IIM Ahmedabad, 1985

Abstract

In this paper the efficiency, savings and inequality effects of income and expenditure taxes are reexamined. It is shown that current welfare is likely to be promoted by a mix of the two taxes rather than either tax alone. Additionally, it is conceivable for ability to save to be at a maximum with such a tax mix. Further, it is distinctly possible that income taxes rather than expenditure taxes encourage saving especially when human saving (education) is taken into account. Finally, income taxes, it is argued, are likely to be more equalitarian than expenditure taxes. Thus, doubts are cast on the desirability of expenditure taxes as compared to income taxes.

The author is indebted to Professors Manu R.Shroff, Anand P. Gupta and G.Srinivasan for helpful discussion. All errors are the responsibility of the author.

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1. INTRODUCTION

In this note we take a fresh look at the income versus expenditure tax debate. Expenditure taxes have recurrently been suggested as an alternative to income taxes by a long and distinguished line of economists going back at least to John Stuart Mill and, in more recent times, Nicholas Kaldor. Advantages have been claimed for expenditure taxes over income taxes on the grounds of efficiency, equity and the enhancement of savings incentives.[1] Disadvantages have been claimed on the grounds of diminished work incentives. Finally, there is no agreement as to the ease with which either tax system may be administered though government advisory commissions have expressed the opinion that expenditure taxes should be no more difficult to administer than income taxes.[2] However, the limited experience with such systems has not proved altogether happy.[3] In this paper we reexamine the efficiency and savings effects of the two taxes and provide a straight forward analysis of income

1. While we briefly present the main claimed advantages of the two taxes to lay a basis for subsequent discussion, useful recent surveys of the income versus expenditure tax debate are in Mieszkowski (1978), Taubman (1978), Chelliah (1980) and Thimmiah (1984). See also Auerbach and Kotlikoff (1983) and Feldstein (1978).

2. U.S.Treasury (1977), Institute for Fiscal Studies (1978), Lodin (1978) and Kaldor (1956).

3. See Thimmiah (1984) for a summary of these experiences in India and Sri Lanka.

distribution effects in a model which allows for human capital. Our analysis leads to the conclusion that the case for expenditure taxes is much weaker than it is commonly believed to be.

Before starting our enquiry, it may be worthwhile to summarise the reasons advanced for the advantages and disadvantages of expenditure taxes referred to above. Income taxes are expected to have adverse efficiency consequences as compared to expenditure taxes. This is so since income taxes but not expenditure taxes drive a wedge between producer and consumer prices of future goods in terms of current goods. There is thus an excess burden from an income tax. (Mieszkowski (1978), Feldstein (1978)). Feldstein (1978) shows that this excess burden may be substantial even if savings are unresponsive to interest rates.

That expenditure taxes but not income taxes satisfy the principle of horizontal equity was basically put forward by Kaldor (1955).[4] The main equity argument is reviewed in Feldstein (1978) and Mieszkowski (1978). In essence horizontal equity is violated since an income tax discriminates against these who postpone consumption out of current income in comparision with those who consume

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4. When horizontal equity is defined in terms of the present value of lifetime income rather than utility.

early given the same present value of lifetime earnings.[5]

This discriminatory property of income taxes, in that savers are penalized, is the essence of the so called 'double taxation of savings' under income taxes that has long been known. Since the rate of return on personal savings is lower under income taxes than expenditure taxes it is presumed that savings is thereby discouraged in an income tax regime as compared to an expenditure tax regime.[6] Buchanan (1959) and Feldstein (1978) point out that it is possible for future consumption to increase and savings to decrease so that, while a switch to expenditure taxes may encourage future consumption, it need not lead to increased saving.

Turning to the advantage enjoyed by income taxes over expenditure taxes, Taubman (1978) points out that expenditure taxes distort the choice between labour and leisure much more than income taxes or, in general the choice between market activities (which are taxed) versus non-market activities (which are untaxed). This is then a source of inefficiency which will have an excess burden. Empirical estimates as to the relative sizes of the excess burdens of

5. For a contrasting view based on the ability to pay concept, see Chelliah (1980).

6. Chelliah (1980) distinguishes between the concepts of ability to save and willingness to save. He concludes that "There is no doubt that if the income tax is replaced by the expenditure tax the ability to save would increase." He further concludes that willingness to save will also rise *ceteris paribus*.

both taxes are not available to the best of our knowledge (though absolute estimates on the excess burden of income taxation are available: See Feldstein (1978) and Feldstein (1983)).

An additional (equity) argument in favour of income taxes is that expenditure taxes will fall heavily on young persons, or, expenditure taxes will be heaviest during that phase of the lifecycle where ability to pay is lowest (Taubman (1978)). This argument has not been given the importance it deserves in the literature given the existence of capital market imperfections.

With regard to administrative convenience, our impression is that the 'jury is still out' on this issue. The reader is referred to Mieszkowski (1978), Taubman (1978) and Feldstein (1978).

In the next four sections we study various aspects of income versus expenditure taxes as follows. In section 2 we go into the question of equivalent tax systems which have as their main tax base either incomes or expenditures. We also look at tax rates implied by equal tax yields under different systems. In section 3 we use the standard two period model with earnings in the first period to develop some new results on savings under the two tax regimes. In section 4 we add human capital to the model to introduce a neglected dimension in the comparision of income and expenditure tax effects. General equilibrium considerations are also introduced at this point. Section 5 contains an explicit analysis of the inequality effects of the two tax systems. Section 6 sums up the discussion in the paper and identifies some areas requiring further study.

In section 2, we show that expenditure taxes implicitly tax inheritances and transfer incomes. Further, in a dummy tax system equivalent to an expenditure tax system, the income tax rate may exceed the income tax rate under an equal yield income tax system (restricting attention to proportional tax regimes). In section 3 we show that mixes of income and expenditure taxes are likely to maximize current welfare rather than either tax alone when there is a revenue constraint and that further, ability to save may also be maximized by such a tax mix. In section 4 we show that income taxes encourage education taking (or human saving). Further, when general equilibrium factor price responses are permitted, expenditure taxes discourage human saving. Thus our analysis casts doubts on the desirability of expenditure taxes over income taxes. This is strengthened by our analysis in section 5 where we show that the inequality reducing properties (both within and over generations) of income taxes as compared to expenditure taxes, which are commonly believed to exist, are supported by our analysis.

2. Equivalent income and expenditure tax systems and equal yield tax rates.

In what follows, we restrict attention to proportional taxes only. We first consider equivalences between income and expenditure tax systems and then move on to a consideration of tax rates under equal yield tax systems.

Let r_i be the gross rate of return on assets in period i; l_i be labour earnings in period i; c_i be consumption in period i; t and x be the income and expenditure tax rates; T_i be transfers (including gifts) in period i; a be initial wealth; and b be bequests. Then, for a k-period lived individual, the lifetime budget constraint is given by

$$a + \sum_{i=1}^{k} p_i(l_i(1-t)+T_i) = \sum_{i=1}^{k} p_i c_i(1+x)+p_k b(1+x) \qquad 2.1$$

where $p_i = \prod_{j=1}^{i}(1/(1+r_j(1-t))$

The equation assumes that transfers are not subject to income taxes and that bequests are subject to expenditure taxes. Modifications necessitated in the analysis when such is not the case will be pointed out.

To find the tax system which has income as its primary base and is equivalent to an expenditure tax system, define $z = x/(1+x)$, divide 2.1 by $(1+x)$ and set t equal to zero to get

$$a(1-z) + \Sigma p_i'(l_i+T_i)(1-z) = \Sigma p_i' c_i + p_k' b \qquad 2.2$$

In 2.2, p_i' is equal to p_i with t set equal to zero.

We conclude that an expenditure tax at rate x is equivalent to (i) an income tax at rate z plus (ii) an initial wealth tax at rate z plus (iii) an asset income subsidy at rate z plus (iv) a tax on transfer income at rate z. If bequest taxes are not included in the expenditure tax regime, then a bequest subsidy at rate z must be added to the equivalent tax system. Three comments are in order. First of all, in an intergenerational context the bequest subsidy and inheritance tax cancel out for all but the first generation faced with an expenditure tax for whom the inheritance tax is still operative. Secondly the equivalence between labour income taxes and consumption taxes that is commonly believed to hold (See Feldstein (1978) and Stiglitz (1983)) with proportional taxes is seen to depend on three assumptions: (i) Bequests are not subject to the expenditure tax; (ii) There are no transfers and (iii) The generation being examined is not the first to be subjected to the expenditure tax. Alternatively, if bequests are taxed as part of the expenditure tax regime, then, for non zero bequests and initial wealth, expenditure and labour income taxes are newer equivalent. Finally, we may comment on the savings implications of 2.2. As compared to an income tax at rate z, exemption of capital income from taxes certainly enhances the incentive to save as is well known. However this has to be weighed against the implicit tax on initial wealth (which is likely to lower savings) and the tax on transfer income (which should lower savings to finance bequests but has otherwise uncertain effects).

If we now define $v = t/(1-t)$, set x to zero in 2.1 and divide through by (1-t) we get

$$a(1+v)+ \Sigma p_i(l_i+T_i(1+v)) = \Sigma p_i c_i(1+v)+ p_k b(1+v) \qquad 2.3$$

From 2.3 we see that an income tax at rate t is equivalent to (i) an expenditure tax at rate v plus (ii) a bequest tax at rate v plus (iii) an inheritance subsidy at rate v, plus (iv) a transfer income subsidy at rate v plus (v) an asset income tax at rate v.

The same sort of comments as in the case of expenditure taxes can be made here as well. It appears from the analysis above that the argument that expenditure taxes promote saving has ignored an important consideration favouring income taxes in that there is an implicit tax on inheritances and transfers under expenditure taxes.

The analysis made so far is incomplete unless the tax rates t and z (or v and x) are compared with reference to the equal yield tax criterion. Therefore, we now turn to an examination of equal yield tax systems.

As is normal, we first look at the case of an all consumption economy. In the one period case Mieszkowski (1978) claims that the tax rates for equal yield income and expenditure taxes are related through the equation $(1-t) = 1/(1+x)$ or $t = z$. This can be derived by eliminating income (y) and consumption from the relations $ty = xc$ and either $y(1-t)=c$ or $y=c(1+x)$. However, this relation is valid only if no tax revenues find their way back into the hands of consumers as transfers. With transfers, the consumer's budget equation becomes $y(1-t)+T=c$ or $y+T=c(1+x)$. In the case where all tax revenues find their way back to the taxpayer we have $T=ty = xc$ so that $t=x$. Thus, in general, in the one period case $z=x/(1+x) \leq t=v/(1+v) \leq x$ for equal yield taxes and an all consumption economy. <u>However if $T > ty$ it is possible for t to exceed x</u> (provided T is such that equal yield income and expenditure taxes exist).

To introduce savings, consider a version of the standard two period model with income only in the first period. If k is second period consumption (plus bequests) we have the lifetime budget constraint as

$$(y(1-t)+T-c)(1+r(1-t)) = k \quad 2.4a$$

$$(y+T-\bar{c}(1+x))(1+\bar{r}) = \bar{k}(1+x) \quad 2.4b$$

2.4a applies in the case of an income tax and 2.4b applies in the case of an expenditure tax. Equal tax yields imply

$$t(y+prs) = x(\bar{c}+\bar{p}\bar{k}) \quad 2.5$$

where $s=(y(1-t)+T-c)$; $p=1/(1+r(1-t))$ and $\bar{p}=1/(1+\bar{r})$.

From 2.4b and 2.5 we get

$$y+T = (\bar{p}\bar{k}+\bar{c})(1+x)=t(1+x)(y+prs)/x$$

If $T/y=b$ and $s/y=a=$ the average propensity to save, we may rearrange this to get $z=t(1+pra)/(1+b)$ 2.6

Clearly, $z > t$ if $T=0$. Also if $T=t(y+prs)$, that is if transfers equal tax collection, we get $z=1>t$. Thus, t can exceed z (and even x) only if transfers are substantially higher than tax revenues. But if $z>t$ obtains, then there is further reason to suspect the savings reducing effects of income taxes. In fact, since transfers are likely to be regressive, savings disincentives under an income tax regime will be felt most by relatively poor persons who, in any case, have low savings propensities.

The extension of this analysis to the k-period case should give rise to similar results and is not attempted here.

To sum up our analysis in this section, one fact seems to stand out. In both the analysis of equivalent systems and equal yield tax rates we have found that a fully specified budget constraint is essential if results are not to be misleading. In particular, initial wealth stocks and transfers need to be accounted for. Once this is done, both strands of the analysis raise doubts as to the savings enhancing potential of expenditure taxes as compared to income taxes. Further, several new insights as to equivalent tax systems and magnitudes of tax rates under equal yields are obtained.

3. Equal yield income and expenditure taxes, savings and welfare.[7]

In this section we use the standard two period partial equilibrium framework to analyse welfare and savings implications of income and expenditure taxes. A somewhat related analysis is that of Feldstein (1978). We show that neither income nor expenditure taxes alone is optimal, a conclusion similar to the result that a mix of wage and rental taxes is optimal obtained by Feldstein (1978). We are also able to provide additional insight as to the optimal tax mix for the achievement of different objectives. Specifically, our results may be (somewhat loosely) summarised as in Figure 3.1, which is self explanatory. We conduct our analysis in four steps. We first show that changing the (income and expenditure) tax mix may be welfare improving or worsening depending on the initial tax mix. Next we show that ability to save and savings *may* increase if the income tax fraction is raised in the tax mix. We then find the savings and current utility maximizing tax policies and show that income taxes are more heavily represented in the current utility maximizing package than in the savings maximizing package. In fact, in the former package, the income tax fraction exceeds 50 percent.

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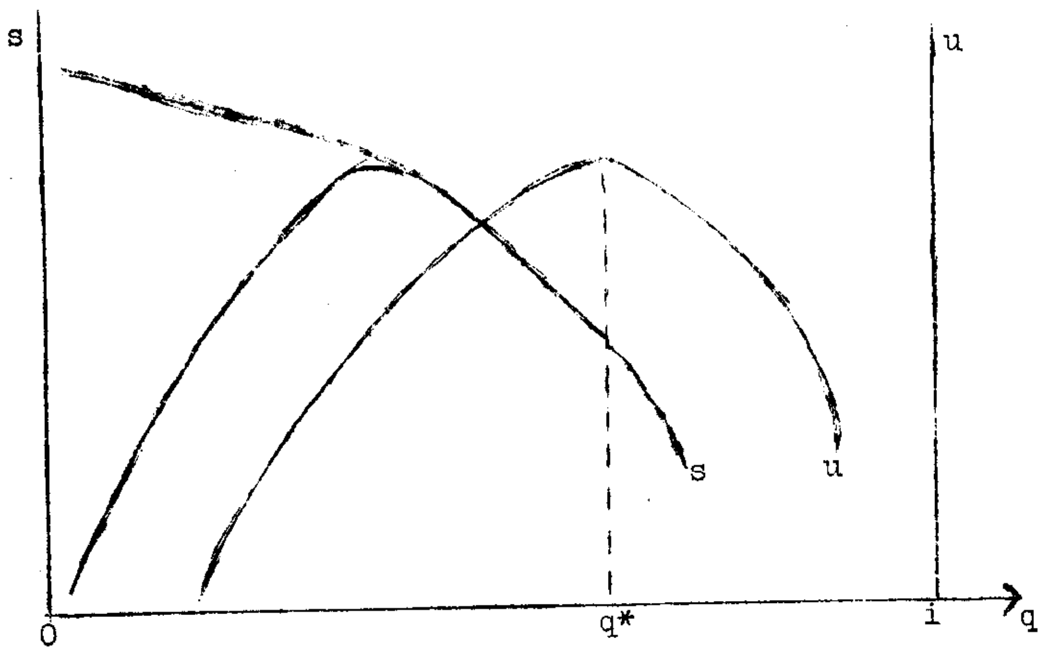
7. We are indebted to G.Srinivasan for pointing out an error in an earlier draft of this section.



q = ratio of income taxes to total (income plus expenditure) taxes.

u = tax payers utility.

s = savings.

q* > 0.5.

Figure 3.1: Savings and tax payers'utility with equal yield income and expenditure tax mixes.

The model we use is the model of equation (2.4) but with both taxes present and no transfers (since transfers will remain unchanged throughout the analysis. Thus we have the budget constraint

$y(1-t)=(c+pk)(1+x)$; $p=1/(1+r(1-t))$ 3.1a

or $s=pk(1+x)$ 3.1b

Also, total tax collection is given by[8]

$$T=T_y+T_c= t(y+prs)+x(c+pk)= \text{constant} \qquad 3.2$$

In (3.2) T_y is the amount of income tax and T_c the expenditure tax yield.

Now consider a small change in tax rates which leaves the tax yield unaltered. If expenditure taxes are increased at the expense of income taxes, we may derive the following expression for the change in the lifetime budget constraint (see Appendix I for details of the derivation of both (3.3) and (3.4)).

$$udx= vdc+wdk \qquad 3.3$$

where $u = (T_y-T_c-z)p^2rk, \; z= (1-p)ty > 0$

$v= y+p^2rs(1+r)(1+x) > 0$ and

$w = py(1-tpr(1+x)+p^2rs(1+x) > 0.$[9]

With regard to w, we may conclude that this term must be strictly positive given that v is positive.. Otherwise, regardless of the sign of u, unlimited increases in c and k would be possible without violating 3.3. However, given that w and v are positive we may conclude that provided u

8. Without risk of confusion, T is used to denote total tax collection in this section since transfers are assumed to be zero.

9. Likewise, if income taxes are increased without altering tax yields, we get the following equation for budgetary changes.

$$-udt = v'dc+w'dk \qquad 3.4$$

where $v' = (c+pk)+(1+x)tprs > 0$

$w' = p(c+pk)-(1+x)tp^2rc > 0$

is positive (resp.negative) a move to greater reliance on expenditure taxes (resp.income taxes must be welfare improving since both c and k can be increased.

The intuition behind this result is best grasped by considering the two extreme cases where either c or k are zero. If k=0 then we have $y(1-t)=c(1+x)$ and $T=by+xc$. In this case $dc/dt=0$ so that no change is possible in consumption when the tax mix changes. If instead $c=0$, then we have $y(1-t)=pk(1+x)$ and $T=t(y+pry(1-t))+xpk$. We now have $dk/dt = r(xk-ty)$. Thus, the source of the ambiguity in the sign of u is traced to the way in which the present value of future tax revenues changes as t changes. If income taxes contribute relatively little to total revenues, then the increase in the income tax rate at the expense of the expenditure tax rate can be relatively small given the increase in p. In fact, as has been shown, for low enough t the taxpayer is made better off since the tax payer's real income increases.

In Appendix 2 numerical examples of ability to save under various tax mixes are given. Ability to save is said to be unchanged if given fixed consumption, savings do not change. It is established there that, given appropriate parameter values, income taxes or a mix of income and expenditure taxes can lead to greater ability to save than income taxes alone.

To further analyse savings behaviour, let the individual maximizes a utility function (with the usual properties guaranteeing interior maxima given the budget constraint) of the form u(c,k) subject to the budget constraint (3.1). The first order condition for a utility maximum is (in addition to the budget constraint).

$$pu_c = u_k \qquad 3.5$$

(Subscripts denote the appropriate partial derivatives and two subscripts will be used for second partials). This has the total differential

$$p^2ru_c(dt/dx)dx \doteq (u_{ck}-pu_{cc})dc+(u_{kk}-pu_{ck})dk$$

Thus, using 3.3 we see that

$$\frac{dc}{dx} = \frac{un+jt_xw}{vn+mw} \gtrless 0$$

$$\frac{dk}{dx} = \frac{mu-vjt_x}{vn+mw}$$

where $t_x = dt/dx < 0$; $j = p^2ru_c > 0$

$m = (u_{ck}-pu_{cc}) > 0$ and $n = -(u_{kk}-pu_{ck}) > 0$.

Now, since s=pk(1+x), we can write

$$\frac{ds}{dx} = p^2rk(1+x)t_x+pk+p(1+x)\frac{dk}{dx} \qquad 3.6$$

This expression has one positive term (pk) one negative ter ($p^2rk(1+x)t_x$) and one term of indeterminate sign (the term involving dk). We conclude that <u>it is an empirical question</u>

as to whether a switch to greater reliance on expenditure taxes (or income taxes) leads to greater saving.[10] This is true even when all manner of market imperfections and general equilibrium considerations are controlled for and the labour leisure choice is omitted. This result has been obtained by Feldstein (1978).

Our analysis need not, however, stop here. Since we find that there are ranges in which both income and expenditure tax increases (at the cost of the other tax) are welfare improving we may ask what pair of tax rates would maximize welfare. We may then examine the relation these rates bear to the savings maximizing rates.

To find the optimal tax rates, we maximize $u(c,k)$ with (3.1) and (3.2) as constraints, using c, k, x and t as choice variables. If q_1 is the Lagrangian multiplier associated with (3.1) and q_2 is the Lagrangian multiplier associated with (3.2), then the following expressions give the first order conditions associated with the choice of t and x respectively (independent of the specific utility function):

$$-q_1(y+p^2rk(1+x)) = q_2(y+p^2rk(1+x)+tp^3r^2k(1+x)+xp^2rk) \qquad 3.7$$

$$-q_1(c+pk) = q_2(c+pk) = q_2(c+pk+tp^2rk) \qquad 3.8$$

If we solve these equations so as to eliminate q_2/q_1 the condition reduces to (see the appendix, equation a1.3) $u = 0$ as should by now be expected. Thus, the optimal tax rates are such that $T_y = T_x + z$. 3.9

10. In specific cases, for example if consumption expenditure is a fixed fraction of disposable income, we must have $ds/dx > 0$.

We may immediately conclude that <u>at the optimum, income taxes make up more than 50 percent of tax revenues.</u> (Note that $T_y - z = tp(y+rs) > 0$ so that $T_x > 0$). To find the optimum tax rates for given tax revenues T, we solve 3.2 and 3.9 to get

$$t = \frac{T}{y(1+p)+2prs} . \qquad x = \frac{pT(y+rs)}{(c+pk)} .$$

In contrast, to maximize saving, we maximize $pk(1+x)$ subject to 3.1, 3.2 and the first order condition 3.5. That is to maximize savings, we solve the problem

$$\max_{x,t} \quad pk(1+x)$$

subject to

$$pu_c = u_k$$

$$T = T_y + T_x$$

$$y(1-t) = (c+pk)(1+x).$$

Accordingly, set up the Lagrangian

$$\mathcal{L} = pk(1+x)+a_1(pu_c-u_k)+a_2(T-T_y-T_x)+a_3(y(1-t)-(c+pk)(1+x)).$$

The first order conditions for a maximum are given by

$$\frac{\partial \mathcal{L}}{\partial t} = p^2rk(1+x)+a_1(j-\frac{m\partial c}{\partial t}+\frac{n\partial k}{\partial t})-a_2(\frac{\partial T_y}{\partial t}+\frac{\partial T_x}{\partial t})-a_3(y+p^2rk(1+x))=0$$

$$\frac{\partial \mathcal{L}}{\partial x} = pk+a_1(-\frac{m\partial c}{\partial x}+n\frac{\partial k}{\partial x})-a_2(\frac{\partial T_y}{\partial x}+\frac{\partial T_x}{\partial x})-a_3(c+pk) = 0.$$

Now both expressions $a_1(.)$ are equal to zero due to utility maximization. This means that the conditions above reduce to

$$\frac{(\frac{\partial T_y}{\partial x}+\frac{\partial T_x}{\partial x})p^2rk(1+x)-(\frac{\partial T_y}{\partial t}+\frac{\partial T_x}{\partial t})pk}{a_3} - u = 0. \qquad 3.10$$

Now a_3, the marginal utility of income, is positive. We also have

$$\frac{\partial T_y}{\partial_x} + \frac{\partial T_x}{\partial_x} \quad p^2rk(1+x) - \frac{\partial T_y}{\partial_t} + \frac{\partial T_x}{\partial_t} \quad pk \quad < 0. \qquad 3.4$$

This is so since 3.11 is equivalent to (see the appendix)

$$(1+x)pr(c+pk+ tp^2rk)-(y+p^2rk(1+x)+tp^3r^2k(1+x)+p^2rkx) < 0$$

This expression simplifies to

$$(1+x)prc-y-p^2rkx < 0$$

Now, if we add and subtract $(1+x)p^2rk$, then, noting that $(1+x)(c+pk) = y(1-t)$ and $pr(1-t)-1 = -p$, we obtain $-py - (1+2x)p^2rk < 0$ which is clearly true. We may conclude that, at the savings optimum, $u < 0$. Thus the savings maximizing and current welfare maximizing tax rates are not identical. In fact, we may recall, from our discussion of (3.3) that this condition obtains when income taxes are lower than is required for a utility maximum. Therefore, as intuition would suggest, the tax mix which maximizes the savings rate gives greater weight to expenditure taxes than the mix which maximizes utility for a given tax yield.[11]

The conclusion we derive from the preceding analysis is that neither expenditure taxes nor income taxes alone are always optimal. If saving *per se* is not desired, then the tax system should be weighted in favour of income taxes (with more than 50 percent income taxes). If the tax system is

11. A corner solution is possible. See footnote 10.

is to encourage savings (if the utility of future generations weighs heavily with policy makers) then the tax system should give more weight to expenditure taxes than in the previous case.

To sum up, even when labour-leisure choices are assumed away an expenditure tax is not superior to an income tax either on grounds of welfare or savings maximization for some values of parameters.

In fact a mixture of the two taxes, as described above, is likely to be optimal in both situations.

4. Income and Expenditure taxes in a model with education and savings.

Our objective in this section is to elaborate on the framework of section 3 by introducing human capital. The model thus developed will be used in the analysis of savings and distributional effects of income and expenditure taxes in this and the next section. The labour-leisure choice is still ignored since its inclusion could only strengthen the case for income taxes. The model is given by the following equations:

$$\max_{c,k} \; u(c,k)$$

subject to $a(1+r_1(1-t))+w_1l_1(1-t)+pw_2l_2(1-t) = (c+pk)(1+x)$.

The notation used is as before except that w_i is now the wage rate in the ith period, l_i is the labour supply in the ith period in efficiency units and $p=1/(1+r_2(1-t))$.

We now add a mechanism for the determination of labour supply. The tax payer is assumed to have an initial endowment of human capital. Human capital increases the efficiency of labour time. Furthermore, by refraining from work (partially or totally) in the first period, the human capital stock in the second period can be enlarged through human capital augmenting activities (i.e. 'education' which we denote by e). Therefore, there are now two assets in the economy.[12] We assume that an index of human capital (h) can be so constructed, that labour time in efficiency units may be written as the human capital index multiplied by labour time. Accordingly, we have $l_1=h(1-e)$ and $l_2=(h+\Delta h)$. In postulating a relationship between e and Δh we choose the simplest specification which will lead to education time increasing (resp.decreasing) whenever the second period reward per efficiency unit of labour increases (resp.decreases), in present value terms, relative to the first period reward. This is accomplished by the human capital production function

2. To avoid differences in interest rates for borrowing and lending with income taxes we assume that individuals have a non-negative net worth at the end of the first period. This allows a uniform specification of the budget equation for all individuals as opposed to seperate equations for borrowers and lenders when there are income taxes. Since borrowers, in the sense of negative net worth, are likely to be in a minority in any economy, little damage to the results will result when we take up the analysis of an n-agent economy.

$\Delta h = g(e)$, $g' > 0$, $g'' < 0$.[13]

The final form of the model developed above is thus

$$\max_{c,k,e} \quad u(c,k) \qquad 4.1$$

subject to

$$m' = a(1+r_1(1-t))+w_1h(1-e)(1-t)+pw_2(h+g)(1-t) = (c+pk)(1+x), \text{ and} \qquad 4.2$$

$$g=g(e),\ g' > 0,\ g'' < 0. \qquad 4.3$$

Note that first period income (y) is given by $y=ar_1+w_1h(1-e)$. The demand for education may now be derived from the first order condition $w_1h = pw_2g'$ to be

$$e= e(w_1, w_2,r_2,t)\ e_1 < 0;\ e_2 > 0;\ e_3 < 0;\ e_4 > 0 \qquad 4.5$$

In 4.5 $e_1 = -\alpha/w_1$, $e_2=\alpha/w_2$, $e_3=-(1-t)p\alpha$ and $e_4= pr_2\alpha$, where $\alpha = -g'/g''$. The result of interest is that <u>e_4 is positive since increasing income tax rates raise the present value of future labour income. Therefore, to the extent that education may be treated as saving (in human terms), we see that there is additional reason to doubt the efficiency of expenditure taxes in promoting saving relative to income taxes.</u>

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13. The highly special nature of this specification, in particular the omission of h in the production function, non-depreciation, and the omission of costs of education taking will not be exploited in obtaining specific results. Therefore, our analysis will lead to results which are qualitatively similar to those in a more general model. In a more general exercise a market for education should be explicitly modelled. Such a model is not required for our purposes.

Turning to the demand for c and k, these demand functions may be written as

$c=c(p,m),\ c_p > 0;\ c_m > 0$ 4.6

$k=k(p,m),\ k_p < 0;\ k_m > 0$ 4.7

where $m = m'/(1+x) = m(w_1, w_2, r_1, r_2, t, x)$ [14] 4.8

m_1, m_2, m_3, are all positive and m_4, m_5 and m_6 are negative. c_m and k_m are positive since we assume that both c and k are normal goods. The responses of the demand for c and k to changes in r_2 and t present problems. An increase in r_2, lowers p and m. Thus, c decreases, but k may increase or decrease. Conversely, an increase in t, raises p but causes m to decrease. Therefore, k will decrease, but c may increase or decrease. However, our assumption of non negative net worth leads to a positive income change overall when r_2 increases so that k increases with r_2. The response of c to a changed income tax rate remains uncertain.

The derived labour supply and savings responses, which are of importance to our analysis below, may now be studied. For labour supply responses, we have

$dl_1 = -hde$ and $dl_2 = g'de$ so that we may write

$l_1 = l_1(w_1, w_2, r_2, t),\ l_{11} > 0,\ l_{12} < 0,\ l_{13} > 0,\ l_{14} < 0$. 4.9

$l_2 = l_2(w_1, w_2, r_2, t),\ l_{21} < 0,\ l_{22} > 0,\ l_{23} < 0,\ l_{24} > 0$. 4.10

and $dl_1 = -h/g'dl_2$ 4.11

14. e does not figure as an argument due to maximization and the envelope theorem.

Turning to the determinants of savings, we have, first of all

$$s=y_1(1-t)+a-c(1+x)=pk(1+x)-pw_2l_2(1-t)=s(w_1,w_2,r_2,t,r_1,x)$$

The partial derivatives of the savings function are as follows.

$$s_1=(l_1(1-c_m)+w_1l_{11})(1-t)>0$$

$$s_2=(w_1l_{12}-c_mpl_2)(1-t)<0$$

$$s_3=(c_mw_2l_2(1-t)+c_p(1+x))p^2(1-t)+(1-t)w_1l_{13}>0$$

$$s_4=-y(1-c_m)+w_1(1-t)l_{14}-(1+x)c_pp^2r+c_mp^2w_2l_2 \gtrless 0$$

$$s_5=a(1-c_m)(1-t)>0$$

$$s_6=-(c+mc_m)<0$$

In assigning signs to s_1 and s_4 we have assumed that $0<c_m<1$. Also, looking at s_4, since first period disposable income has fallen we shall assume $s_4<0$, especially since second period consumption (plus bequests), k, decreases when t increases.

We thus have

$$s=s(w_1,w_2,r_2,t,r_1,x);\ s_1>0;\ s_2<0;\ s_3>0;\ s_4<0;\ s_5>0;\ s_6<0 \qquad 4.12$$

We formally state the assumptions made above before proceeding further.

<u>Assumption 4.1</u> (i) Both c and k are normal; further, $0<c_m<1$.

(ii) Net worth at the end of the first period is non negative.

(iii) $ds/dt<0$.

The model of consumer behaviour with human capital and saving is now fully developed. In particular, equations 4.8 to 4.12 are important in the analysis we now undertake. The focus of analysis in this section is the savings consequences of income and expenditure taxes when wage and interest rates are endogenous. Before undertaking this task we briefly argue that the results of the previous section, with regard to savings and current utility carry over to this section and that further, expenditure taxes are less likely to promote savings once human savings are included in total savings.

To examine savings behaviour in this model - under each tax regime - consider the expression for the total tax yield.

$$T=t(y_1+py_2)+ x(c+pk). \qquad 4.13$$

where $y_2=(a+s)r_2+w_2l_2$. Using 4.2, we may rewrite 4.13 as

$$T=(t+z(1-t))(y_1+pw_2l_2)+ptr_2(a+s)+za \qquad 4.14$$

As before, $z=x/(1+x)$.

If we differentiate 4.14 with respect to x and set $dT= 0$ we obtain

$$0= T_1t_x+T_2+ ptr_2s_x \qquad 4.15$$

where $t_x= dt/dx < 0; s_x= s_4t_x+ s_6$; T_1 is the rate of change of T with respect to t if savings are unchanged and T_2 is the analogous rate of change with respect to x. T_1 and T_2 are both positive. s_x is, of course, the change in savings if expenditure taxes are increased so as to leave T unchanged. If we rearrange 4.15 we see that

$$s_x \gtrless 0 \text{ as } -t_x \gtrless \frac{T_2}{T_1} . \qquad 4.16$$

That is, s_x is more likely to be positive, if T_2 is relatively small and T_1 is relatively large. Clearly, as before, this occurs when expenditure taxes make up a small fraction of total taxes. Thus, the savings results of the previous section carry over to this more general model. That the savings and utility maximizing tax regimes are not identical is also easy to verify. In fact $-t_x = (\partial m/\partial x) \div (\partial m/\partial t)$ gives the utility maximizing tax mix. It is easy to verify that total (human plus non human) savings will increase by less (or decrease by more) than only non-human savings if the expenditure tax fraction is increased. An appropriate measure of human savings is first period earnings foregone. Noting that foregone earnings are not taxed, the expression for total savings is $s + w_1 he = \tilde{s}$. Since c_4 is positive, clearly $s_x < s_x$. Thus, in the presence of education opportunities it is possible that income taxes rather than expenditure taxes encourage savings.

In order to introduce mechanisms for the endogenous determination of wages and interest rates, we assume that output in each period is the result of a production process to which assets and labour are inputs. The production functions have the usual linear homogeneity, diminishing marginal product and factor complementarity properties and further, the Inada boundary conditions hold. Also, factor payments equal the marginal products of the respective factors. Finally, we assume that there are n agents who differ with respect to their endowment vector(a,h). The aggregation of factor supplies over the n agents is indicated with the subscript n (eg l_{1n} or a_n). Noting that initial asset holdings are exogenous, we have the following logarithmic differential equations

$$\hat{w}_1 = -\theta_1 \hat{l}_{1n}/\sigma_1, \qquad 4.17$$

$$\hat{r}_1 = (1-\theta_1)\hat{l}_{1n}/\sigma_1, \qquad 4.18$$

$$\hat{w}_2 = \theta_2(\lambda\hat{s}_n + \mu\hat{l}_{1n})/\sigma_2, \quad \text{and} \qquad 4.19$$

$$\hat{r}_2 = -(1-\theta_2)(\mu\hat{l}_{1n} + \lambda\hat{s}_n)/\sigma_2. \qquad 4.20$$

where θ_i is the capital share of output in period i,

σ_i is the elasticity of substitution in period i,

$\mu = pw_2 l_{2n}/l_{1n}w_1$ and $\mu l_{1n} = -l_{2n}$

$\lambda = s_n/(s_n+a_n)$.

A caret over a variable denotes a logarithmic differential. if α_i denotes the absolute elasticity of savings with respect to the i^{th} argument of 4.12 and β_i denotes the first period labour supply elasticity with respect to the i^{th} argument in 4.8, we also have the equations

$$\hat{s}_n = \alpha_1\hat{w}_1 - \alpha_2\hat{w}_2 + \alpha_3\hat{r}_2 - \alpha_4\hat{t} + \alpha_5\hat{r}_1 - \alpha_6\hat{x} \qquad 4.21$$

$$\hat{l}_{1n} = \beta_1\hat{w}_1 - \beta_2\hat{w}_2 + \beta_3\hat{r}_2 - \beta_4\hat{t} \qquad 4.22$$

Solving 4.21 and 4.22 using 4.17 to 4.20 we get equations 4.23 and 4.24.

$$\hat{s}_n = -S_t\hat{t} - S_x\hat{x} \qquad 4.23$$

$$\hat{l}_{1n} = -L_t\hat{t} + L_x\hat{x} \qquad 4.24$$

where

$$S_t = \sigma_2\sigma_2(\alpha_5\beta_4(1-\theta_2) + \sigma_1\alpha_4) + \theta_1\sigma_2(\beta_1\alpha_4 - \alpha_1\beta_4) + \sigma_1\mu(1-\theta_2)$$
$$(\beta_3\alpha_4 - \alpha_3\beta_4) + \sigma_1\mu\sigma_2(\beta_2\alpha_4 - \alpha_2\beta_4))/D,$$

$$S_x = \alpha_6\sigma_2(\sigma_1\sigma_2 + \beta_1\theta_1\sigma_2 + \beta_2\theta_2\mu\sigma_1 + \beta_3\mu(1-\theta_2)\sigma_1)/D,$$

$$L_t = (\beta_4\sigma_2 + \lambda\theta_2(\beta_4\alpha_2 - \alpha_4\beta_2) + \lambda(1-\theta_2)(\beta_4\alpha_3 - \alpha_4\beta_3))\sigma_1\sigma_2/D,$$

$$L_x = \lambda(\beta_2\theta_2 + \beta_3(1-\theta_2))(\sigma_1\sigma_2 + \beta_1\theta_1\sigma_2 + \beta_2\theta_2\mu\sigma_1 + \beta_3(1-\theta_2)\mu\sigma_1)/D, \text{ and}$$

$$D = (\sigma_2 + \alpha_2\theta_2\lambda)(\sigma_1\sigma_2 + \beta_1\theta_1\sigma_2 + \beta_2\theta_2\mu\sigma_1 + \beta_3(1-\theta_2)\mu\sigma_1) +$$
$$\lambda\sigma_2\alpha_3(1-\theta_2)(\sigma_1 + \beta_1\theta_1) + \sigma_2\lambda(\beta_2\theta_2 + \beta_3(1-\theta_2)(\alpha_5(1-\theta_1) - \alpha_1\theta_1).$$

We will assume that D is positive since only the last term of D in the expression above is negative. Among the several sufficient conditions for this are a low first period wage elasticity of savings or that current savings are small relative to initial assets or that the elasticity of substitution in the second period is low. With this assumption both S_x and L_x are unambiguously positive. Since first and second period labour supplies are negatively related this means that an increase in the expenditure tax rate causes both savings and second period labour supply to decrease ceteris paribus. It is likely, as we argue below, that both S_t and L_t are positive. If this is the case, then raising expenditure taxes at the expense of income taxes may or may not lead to increased non-human saving but definitely leads to reduced second period labour supply. Thus, it would appear that the general equilibrium analysis reinforces the case against expenditure taxes.

Turning to an analysis of S_t and L_t, the ambiguity in their signs results from terms of the form $\beta_4\alpha_j-\alpha_4\beta_j$, j=1,2,3. These expressions can be manipulated to get expressions of the following type which will have the same sign†:

$$\left|\frac{\partial l_{1n}}{\partial t}\right|\left|\frac{\partial s_n}{\partial j}\right| - \left|\frac{\partial s_n}{\partial t}\right|\left|\frac{\partial l_{1n}}{\partial j}\right|, \quad j= w_1,\ w_2,\ r_2$$

$$\frac{\left|\frac{\partial s_n}{\partial j}\right|}{\left|\frac{\partial s_n}{\partial t}\right|} - \frac{\left|\frac{\partial l_{1n}}{\partial j}\right|}{\left|\frac{\partial l_{1n}}{\partial t}\right|} \quad , \; j = w_1, \; w_2, \; r_2. \qquad 4{:}25$$

Thus the signs of these terms depends on the difference between the absolute slopes of level curves of the s_n and l_{1n} functions in (t,j) space. If 4.25 is positive a perverse savings response becomes more likely and if negative a perverse labour supply response becomes likely. In addition, S_t and L_t each have a term that is unambiguously positive. To study the most likely signs of S_t and L_t, consider the behaviour of the system if t is increased. At constant factor rewards, an increase in t results in a smaller first period labour supply, a greater second period labour supply and less saving (from 4.8 to 4.12). But these changes in factor supplies cause r_1 and w_2 to fall but w_1 and r_2 to increase. The movements in w_1, w_2 and r_2 thus dampen the initial reallocation of labour supply. If this response to changing factor prices does not overwhelm the initial response to the increased income tax rate, L_t will be positive. We may, therefore, treat this as the most likely case. Turning to savings, three of the four movements in factor prices (i.e all but the change in r_1) lead to offsetting increases in saving. However, as with labour supply, a positive S_t may be taken to be the most likely case. The possibility of a perverse movements in either factor should nonetheless be borne in mind.[13] It may be worthwhile to list the various response patterns and their implications. This is done in Table 4.1. The table shows clearly that a move to expenditure taxes is never unambiguously savings promoting, but with a perverse non-human savings response and a normal labour supply response a move to income taxes unambiguously promotes total savings.

Table 4.1: General equilibrium responses of a move to greater reliance on expenditure taxes.

Case	$S_t > 0; L_t > 0$ Normal case	$S_t > 0$ $L_t < 0$	$S_t < 0$ $L_t > 0$	$S_t < 0$ $L_t < 0$
s_n	?(+)	?	−	−
l_{1n}	+	?	+	?

13. Sufficient conditions for S_t and L_t to be positive can be derived algebraically. However, the expressions so obtained are difficult to interpret and thus serve no useful purpose.

To sum up the analysis in this section, we have shown that the inclusion of human capital accumulation possibilities leads to a further weakening of the case for expenditure taxes. The savings and efficiency effects of different tax regimes, it would appear, can only be determined empirically. What is clear is that expenditure taxes discourage human capital accumulation whereas income taxes encourage it.

5. Inequality and tax regimes

Having studied the efficiency and savings effects of taxes, we comment briefly on the distributional effects of the taxes. We will thus have studied all the economic areas which are of interest in an examination of the impact of taxation barring a priori equity considerations.

Based on the earlier analysis, one comment can already be made. Since expenditure taxes probably encourage non-human saving at the expense of human saving, expenditure taxes may lead to intergenerational perpetuation of inequality compared with income taxes since non-human assets are more easily bequeathed than human assets. Against this however is the possibility of a lower intergenerational absolute income path with income taxes precisely because human assets cannot be bequeathed.

To examine inequality, we look at the changes in the entire stream of lifetime consumption as measured by the "income" term (m) of equation 4.8. If we use, the superscript j to denote the poorest j individuals, then inequality according to the Lorenz criterion increases, remains unchanged or decreases by this measure as

$$d\left(\frac{m^j}{m^n}\right) \gtrless 0 \quad \text{for } j = 1, \ldots, n-1$$

$$\text{Now, } \frac{m^j}{m^n} = \frac{a^j + y_1^j(1-t) + pw_2 l_2^j(1-t)}{a^n + y_1^n(1-t) + pw_2 l_2^n(1-t)} \qquad 5.1$$

Differentiating 5.1 with respect to x and rearraning gives us

$$m_n t_x(y^n + pw_2 l_2^n)\left(\frac{y^j + pw_2 l_2^j}{y^n + pw_2 l_2^n} - \frac{m^j}{m^n}\right) + m_n(1-t)(a^j\tilde{r}_1 + l_1\tilde{w}_1)$$

$$+ m_n(1-t)p^2 r_2^2 t_x \left(\frac{w_2 l_2^j}{w_2 l_2^n} - \frac{m^j}{m^n}\right) + m_n(1-t)p\left(\frac{u^j}{u^n} - \frac{m^j}{m^n}\right)$$

where $u^j = \tilde{w}_2 l_2^j - p(1-t)w_2 l_2^j \tilde{r}_2$ and where $\tilde{q} = q_t t_x + q_x$ for $q = w_1, w_2, r_1$ and r_2.[16] If we restrict attention to the normal case of section 4, then we have $\tilde{w}_1 < 0$; $\tilde{r}_1 > 0$; $\tilde{w}_2 > 0$; $\tilde{r}_2 < 0$. Further, $(a^n\tilde{r}_1 + l_1^n\tilde{w}_1) = \tilde{y}_1 - w_1 l_1^n = 0$. But since the poor are likely to have a higher proportion of labour to capital income, we can conclude that $a^j\tilde{r}_1 + l_1^j\tilde{w}_1 < 0$ for $j=1, \ldots, n-1$. Thus of the four terms

16. Changes in labour supplies drop out due to the envelope theorem.

above, since the poor are likely to have less initial assets, the first bracketed term will be positive so that the first term is negative. The second term is negative. The third term is negative and only the fourth term, which shows the increase in the present value of second period labour income due to increased factor prices is likely to be positive. That is, since the second period capital labour ratio is higher, the present value of second period wages increases and since wage income is likely to make up a larger fraction of a poor persons total 'income', this is likely to lower inequality. We note that a perverse savings response will ensure that greater inequality will occur with a move to expenditure taxes since, in this case, even the fourth term will be negative. However, even if this is not so the common presumption of rising inequality when expenditure taxes replace income taxes is likely to hold true though the question is, once more, empirical.[17]

6. Conclusions and Further Questions

The analysis in this study casts doubt on the efficiency and savings promoting properties of expenditure taxes as compared to income taxes. In a second best world with a revenue constraint it is likely that a mix of both taxes leads

17. See Auerbach and Kotlikoff (1983) for some interesting simulations. Also, it may be pointed out that Allingham (1979) has an example in which progressive income taxes raise inequality.

to greater efficiency than either tax alone. Further, three arguments have been advanced to show that the savings promoting properties of expenditure taxes are suspect. These are (i) Expenditure taxes implicitly tax assets and transfers. (ii) The income tax rate in the expenditure tax equivalent tax system is likely to be higher than the equal yield income tax rate. (iii) Not only is it likely that income taxes promote human savings, but, when general equilibrium factor price responses are looked at, expenditure taxes actively discourage human savings[1]. Finally, in section 5 we show that the redistributive properties of income taxes are likely to remain intact both within and across generations as compared to expenditure taxes.

Two directions in which additional research may be fruitful are thus suggested. Firstly, it was suggested in the introduction that the principle of horizontal equity might be violated by expenditure taxes (given that they fall disproportionately on the young) in the presence of capital market imperfections. In addition, the importance of the labour supply response identified in this study will depend on the operation of the market for education to a large extent - especially in view of the discrimination which is popularly believed to afflict this market. Thus a study of the two taxes in a world with imperfect markets is clearly suggested.

Secondly, we have suggested that a mix of taxes may be savings maximizing and some other mix of taxes may be current welfare maximizing. Empirical work to determine these mixes clearly has useful policy implications. We hope to look at these questions at a future date.

Appendix I: Derivation of equations (3.3) and (3.4)

We require the lifetime budget constraint $y(1-t)=(c+pk)(1+x)$ and the corresponding savings definition $y(1-t)-c(1+x)=s = pk(1+x)$.

We have also the tax yield equation

$T=t(y+prs)+x(c+pk)= t(y+p^2rk(1+x))+x(c+pk)$.

To derive equations (3.3) and (3.4) we totally differentiate the lifetime budget constraint and the tax yield equation (setting $dT=0$) and substitute for dt or dx from the latter into the former equation.

The total differentials of the budget equation is

$$-(y+p^2rk(1+x))dt = (c+pk)dx+(1+x)dc + p(1+x)dk \qquad A1.1$$

Likewise, the total differential of the yield equation is

$$dT = 0 = Ddt + Fdx+xdc +(xp+tp^2r(1+x))dk, \text{ where} \qquad A1.2$$

$D= y+p^2rk(1+x)+ tp^3r^2k(1+x)+ xp^2rk$ and

$F= c+pk+tp^2rk$

From A1.2 we can solve for dx or dt to get

$$dt = -\frac{Fdx}{D} - \frac{xdc}{D} - \frac{(xp+tp^2r(1+x))dk}{D} \text{ and}$$

$$dx= -\frac{D}{F} dt - \frac{x}{F} dc -\frac{(xp+tp^2r(1+x)dk}{F}$$

We first consider equation 3.3.substituting for dt in A1.1 gives

$udx= vdc+wdk$, where

$u= (y+p^2rk(1+x))F-D(c+pk)$, $v=(1+x)D-x(y+p^2rk(1+x))$ and

$w=p(1+x)D - (xp+tp^2r(1+x))(y+p^2rk(1+x))$

u,v and w may be simplified as follows

$$u = (y+p^2rk(1+x))(c+pk+tp^2rk)-(c+pk)(y+p^2rk(1+x)+ tp^3r^2(1+x)+ xp^2rk) \qquad A1.3$$

cancel out $(y+p^2rk(1+x))(c+pk)$ to get

$$(y+p^2rk(1+x))tp^2rk - (c+pk)(tp^3r^2k(1+x)+ xp^2rk)$$

factor out p^2rk to get

$$p^2rk(ty+ tp^2rk(1+x)-tpr(1+x)(c+pk)-x(c+pk))$$

since $(c+pk)(1+x)=y(1-t)$ and since $1-pr(1-t) = p$ we get

$$p^2rk(pty+ tp^2rk(1+x) - x(c+pk))$$

making use of the equation for T, we can rewrite this as

$$u = p^2rk(T_y-T_c-z) \text{ where } z = (1-p)ty > 0$$

We can, similarly, solve for v and w

On doing so we get

$$v=y+p^2rs(1+r)(1+x) > 0 \text{ and}$$

$$w=py(1-tpr(1+x)+p^2rs(1+x) > 0.$$

Likewise we have, for equation 3.4 (on substituting for dx):

$$-udt = v'dc+ w'dk, \text{ where}$$

$$v' = (1+x)F-x(c+pk) = (c+pk) +(1+x)tp^2rk > 0 \text{ and}$$

$$w' = p(1+x)F-(xp+ tp^2r(1+x))(c+pk) = p(c+pk)-(1+x)tp^2rc > 0.$$

Appendix II : Ability to Save, interest rates and Taxes: Some examples

In this Appendix we present examples which show the following

1. Ability to save may be higher, equal or lower with an income tax as compared to an equal yield expenditure tax (Tables A1, and A2).
2. It is possible for the ability to save to be at a maximum either with expenditure taxes alone or with a mix of income and expenditure taxes if yields are held constant (Table A3).

With regard to the possibility of the ability to save increasing monotonically with income tax rates, no example or disproof has been found. All the examples constructed hold consumption (or the average propensity to consume in the first period) constant and examine the changes in savings which must then occur at different taxes. It may be recalled that the term "ability to save" has been defined in precisely this fashion in the Text. The model, in all cases, is that of section 3 of the paper.

Table A1 gives, first of all, the expenditure tax rate (x) and, secondly, the limiting income tax rate (t) which obtains when the interest rate is infinite. SX and St are the associated savings rates. By savings rate is meant the ratio of savings to initial income (y). An asterisk against St indicates that it exceeds SX.

Each row in table A2 gives a case similar to that in table 1 except that the interest rates are now finite (except for the last row). In all the cases of Table A2 St exceeds SX.

Table A3 contains six examples of how ability to save varies with the tax mix. The first three differ only with respect to the interest rate used. The interest rates have been chosen so that St is alternately greater than, equal to and less than SX. The next two examples are constructed using what are, perhaps, more reasonable values of consumption and tax collection. In these five examples, ability to save is maximized with a mixture of income and expenditure taxes. In the sixth example, ability to save is maximized when only expenditure taxes are levied. In this example, values of second period consumption are also presented to show that they remain u-shaped.

We now present the formulae used in computations. It is to be noted that y has been set equal to 1 throughout so that (for example) consumption and the average propensity to consume are identical.

For table A1, t is calculated from

$T - (T+2-C)t + 2t^2 = 0$ and X is calculated from $x = T/(1-T)$

St and SX are then calculated from $St = 1-C-t$ and $SX = 1-C(1+X)$.

When r is not infinite as in tables A2 and A3 t is calculated from

$T(1+r) - ((T+2-C)r+1)\ t + 2rt^2 = 0$ when X=0 and

$(T-2)(1+r) - ((T+w)r + (1+r) - 2(1+2r))t + r(2-Z)t^2 = 0$ when

$x \neq 0$. In this formula $Z = x(1+X)$ and $w = 1-C(1+x) = t+S$.

The different values of x are considered one by one with values from zero to $x = T(1-T)$.

Table A1 : Ability to save with income and expenditure taxes: Selected examples

C	T	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
C	x	0.1111	0.25	0.4286	0.6667	1.0	1.5	2.3333	4.0
0.1	Sx	0.89	0.875	0.857	0.833	0.8	0.75	0.67	0.5
	St	0.8472	0.7941	0.7405	0.6864	0.6317	0.576	0.5193	0.4608
	t	0.0528	0.1059	0.1595	0.2136	0.2683	0.324	0.3807	0.4392
0.2	Sx	0.778	0.75	0.714	0.67	0.6	0.5	0.3333	
	St	0.7441	0.6873	0.6294	0.5702	0.5089	0.4449	0.3513*	
	t	0.0559	0.1127	0.1706	0.2298	0.2911	0.3551	0.4484	
0.3	Sx	0.67	0.625	0.571	0.5	0.4	0.25		
	St	0.6405	0.5794	0.5162	0.45	0.3791	0.3*		
	t	0.0595	0.1206	0.1838	0.25	0.3209	0.4		
0.4	Sx	0.56	0.5	0.429	0.33	0.2			
	St	0.5364	0.4700	0.4	0.3236	0.2351*			
	t	0.0636	0.1298	0.2	0.2764	0.3649			
0.5	Sx	0.45	0.375	0.286	0.167				
	St	0.4317	0.3589	0.2791	0.1851*				
	t	0.0683	0.1411	0.2209	0.3149				
0.6	Sx	0.33	0.25	0.143					
	St	0.326	0.2449	0.15*					
	t	0.074	0.1551	0.25					
0.7	Sx	0.22	0.125						
	St	0.2193	0.1266*						
	t	0.0807	0.1734						
0.8	Sx	0.11							
	St	0.1108*							
	t	0.0892							

Notes: 1. Initial income has been normalized to 1.
2. C is first period consumption.
3. T is the present value of the tax yield.
4. x and t are the income and expenditure tax rates. Sx and St are the associated abilities to save.
5. t and St correspond to the limiting case of an infinite interest rate on saving.

Table A2 : Ability to save with income and expenditure taxes: Further examples

C	T	r	x'	t	SX	St
0.2	0.7	10	2.3333	0.4564	0.3436	0.3436
0.3	0.65	3	1.8571	0.5417	0.1429	0.1583*
0.4	0.55	3	1.2222	0.4383	0.1111	0.1162*
0.7	0.25	∞	0.3333	0.2289	0.067	0.0711*

Notes: 1. See Table A1, notes 1-4.

2. St > SX in the last row provided r greater than 6.

Table A3 : Ability to save and the income tax - expenditure tax mix: Some examples (all figures in percentages)

Example 1 : T=50 C=40 x	(a) r=300 t	s	(b) r=500 t	s	(c) r=700 t	s	Example 2 : T=30 C=60 x	(a) r=600 t	s	(b) r=900 t	s	Example 3: T=10 C=70, r=500 x	t	s	K
0	41.67	18.33	40.00	20.00	39.13	20.87	0	25.98	14.02	25.70	14.30	0	8.38	21.62	120.8
20	32.26	19.54	31.23	20.77	30.61 (88.4)	21.39	10	19.81	14.19	19.61	14.39	3	5.83	22.07	122.4
40	23.86	20.14	23.02 (78.0)	20.97	22.62	21.38	20	13.71	14.29	13.59 (85.7)	14.41	6	3.72	22.08	121.1
60	15.42 (68.3)	20.58	15.18	20.32	14.93	21.07	30	7.68 (80.25)	14.32	7.62	14.38	9	1.53	22.16	120.3
80	7.76	20.24	7.53	20.47	7.43	20.57	40	1.71	14.29	1.69	14.31	11.11	0.00	22.3	120.3
100	0.00	20.00	0.00	20.00	0.00	20.00	42.86	0.00	14.286	0.00	14.286				

Notes: 1. T = Tax yield as a percentage of first period income.
C = First period consumption as a percentage of first period income.
S = Ability to save as a fraction of first period income.
t, x = Income and expenditure tax rates.
r = Interest rate on savings.
K = Second period consumption (cum bequests) as a percentage of first period income.

2. Figures in parentnesis are percentage of taxes collected through income taxation. Only reported in the maximum ability to save cases.

3. Since the first period is of the order of 30 years, reasonable interest rates which correspond to annual interest rates between 2 and 12 per cent are between 64% and 3000%.

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