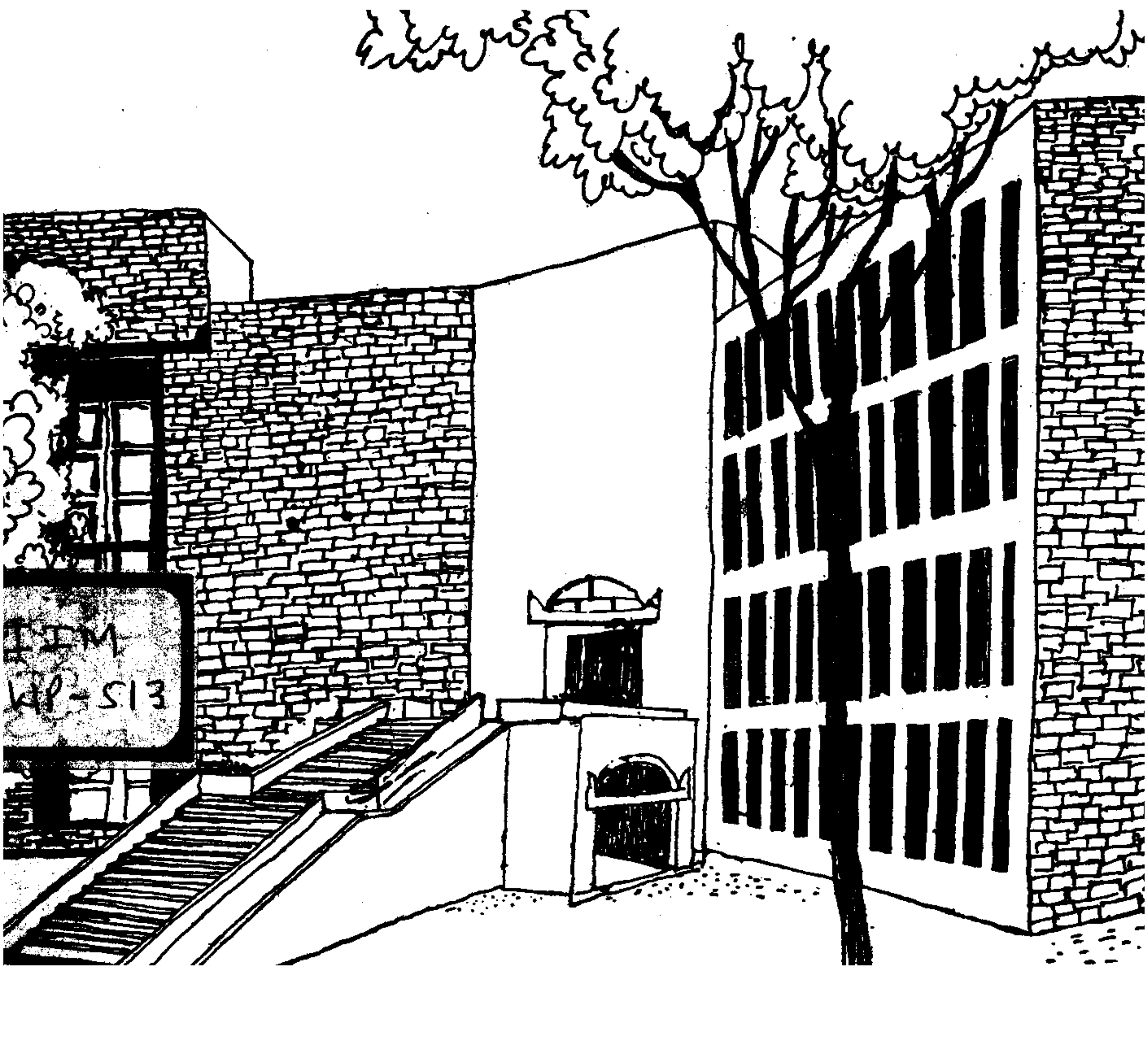
विद्याविनियोगाद्विकासः
IIM
AHMEDABAD

W. P. 513

Working Paper



MULTI-ITEM INVENTORY WITH MULTIPLE RESTRICTIONS

By

Arabinda Tripathy

W P No. 513
June 1984

INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD-380015
INDIA

'Multi-Item Inventory with Multiple Restrictions'

Arabinda Tripathy
(Indian Institute of Management, Ahmedabad)
India

Abstract: The normal procedure to deal with multi-item inventory situation with restrictions is to obtain the e.o.q. of all the items individually and to check for violation of restrictions assuming that the replenishment of all the items may occur simultaneously. In case of violation of restrictions the problem is solved by using the well known lagrangean multiplier method. The lagrangean multiplier method is quite tedious even for one restriction and results in only fifty percent utilization of the constrained resource. The equal order interval for multiitem inventory with one restriction results in more than fifty percent utilization of the constrained resource and it is also computationally very simple. The principle of equal order interval has been used here for two restrictions situation and an working procedure for the same has been evolved.

(Key words - Lagrange multiplier, equal order interval, multi-item inventory)

1. Introduction:- The usual lot size inventory model assumes that there is no interaction between the items and determines the replenishment policy of the items individually. More often than not the assumption that there is no interaction between the items is not valid. The interactions can be of many types. For example, the demand of one product is dependent on that of the other, one product is a substitute of the other, limitation of warehouse space to be shared by all the items, limits on the maximum amount that can be used for financing the inventory of all the items, maximum number of orders that can be placed in a time period and so on. The interaction between items caused by restriction on warehouse space, restriction of funds for financing inventory etc. are discussed here. Many authors have dealt with the problem, specially in relation to one restriction. (1,2,3,4,5,6,7). The interaction between items due to two constraints makes the problem much more difficult to solve, [3].

The analysis presented here considers two constraints, both limiting the maximum amount of inventory holding in a multi-item inventory situation. Both the constraints are linear with respect to the quantity of the items. In particular the warehouse space and monetary restrictions on inventory holding have been considered here.

Let

i	=	the designation of the item where there are n items.
R_i	=	demand per unit time of the i^{th} item
C_i	=	order cost per order of the i^{th} item

I_i = inventory carrying cost per unit of i th item per unit time

P_i = unit cost of item i.

W_i = unit volume of item i.

M = maximum amount of money that can be utilised for holding inventory of all the items.

V = maximum volume (space) available for holding inventory of all the items.

Q_i = order quantity of item i

$\bar{Q}_i$ = economic order quantity of item i

We consider here a purely deterministic situation and the replenishment of items to be instantaneous.

2. Single Constraint Situation:

In the single constraint situation, there is either a limit on the available space or a limit on the maximum amount of money that can be used for holding inventory of all the items.

Initially we ignore the existence of a constraint. In such a case there is no interaction between the items and the optimal inventory policy for individual items will also be optimum for the entire system.

The optimum inventory policy of individual item is to follow the economic order quantity in order to minimise the total variable cost. The economic order quantity for individual item is given by,

$$Q_i = \sqrt{\frac{2R_iC_i}{I_i}} \qquad (1)$$

In the above analysis it has been assumed that there is no interaction between the items, hence, there is a possibility that a time will come when all the n items, will be replenished simultaneously. The maximum amount of money required (assuming that the single constraint is the monetary constraint) in such a situation is

$M_{e.o.q.}$ = Maximum amount of money required when e.o.q. policy is followed for all the items.

$$= \sum_{i=1}^{n} \bar{Q}_i * P_i \qquad (2)$$

The e.o.q. policy for individual item will still be optimum for the system with single constraint on the monetary limit if

$$M_{e.o.q.} \leq M \qquad (3)$$

(A similar analysis can be made for a single constraint of space limit).

However if the above condition is not satisfied the solution is infeasible and we have to find out new optimum solution.

3. Lagrangean Multipliers:

The problem of minimising the total variable cost in an inventory system with one constraint can be stated mathematically as follows:

Minimise Total Cost = Order Cost + Inventory Carrying Cost

P1 i.e. Minimise Total Cost

$$T.C. = \sum_{i=1}^{n} \frac{R_i C_i}{Q_i} + \frac{1}{2} \sum_{i=1}^{n} I_i Q_i \qquad (4)$$

$$\text{s.t} \quad \sum_{i=1}^{n} P_i Q_i \leq N \qquad (5)$$

$$Q_i \geq 0$$

Using lagrange multiplier ' λ ' to dualise the constraint and incorporating in the objective function we get,

LP1 Minimise

$$\text{Total Cost} = \text{T.C.} = \sum_{i=1}^{n} \frac{R_i C_i}{Q_i} + \frac{1}{2} \sum_{i=1}^{n} I_i Q_i + \sum \left(\sum_{i=1}^{n} P_i Q_i - M \right) \qquad (6)$$

s.t.

$$Q_i \geqq 0$$

$$\lambda \geqq 0$$

Taking partial derivative with respect to the variables we get, for optimality

$$\frac{\partial \text{ T.C.}}{\partial \lambda} = 0 \; ; \; \frac{\partial \text{ TC}}{\partial Q_i} = 0$$

$$\text{or} \quad \sum_{i=1}^{n} P_i Q_i - M = 0 \qquad (7)$$

$$\text{and} \quad Q_i^* = \sqrt{\frac{R_i C_i}{\frac{1}{2} I_i + \lambda P_i}} \qquad (8)$$

This method of analysing the constrained inventory system with lagrange multipliers is very common and many authors have dealt with this type of analysis.

Substituting the value of Q_i^* from (8) in the L.H.S. of (7) we get the function

$$\sum_{i=1}^{n} P_i \frac{R_i C_i}{\frac{1}{2} I_i + \lambda P_i} - M \; ;$$

which is a monotone decreasing function of λ ; consequently there is unique $\lambda^* \geqq 0$ such that (7) is satisfied.

It can be seen that the average amount of money required for holding inventory with replenishment quantities computed by lagrangean multipliers is only 50% of the maximum permissible limit. That is if M is the maximum amount of money available for holding inventory, the average requirement of money will be $\frac{1}{2}M$ when the replenishment quantities are computed using the lagrange multipliers.

4. <u>Equal Order Interval:</u> In the equal order interval method, the order interval, i.e, the time between replenishments, of all the items is the same. Hence, whatever initial temporal position the items are given will be maintained with time. With 't' as the common order interval of all the items, the total variable cost

$$T.C. = \sum_{i=1}^{n} \frac{C_i}{t} + \frac{1}{2} \sum_{i=1}^{n} I_i R_i t \qquad (9)$$

For the T.C. to be minimum,

$$\frac{\partial T.C.}{\partial t} = 0$$

which results in t_{opt} = optimal value of $t = \sqrt{\dfrac{2 \sum_{i=1}^{n} C_i}{\sum_{i=1}^{n} R_i I_i}}$ (10)

The maximum amount of money required for holding inventory by using same order interval of t_{opt} for all the items, as computed in [6].

M_{opt} = Maximum amount of money required with equal order interval of t_{opt}

$$= \frac{1}{2} t_{opt} \left\{ \sum_{i=1}^{n} P_i R_i + \frac{\sum_{i=1}^{n} P_i^2 R_i^2}{\sum_{i=1}^{n} P_i R_i} \right\} \qquad (11)$$

t_{opt} is the desired equal order interval if

$$M_{opt} \leqq M \qquad (12)$$

New equal order interval has to be computed if condition (12) is not satisfied.

Let t_m be the equal order interval taking into account the monetary constraint, then from (11)

$$t_m = \frac{M}{M_{opt}} * t_{opt}$$

With t_m as the order interval of all the items, the staggering of the replenishments of the items need to be decided. In an optimal staggering, the requirement of the constrained resource by the replenishment of an item is the amount of constrained resource released due to the consumption of all the items during the period between the previous replenishment in the system and the present replenishment of the item. (Figure-1).

If t_i^m is the interval between the replenishment of the i^{th} item and the previous replenishment in the system, then [6]

$$t_i^m = t_m \frac{P_i R_i}{\sum_{i=1}^{m} P_i R_i} \qquad (13)$$

Obviously $\sum_{i=1}^{n} t_i^m = t_m$

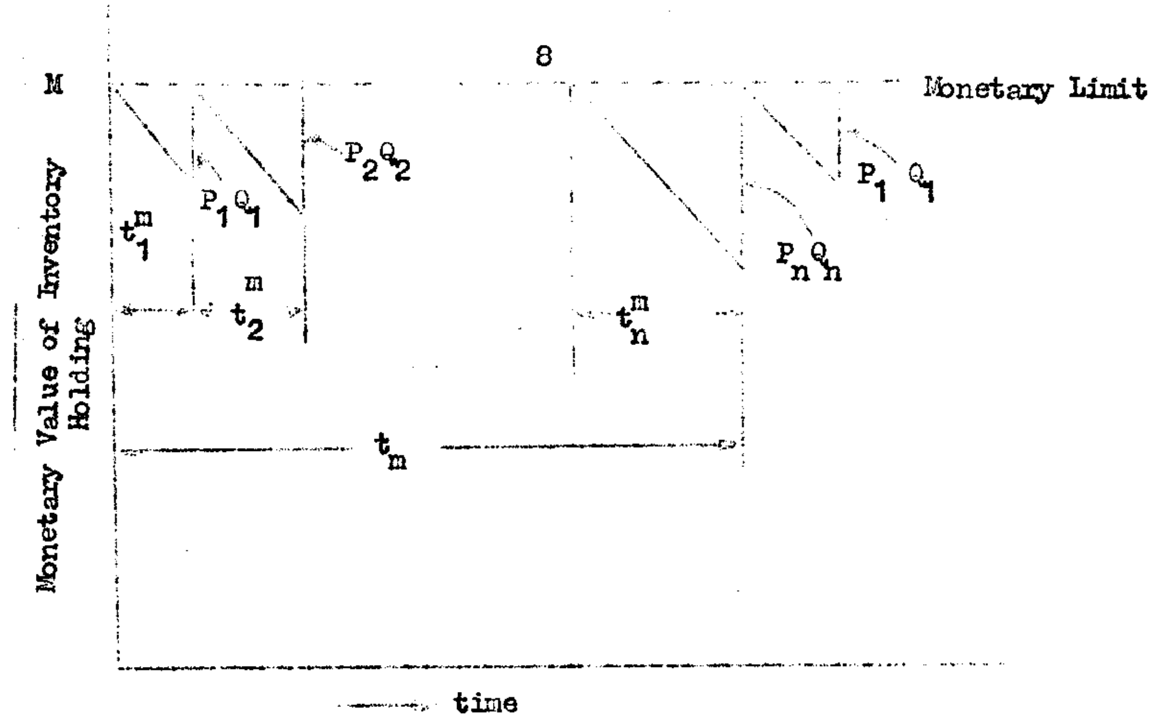


Fig. 1: Monetary Value of Inventory Holding Over Time

The average utilization of constrained resource with t_m as the order interval for all the items works out to be [6]

$$\text{Average utilization of constrained resource} = \frac{\sum_{i=1}^{n} P_i R_i}{\sum_{i=1}^{n} P_i R_i + \dfrac{\sum_{i=1}^{n} P_i^2 R_i^2}{\sum_{i=1}^{n} P_i R_i}} \qquad (14)$$

It can be seen by inspection that the right hand side of (14) is ≥ 0.5, i.e. the utilization of the constrained resource will not be less than 50%, where as it is 50% in case of lagrange multiplier.

5. Two Constraint Situation:

In a two constraint situation, taking one of them to be the maximum monetary limit and the other constraint to be the maximum warehouse space, both of which are directly proportional to the order quantity of the items, the problem can be mathematically stated as follows:

[P2] Minimise total cost = T.C. = $\sum_{i=1}^{n} \frac{R_i C_i}{Q_i} + \frac{1}{2} \sum_{i=1}^{n} I_i Q_i$

Subject to, $\sum_{i=1}^{n} P_i Q_i \leq M$ (5')

$\sum_{i=1}^{n} W_i Q_i \leq V$ (15)

$Q_i \geq 0$

As earlier the policy of using economic order quantities $\bar{Q}_i$ for all the items (i = 1,- - - - - - - - n) will be optimal for the system represented by the problem [P2], if in addition to satisfying the inequality (3), the inequality

$$V_{e.o.q.} \leq V \qquad (16)$$

is satisfied.

Where,

$V_{e.o.q.}$ = Maximum space required when e.o.q. policy is used for all the items

$$= \sum_{i=1}^{n} \bar{Q}_i * W_i \qquad (17)$$

However, if the inequalities (3) and (16) are not satisfied by the $\bar{Q}_i$, e.o.q. solution is infeasible with respect to problem [P2] and new Q_i values need to be determined. One of the option in this case is to use lagrange multiplier method as has been used for problem [P1]

The lagrangean problem with lagrange multipliers λ_1 and λ_2 for the constraints (5') and (15) respectively in problem [P2] is

[LP2] Minimise T.C. $= \sum_{i=1}^{n} \frac{R_i C_i}{Q_i} + \frac{1}{2}\sum_{i=1}^{n} I_i Q_i + \lambda_1 \left(\sum_{i=1}^{n} P_i Q_i - M \right)$

$$+ \lambda_2 \left(\sum_{i=1}^{n} W_i Q_i - V \right) \qquad (18)$$

s.t

$$Q_i \geqq 0$$
$$\lambda_1 \geqq 0$$
$$\lambda_2 \geqq 0$$

For optimality:

$$\frac{\partial TC}{\partial Q_i} = 0$$

i.e. $$Q_i^* = \sqrt{\frac{R_i C_i}{\frac{1}{2} I_i + \lambda_1 P_i + \lambda_2 W_i}} \qquad (19)$$

$$\frac{\partial TC}{\partial \lambda_1} = 0$$

i.e. $$\sum_{i=1}^{n} P_i Q_i - M = 0 \qquad (7')$$

$$\frac{\partial TC}{\partial \lambda_2} = 0$$

i.e. $$\sum_{i=1}^{n} W_i Q_i - V = 0 \qquad (20)$$

The difficulties in numerically evaluating Q_i^* increases considerably when the number of constraints imposed on the system is more than one. In case of two constraints (as in [P2]) and when the $\overline{Q}_i$ is not a solution to the problem, the procedure is to determine Q_i taking only one constraint by adopting a procedure similar to that used for solving [P_1]. The solution so obtained is checked for feasibility for the second constraint. If feasible it is the desired solution. In infeasible, the problem is again solved for Q_i by considering the second constraint only. The new values of Q_i are checked for feasibility in the first constraint. If feasible, it is the desired solution. If infeasible, we have to solve the problem by taking into account both the constraints. This is considerably difficult to solve. More often than not a trial and error technique is used. However, techniques like Networks method, the method of steepest descent, subgradient optimization etc. can also be used.

The equal order interval method can also be used in the case of two constraints problem [P2]. The two constraints equal order interval problem is also represented by (9) as in case of single constraint problem. The t_{opt} and M_{opt} are also obtained respectively by (10) and (11). The maximum space requirement in this case will be given by

V_{opt} = maximum space requirement with equal order interval of t_{opt}

$$= \frac{1}{2} t_{opt} \left\{ \sum_{i=1}^{n} W_i R_i + \frac{\sum_{i=1}^{n} W_i^2 R_i^2}{\sum_{i=1}^{n} W_i . R_i} \right\} \qquad (21)$$

In this two constraint problem, apparently t_{opt} is the desired equal order interval if

$$M_{opt} \leqq M \qquad (12')$$

and

$$V_{opt} \leqq V \qquad (22)$$

If any or both of (12') and (22) are not satisfied, then new equal order interval need to be computed. In this case a procedure similar to that for lagrange multiplier can be adopted. That is, we can compute equal order interval by considering only one constraint at a time and check for feasibility of the second constraint. In case it is not feasible compute equal order interval by considering the second constraint check for the feasibility of the first constraint. However, the computational aspects of equal order interval has some special characteristics.

As has been discussed earlier,

t_m = equal order interval with monetary constraint

$$= \frac{M}{M_{opt}} \; t_{opt}$$

Similarly,

t_v = equal order interval with space constraint

$$= \frac{V}{V_{opt}} \; t_{opt}$$

Since, both V and M are directly proportional to the equal order interval, the lower of the t_m and t_v can be taken to be the desired equal order interval, which will satisfy both the constraints. The replenishment intervals between items are computed by relations similar to (13) on the basis of the constraint which is used for computing the desired equal order interval. The possibility of violating the constraints occur at the time of replenishments of the items. Therefore, none of the two constraints should be violated during the replenishments of any of the n items which are staggered in equal order interval operating policy.

Let t_m be the lower of t_n and t_v and as such it is the desired equal order interval, on the assumption that this will result compliance with both the constraints. The inter-item replenishment interval t_i^m can now be obtained by using the relationship (13). The use of t_m and t_i^m as the operating parameters for the equal order interval policy of the inventory system ensures that the monetary constraint will not be violated. However, it is likely, that the space constraint may be violated during the replenishment of one or more items with the above operating policy. This results in the problem being still unsolved. Due to this characteristics of the equal order interval policy, even if (12') and (22) are satisfied by t_{opt}, still the limits may be exceeded during the replenishment of the items, which need to be ascertained before using t_{opt}.

6. <u>Example:</u> Table 1 gives the necessary data for the problem. Some part of the data is same as in Hadley and Whitin [3]

Table 1: Data for multiitem two constraints inventory system

Item No.	i	1	2	3	4
Annual Demand	R_i	1000	500	2000	1500
Order Cost	C_i	50	75	100	125
Inventory Carrying Cost	I_i	4	20	10	14
Unit Cost	P_i	20	100	50	80
Unit Volume	W_i	2	1	3	3.5
Economic Order Quantity	$\overline{Q}_i$	158	61	200	163.7

A three item inventory system with item no.1,2 and 3 is being considered, with M = 14,000 and V = 700.

$$M_{e.o.q.} = \sum_{i=1}^{3} \overline{Q}_i \; P_i = 19,260$$

$$\text{i.e. } M_{e.p.q.} > M$$

$$V_{e.o.q.} = \sum_{i=1}^{3} \overline{Q}_i \; W_i = 977$$

$$\text{i.e. } V_{e.o.q.} > V$$

So both the constraints are violated with e.o.q. policy.

Optimal equal order interval,

$$t_{opt} = \sqrt{\frac{\sum_{i=1}^{3} 2\, C_i}{\sum_{i=1}^{3} R_i\, I_i}} = 0.115$$

With this equal order interval using (11) and (21)

$M_{opt} = 14,138$

and $V_{opt} = 761$

i.e. $M_{opt} > M$ and $V_{opt} > V$

Hence, t_{opt} is not the desired equal order interval and new equal order interval need to be computed with consideration of both the constraints, which are as follows:

$t_m = 0.1139$

$t_v = 0.1058$

Since $t_v < t_m$, we can use t_v as the equal order interval. The maximum space requirement with t_v will be equal to V the limit and the maximum money requirement as computed by (11) by substituting t_{opt} by t_v will be less than M the permissible limit. The inter-item replenishment interval t_i^v computed by using t_v are as follows:

$t_1^v = 0.0249$

$t_2^v = 0.0062$

$t_3^v = 0.0747$

The requirement of space and money after the replenishment of each item with t_v as the common order interval and using the corresponding inter-item replenishment interval are:

m_1	-	10516
v_1	-	700
m_2	-	14752
v_2	-	700
m_3	-	12633
v_3	-	700

where m_i - total money in inventory after the replenishment of the i^{th} item and v_i - total space after the replenishment of the i^{th} item.

As can be seen the space requirement reaches the limiting value of 700 after each replenishment as per our expectation, but the money value after each replenishment is different, and in fact, contrary to our expectation, the money value after replenishment of the 2nd item is 14752 which is more than the limit of 14000. This can be explained by the absence of any relationship between P_i and W_i of an item 'i'. Because of this the monetary value after the replenishment of an item when t_v is the operating order interval and vice-versa will depend on the sequence of replenishment of the items. The number of different sequence possible with n items is $(n-1)!$. The Table-2 below furnishes the details of money and space requirements after each replenishment for both the situation of money constraints and space constraint being the operating principle.

Table – 2: Money and Space Requirement For Different Sequence in Equal Order Interval with Three Items

	Money Constraint		Space Constraint	
Sequence	1-2-3	1-3-2	1-2-3	1-3-2
m_1	14000	14000	10516	13631
v_1	848*	680	700	700
m_2	14000	14000	14752*	15748*
v_2	620	566	700	700
m_3	14000	14000	12633	11512
v_3	734	794*	700	700

* Maximum value of the constraint not considered for the computation of equal order interval.

The total number of possible sequence with three items is (3-1)! , i.e 2. It can be seen from Table - 2, that the maximum value of the constraint not used for computing the equal order interval is different for different sequence and the maximum occurs not necessarily during the replenishment of the same item.

7. Four <u>Items Situation:</u> Considering all the four items of Table-1, we have the following situation, assuming

$M = 16,000$ and $V = 800$;

For e.o.q policy

$M_{e.o.q.} = 32,356 > M$

$V_{e.o.q.} = 1,550 > V$

Following equal order interval policy

$$t_{opt} = 0.1128$$

$M_{opt} = 21665$; $V_{opt} = 1054$

Here also $M_{opt} > M$ and $V_{opt} > V$

The requirement of money and space using t_m and t_v for different sequence (a total of 6 sequence in this case as (4-1)! = 6) are presented in Table-3.

As in case of three items, here also the maximum value of the constraint not used for computing the equal order interval is different for different sequence and the maximum occurs not necessarily during the replenishments of the same item. Further it can be conjectured from the behaviour of the equal order interval that even if only one constraint is

Table - 3: - Money and Space Requirements For different Sequence in equal order interval with four items (Problem No.1)

Sequence	1-2-3-4	1-2-4-3	1-3-2-4	1-3-4-2	1-4-3-2	1-4-2-3
Money Constraint						
m_1	16000	16000	16000	16000	16000	16000
v_1	840*	896*	768	709	766	838*
m_2	16000	16000	16000	16000	16000	16000
v_2	683	740	717	621	677	644
m_3	16000	16000	16000	16000	16000	16000
v_3	789	808	873*	815*	834*	750
m_4	16000	16000	16000	16000	16000	16000
v_4	752	703	680	778	729	800
Space Constraint						
m_1	14701	13483	16261	17524	16306	14746
v_1	800	800	800	800	800	800
m_2	18082*	16864	17356	19408*	18190*	18916*
v_2	800	800	800	800	800	800
m_3	15796	15367	13975	15238	14809	16630
v_3	800	800	800	800	800	800
m_4	16585	17653*	18145*	16027	17095	15535
v_4	800	800	800	800	800	800

* Maximum value of the constraint not considered for the computation of equal order interval.

effective, using the corresponding equal order interval and the inter-item replenishment interval, may result in violation of the in-effective constraint after the replenishment of some of the items in some of the sequences. E.g. if the monetary constraint is effective and the space constraint is not, then the use of t_m and the corresponding inter-item replenishment interval may result in violation of space constraint during the replenishment of some of the items in some of the replenishment sequences. This necessitates in two constraint multi-item inventory situation, to find out a sequence of replenishment of items with an equal order interval (t_m or t_v) and respective inter-item replenishment intervals, such that none of the constraints are violated. In case such a situation does not exist (as in the two cases discussed in Table-2, and Table-3), it will be worthwhile to find out the sequence with equal order interval and corresponding inter-item replenishment intervals such that the maximum value of the constraint not considered for the computation of the order interval is the least. This sequence can be used to generate a lower cost solution with better utilization of the constrained resources. Table No.4 to 9 incorporates similar information as in Table-3, for six other test problems with four items. The summary of the text problem results have been furnished in Table-10. The following observations can be made from Table-10.

i) The sequences which result in lowest maximum of the resource constraint not used for computing order interval are reverse of one another for the two constraints. E.g. sequence 1-3-4-2 in Problem 1 results in lowest maximum of space requirement when order interval is computed by considering space constraints. These two sequences are reverse of one another. Same is the case with the other problems. This in a sense suggests that the desired sequence is dependent to some degree on the ratio P_i/W_i (or W_i/P_i as the case may be).

Table No.4 - Four Item Situation - Problem No.2

i	1	2	3	4
P_i	10	80	30	150
W_i	5	2	4	3

Sequence	1-2-3-4	1-2-4-3	1-3-2-4	1-3-4-2	1-4-3-2	1-4-2-3
Money Constraint						
m_1	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_1	756.94	1048.11*	707.46	716.02	1007.19*	1056.67*
m_2	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_2	679.86	971.03	929.17	432.47	723.63	474.33
m_3	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_3	978.65*	764.55	1006.24*	1014.81*	800.71	773.11
m_4	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_4	473.38	465.77	423.9	509.54	501.93	551.41
Space Constraint:						
m_1	13445.31	8035.35	14364.64	14205.53	8795.58	7876.24
v_1	800.00	800.00	800.00	800.00	800.00	800.00
m_2	14877.35	9467.4	10245.30	19474.04*	14064.09	18696.13*
v_2	800.00	800.00	800.00	800.00	800.00	800.00
m_3	9325.96	13303.87	8813.26	8654.14	12632.04	13144.75
v_3	800.00	800.00	800.00	800.00	800.00	800.00
m_4	18713.81*	18855.25	19633.15*	18041.99	18183.43*	13144.75
v_4	800.00	800.00	800.00	800.00	800.00	800.00

* Maximum value of the constraint not considered for the computation of equal order interval.

Table No. 5 — Four Item Situation — Problem No. 3

i	1	2	3	4
P_i	25	80	60	175
W_i	2	6	8	10

Sequence	1-2-3-4	1-2-4-3	1-3-2-4	1-3-4-2	1-4-3-2	1-4-2-3
Money Constraint m_1	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_1	1134.96	1403.7 *	1103.61	1124.61	1393.34*	1424.69
m_2	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_2	1121.81	1390.54	1401.74	1101.51	1370.24	1090.32
m_3	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_3	1433.093*	1380.606	1414.09*	1435.09*	1383.40	1401.6
m_4	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_4	1111.87	1069.32	1080.52	1114.67	1072.12	1103.47
Space Constraint						
m_1	10708.23	8608.14	10953.24	10789.17	8689.09	8444.08
v_1	800.00	800.00	800.00	800.00	800.00	800.00
m_2	10811.05	8710.96	8623.46	10969.65*	8869.56	11057.15*
v_2	800.00	800.00	800.00	800.00	800.00	800.00
m_3	8378.45	8788.62	8520.64	8356.57	8766.74	8624.55
v_3	800.00	800.00	800.00	800.00	800.00	800.00
m_4	10888.71*	11221.22*	11133.72*	10866.83	11199.34*	10954.33
v_4	800.00	800.00	800.00	800.00	800.00	800.00

* Maximum value of the constraint not considered for the computation of equal order interval.

Table No.6 - Four Item Situation - Problem No.4

i	1	2	3	4
P_i	15	125	80	50
W_i	4.5	8	4	7

Sequence	1-2-3-4	1-2-4-3	1-3-2-4	1-3-4-2	1-4-3-2	1-4-2-3
Money Constraint						
m_1	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_1	1578.19*	1318.37	1611.87*	1526.17*	1266.34	1232.66
m_2	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_2	1472.94	1213.11	1068.77	1205.29	1025.46	1429.64
m_3	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_3	1035.09	1077.49	1174.02	1088.31	1130.72	991.78
m_4	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_4	1337.31	1515.34*	1370.99	1390.54	1568.57*	1534.89*
Space Constraint						
m_1	7259.18	9098.45	7020.75	7627.46	9466.73	9705.16
v_1	800.00	800.00	800.00	800.00	800.00	800.00
m_2	8004.25	9843.53	10865.36*	9332.62	11171.90*	8310.80
v_2	800.00	800.00	800.00	800.00	800.00	800.00
m_3	11103.78*	10803.62*	10120.28	10726.98*	10426.82	11410.33*
v_3	800.00	800.00	800.00	800.00	800.00	800.00
m_4	8964.34	7704.09	8725.91	8587.54	7327.3	7565.72
v_4	800.00	800.00	800.00	800.00	800.00	800.00

* Maximum value of the constraint not considered for the computation of equal order interval.

Table 7: - Four Item Situation - Problem No.5

i -	1	2	3	4
P_i	25	90	100	60
W_i	1	5	8	9

Sequence	1-2-3-4	1-2-4-3	1-3-2-4	1-3-4-2	1-4-3-2	1-4-2-3
Money Constraint						
m_1	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_1	1581.81	1358.24	1542.77	1474.91	1251.34	1290.38
m_2	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_2	1478.01	1254.45	1289.93	1557.41	1333.85	1521.93
m_3	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_3	1328.97	1440.75	1393.73	1325.86	1437.64	1372.88
m_4	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_4	1664.31*	1589.79*	1625.28*	1661.21*	1586.69*	1625.72*
Space Constraint						
m_1	7838.38	9152.16	8067.77	8466.6	9780.38*	9550.99*
v_1	800.00	800.00	800.00	800.00	800.00	800.00
m_2	8448.35	9762.13*	9553.602*	7981.75	9295.53	8190.29
v_2	800.00	800.00	800.00	800.00	800.00	800.00
m_3	9324.21*	8667.31	8943.63	9342.45*	8685.56	9066.14
v_3	800.00	800.00	800.00	800.00	800.00	800.00
m_4	7353.53	7791.46	7582.92	7371.78	7809.71	7580.32
v_4	800.00	800.00	800.00	800.00	800.00	800.00

* Maximum value of the constraint not considered for the computation of equal order interval.

Table No. 8 - Four Item Situation - Problem No. 6

i	-	1	2	3	4
P_i	-	25	75	120	100
W_i	-	1	2	3	4

Sequence:	1-2-3-4	1-2-4-3	1-3-2-4	1-3-4-2	1-4-3-2	1-4-2-3
Money Constraints						
m_1	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_1	526.47*	466.24	528.15*	519.59*	459.55	457.87
m_2	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_2	518.39	458.15	448.12	508.35	448.11	518.39
m_3	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_3	446.44	454.81	456.206	447.84	456.206	446.44
m_4	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_4	515.04	526.75*	516.71	516.43	528.15*	526.47
Space Constraint						
m_1	23807.41	27007.41	23718.52	24162.97	27362.9	27451.80
v_1	800.00	800.00	800.00	800.00	800.00	800.00
m_2	24237.04	27437.04	27970.37*	24770.37	27970.37*	24237.00
v_2	800.00	800.00	800.00	800.00	800.00	800.00
m_3	28059.26*	27614.81*	27540.74	27985.19*	27540.74	28059.20
v_3	800.00	800.00	800.00	800.00	800.00	800.00
m_4	24414.81	23792.59	24325.93	24340.74	23718.52	23807.41
v_4	800.00	800.00	800.00	800.00	800.00	800.00

* Maximum value of the constraint not considered for the computation of equal order interval.

Table 9: - Four Item Situation - Problem No.7

i	1	2	3	4
P_i	30	80	150	180
W_i	2	2	3	4

Sequence:	1-2-3-4	1-2-4-3	1-3-2-4	1-3-4-2	1-4-3-2	1-4-2-3
Money Constraint:						
m_1	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_1	391.23	381.21	398.74*	404.17*	394.15*	386.64*
m_2	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_2	401.808*	391.79*	368.41*	358.40	348.38	381.77
m_3	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_3	360.904	335.44	357.84	363.26	337.808	340.86
m_4	16000.00	16000.00	16000.00	16000.00	16000.00	16000.00
v_4	345.46	376.34	352.97	347.82	378.71	371.20
Space Constraint						
m_1	32296.66	33225.31	31600.17	31097.14	32025.80	32722.30
v_1	800.00	800.00	800.00	800.00	800.00	800.00
m_2	31316.41	32245.06	34411.93	35340.59	36269.25	33173.72
v_2	800.00	800.00	800.00	800.00	800.00	800.00
m_3	35108.42	37468.77*	35392.19	34889.16	37249.50*	36965.74*
v_3	800.00	800.00	800.00	800.00	800.00	800.00
m_4	36540.11*	33676.74	35843.61*	36320.84*	33457.48	34153.98
v_4	800.00	800.00	800.00	800.00	800.00	800.00

* Maximum value of the constraint not considered for the computation of equal order interval.

Table No. 10 – Summary of test problem results

1. Problem No.	1	2	3	4	5	6	7
2. P_i/W_i Item No. 1	10	2	12.5	3.33	25	25	15
3. " 2	100	40	13.33	15.63	18	37.5	40
4. " 3	16.67	7.5	7.5	20	12.5	40	50
5. " 4	22.86	50	17.5	7.14	6.67	25	45
6. Sequence with increasing P_i/W_i	1-3-4-2	1-3-2-4	1-2-4-3	1-4-2-3	1-4-3-2	(1-4-2-3 (1-2-3-4	1-2-4-3
7. Sequence with decreasing P_i/W_i	1-2-4-3	1-4-2-3	1-3-4-2	1-3-2-4	1-2-3-5	(1-3-2-4 (1-4-3-2	1-3-4-2
8. Value of constraint not considered							
i) Money constraint seq. (6)	815	1006	1404	1535	1587	(526 (526	392
ii) Money Constraint lowest maxm.	815	979	1393	1515	1587	520	387
iii) Sequence for (ii)	1-3-4-2	1-2-3-4	1-4-3-2	1-2-4-3	1-4-3-2	1-3-4-2	1-4-2-3
iv) Space Constraint seq. (7)	17653	18696	10970	10865	9324	(27970 (27970	36321
v) Space Constraint lowest maximum	17653	18183	10809	10727	9324	27615	35044
vi) Sequence (v)	1-2-4-3	1-4-3-2	1-2-3-4	1-3-4-2	1-2-3-4	1-2-4-3	1-3-2-4
vii) Money Constraint Highest maximum	896	1057	1436	1612	1664	528	404
viii) Space Constraint Highest Maximum	19408	19633	11221	11410	9780	28059	37469
9. $\frac{8(i) - 8(ii)}{8(ii)} \times 100$	0	2.76	0.79	1.32	0	1.15	1.29
10. $\frac{8(vii) - 8(ii)}{8(v)} \times 100$	9.94	7.97	3.09	6.40	4.85	1.54	4.39
11. $\frac{8([illegible]) - 8(v)}{8(v)} \times 100$	0	2.82	0.74	1.29	0	1.29	1.33
12. $\frac{8(viii) - 8(v)}{8(v)} \times 100$	9.94	7.97	3.05	6.37	4.89	1.61	5.53
13. t_m	0.0833	0.0638	0.0501	0.0752	0.0639	0.0505	0.0356
14. t_v	0.0856	0.0654	0.0319	0.0460	0.0344	0.0[illegible]0	0.0707

ii) The sequence in the increasing order of P_i/W_i (or W_i/P_i) is the best result of the six possible sequences in two out of the seven test problems, second in four out of the seven test problems and in only one case it comes out to be the third.

iii) The maximum deviation between the two extreme sequences is 9.94% in the test problems considered here.

iv) The maximum deviation of the increasing order of P_i/W_i sequence is 2.76% from the best solution.

v) The deviation in (iv) is lower then the policy is based on lower of the two (t_m and t_v) order interval is considered.

The finding shows that the sequence in the increasing order of P_i/W_i (or with W_i/P_i as the case may be) results in either the desired sequence or very close to the desired sequence.

On the basis of the above finding the multi-item, two constraint problem can be solved by the following procedure with equal order interval policy.

Step-1 – Compute t_m and t_v.

Step-2 – Consider lower of the two order intervals t_m and t_v.

Step-3 – Compute the inter-item replenishment interval based on the policy accepted at step-2.

Step-4 – Consider the sequence based on the increasing order of P_i/W_i (or W_i/P_i as the case may be) and find out the maximum value of the requirement of the constrained resource not considered for computing time interval in step-2.

Step-5 – If the requirement arrived at Step-4 is lower than the permissible limit, the desired replenishment policy has been arrived at.

Step-6 – If the requirement arrived at Step-4 is greater than the permissible limit, reduce the order interval proportionately and proceed through Step-3 to Step-5.

In case the number of items are very large, a grouping operation can be performed based on the procedure illustrated in 1,2,6 .

8. Conclusions: In case of two constraints, it becomes very tedious to compute the lagrange multipliers. The equal order interval method is computationally simple. The outcome in equal order interval policy is dependent on the replenishment sequence. The sequence based on the ratio of unit price to unit space (or the reverse of it as the case may be) results in very good solution. The equal order interval policy also results in better utilization of the constrained resources.

9. Acknowledgement: The author acknowledges the computational assistance provided by Sri R.N. Dutta in connection with the study.

REFERENCES

1. Chakravorty, A.K. - 'Multi-item Inventory Aggregation into Groups' - Journal of Opl.Res.Soc. Vol.32, (1981) No.1.

2. Goyal, S.K. - 'A Note on Multi product Inventory Situations with one Restriction' - Journal of Opl.Res.Soc. Vol.19 (1978) No.3.

3. Hadley, G. and Whitin, T.M. - 'Analysis of Inventory Systems' - Prentice Hall Inc.

4. Oral, M. - 'Multi-item Inventory Management with Monetary Objective Function' - AIIE Transactions, Vol.13 (1981) No.1. pp. 41-46.

5. Oral, M. 'Equivalent Formulations of Inventory Control Problems' - AIIE Transactions, Vol.=2 (1981), No.4, PP 325-332.

6. Page, E. and Paul, R.J. - 'Multiproduct Inventory Situations with one Restriction' - O.R.Q., Vol.27 (1976), PP 815-834.

7. Tersine, Richard,J. - 'Principles of Inventory and Materials Management North Holland.