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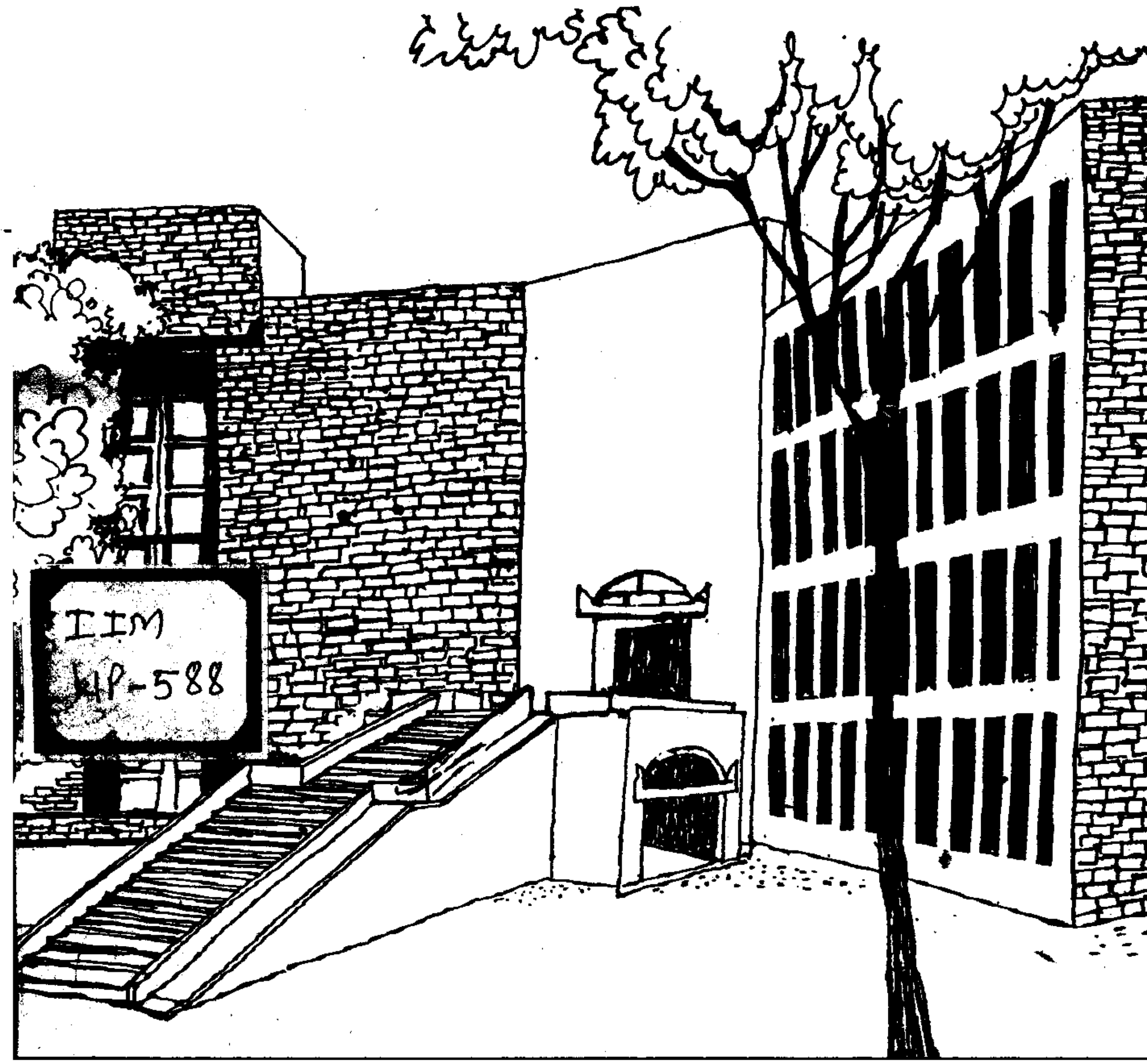

INVESTMENT OPPORTUNITIES AND GORDON'S STOCK VALUATION MODEL – A NOTE

By

V. Raghunathar
G. Srinivasan

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INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD 380015
INDIA

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In finance literature "Gordon's Stock Valuation Model"[1] has come to be widely known, accepted and quoted in journals and text books alike. Fama and Miller (2) call it the constant growth model for market valuation, incorporating investment opportunities under certainty. The model is given as follows:

$$V_0 = \frac{X_1 \ (1-k)}{r - kp} \quad \text{.........} \ (1)$$

Where V_0 = Value of the firm's total stock at time 0

X_t = Net operating cash flows in period t

r = Market rate of return

p = Rate of return on investment in the product market for all future periods

k = Fraction of earnings retained in the business in any period t, so that

$k = I_t/X_t$, where I_t = Investment made in period t

In the above model, $r > kp$ (or $k < r/p$) becomes a necessary condition for the market value to be finite. Gordon and most other authors presume this condition to be satisfied, since the market value of a

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[1] Gordon (3) himself presents a brief history of this model. While Durand (1) observes equation 1 to be a standard actuarial formula, according to Gordon (3) the actuarial literature has no reference to the economic content involved in the derivation of the model. According to him J.B. Williams (5) seems to

firm must indeed be finite. Fama and Miller (2) explain this condition to imply that "in equilibrium the market rate of interest must be such that no firm has opportunities into the indefinite future to invest the proportion k of each period's earnings at a rate of p so that $kp > r$." Consequently, the above model fails to provide a value for an optimum k ; instead it merely provides a limit for k.

In this paper we proceed to show that the constraint $k < r/p$ in the above model does not capture the constraint on the investment opportunities which a firm must necessarily have (for if the investment opportunities could be infinite, given also unrestrained funds at cost r, the firm could at once invest an infinite capital in the available opportunity and have an infinite market value). We shall further show that when the limited or finite investment

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have been the first in 1938 to have attempted a valuation formula on the lines suggested by the model. But the latter is said to have abandoned the equation before it could take the form of equation 1, since he tried to work with varying k and p, which could not yield him a manageable expression. According to Gordon (3) equation 1 appeared for the first time in its current form in Gordon and Shapiro (4). We in our turn accept Gordon's claim and have accordingly referred to the model under his name.

opportunities are explicitly recognized, the limit for k would in fact be lower than r/p. And finally, we shall provide the value for optimal k; the optimal k being that which would just satisfy the opportunities constraint.

We shall illustrate the inadequacy of the constraint $k \leq r/p$ employing a small numerical example.

Let us assume a firm which earns a return of 20% (p) on all its investments. Let the market rate of return be 10% (r) and the initial net cash in flows be Rs.10 million (X_1). According to the above model the market value of our assumed firm approaches infinity as k approaches 50% (the firm's r/p ratio). Thus the firm could choose any k so long as it was less than 50%. Let us assume that the firm chooses its k = 49.5%.

We shall now assess the present value of all future investments implied by the chosen k. In order to do so, we shall first develop a general formula for arriving at the present value of all future investments.

Let us assume that in accordance with k r/p, a firm chooses its k such that

$$k = \Delta(r/p) = r/p', \quad \text{where } \Delta < 1$$

$$\text{so that } p' = p/\Delta$$

Now if I_t = Investment at the beginning of time t,

we have $I_t = kX_t$

$$= rX_t/p' \qquad \ldots\ldots(2)$$

Also we have $X_2 = X_1 + pI_1$

$$= X_1 + p(rX_1/p')$$

$$= X_1(1 + rp/p')$$

Therefore $X_t = X_1 (1 + rp/p')^{t-1} \ldots(3)$

Now the present value of the infinite stream of future investments (PVI) may be expressed as

$$PVI = \sum_{t=1}^{\infty} \frac{I_t}{(1+r)^t}$$

$$= \frac{r}{p'} \sum_{t=1}^{\infty} \frac{X_t}{(1+r)^t} \quad \text{(From Eq.2)}$$

$$= \frac{rX_1}{p'} \sum_{t=1}^{\infty} \frac{(1 + rp/p')^{t-1}}{(1+r)^t} \quad \text{(From Eq.3)}$$

$$= \frac{X_1}{p'-p} \quad \text{(The expression converges since } p < p'\text{)}$$

Substituting for p', we have

$$PVI = X_1\Delta/p(1-\Delta) \qquad \ldots\ldots\ldots(4)$$

Now, in the numerical example considered earlier, $= .99$ ($k = 49.5\%$ implies $\Delta = .99$, since $k = \Delta r/p$). Substituting for Δ, p and X_1 in equation 4, the PVI for our example turns out to be Rs.4,950 million.

Now in case the present value of our investment opportunities (assuming their transferability across periods) happens to be less than Rs.4,950 million, clearly the chosen k of 49.5% would be infeasible, even though it satisfied the condition $k < r/p$ i.e. $k < 50\%$.

Let us now presume that the present value of available investment opportunities to the firm equals Rs.2,000 million. What should the firm's k be so that the present value of the firm's future investments just equals Rs.2,000 million? This is now easily obtained through equation 4 by equating its right hand side to Rs.2,000 million and substituting the values of X_1 and p. This yields a Δ of 97.56% or a k of 48.78%.

In other words, if the present value of investment opportunities be restricted to Rs.2,000 million, the firm cannot have a $k > 48.78\%$. Any k higher than this, even when less than 50% (satisfying the condition $k < r/p$) would imply a present value of investment which violates the opportunities constraint. Thus in this case the feasible $k \leqslant 48.78\%$ and the optimal $k = 48.78\%$.

In order to explicitly incorporate the opportunities constraint into the conditionality for k, we must have

Present value of future investments $\leq$ Present value of all future investment opportunities (say A)

$$\text{or} \quad \sum_{t=1}^{\infty} \frac{I_t}{(1+r)^t} \leq A$$

$$\text{or} \quad \frac{X_1 \Delta}{p(1-\Delta)} \leq A \qquad \text{(From eq.4)}$$

$$\text{or} \quad \frac{X_1 kp/r}{p(1-kp/r)} \leq A$$

$$\text{or} \quad \frac{kX_1}{r-kp} \leq A$$

$$\text{or} \quad k \leq \frac{Ar}{X_1+Ap}$$

It can be easily seen that

$$k \leq \frac{Ar}{X_1+Ap} < \frac{r}{p}$$

Thus we see that in a model incorporating the investment constraint explicitly, $k < r/p$ is only a necessary condition, but not a sufficient one. The explicit incorporation of investment opportunities becomes necessary for following reasons :

a. The traditional model does not help in choosing an optimal k; the optimal k being one which equates the present value of investments to the present value of opportunities available.

b. The traditional model appears to imply that the investment opportunity is a function of k, whereas the causality must doubtless be the other way, as brought out in the numerical example earlier.

In conclusion when finite investment opportunities are explicitly recognised, the optimum k equals $Ar/(X_1+Ap)$, an amount which is less than r/p. Also it is apparent that the smaller the investment opportunities the smaller is the limit of k.

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