T. R. NO. 63

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Technical Report

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MATHEMATICAL MODELS FOR URBAN WATER SUPPLY AND WASTE DISPOSAL SYSTEMS

by

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T. R. No. 63

January, 1975

Indian Institute of Management
Ahmedabad

No. 02

To

Chairman (Research)
IIMA

Technical Report

Title of the report MATHEMATICAL MODELS FOR URBAN WATER SUPPLY AND WASTE DISPOSAL SYSTEMS.

Name of the Author Shishir K. Mukherjee

Under which area do you like to be classified? ....

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Before deciding on a waste disposal plan, the effect on the quality of the receiving waters of waste-water discharges after treatment, in any, must be analysed. A water quality model for predicting pollutant concentrations from hydrological and waste discharge data is presented for this purpose. An iterative solution approach using the network model is suggested for designing an optimal water supply/wastedisposal system which will meet all water demands while maintaining the receiving waters at acceptable quality levels.... Thus design of an optimal system for water sully and wastewater disposal involves interactions between the three given models.

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Date 28-12-74

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Shishir K. Mukherjee

Abstract

Most urban areas are facing an ever increasing demand for fresh water due to population and industrial growth. This paper proposes an integrated approach for optimal design of urban water supply and waste disposal systems. A network model is presented for evaluating alternatives for supplying water from different sources - treated fresh water, ground-water, desalinated sea-water and renovated water - to satisfy future demands for domestic, industrial and public use at minimum cost. This model carries out optimization on the basis of estimated cost functions for various processes including water development, conveyance and treatment.

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* Presented at the Conference on Urban Development, Problems and Issues, held at Calcutta during 27th and 28th September, 1974. under the auspices of the Indian Institute of Management, Calcutta.

Introduction

Due to natural population growth, migration from rural areas and growth in the industrial and commercial sectors, most urban areas are facing an ever increasing demand for water. Rapid urbanisation and consequent rise in the standard of living also has a tendency to increase the per capita water demand for domestic consumption. Finding new sources of fresh water for meeting demands for domestic, industrial and other public uses is becoming increasingly difficult and costly for most urban areas as easily accessible and cheaper sources of fresh water are being exhausted. The problem is further complicated by the need for building expensive treatment plants for handling the growing volume of domestic and industrial wastewater as a result of growing awareness that the receiving waters should be maintained at acceptable quality levels and there should be no health hazards from the discharge of pollutants. Satisfactory planning and implementation of adequate water supply and waste disposal facilities are already taxing the financial and managerial capacity of administrative authorities in almost all cities and towns and there are huge backlogs in all the Indian cities, large or small, in providing adequate water supply and waste disposal fa cilities.

There is a strong need for studying the urban water supply and waste disposal facilities as an integral system for planning purposes and developing a framework within which various alternative solutions could be evaluated from economic, financial and other considerations. The objective of such a study is to evolve an optimal system for water supply and waste disposal which will meet the increasing water demand from domestic, industrial and agricultural uses within an urban area and will maintain the receiving waters for urban wastewater at acceptable quality levels. The analysis and optimization of the resulting system is a difficult process due to the large number of alternatives encountered, the economies-of-scale in water development, transmission and treatment processes and the complex interaction of pollutants in the receiving waters specially for bio-degradable wastes and in tidal estuaries. The development of a single model for simulating and optimizing the complex system might be possible but it would be quite difficult to obtain realistic solutions for such a model under the prevailing system of data availability and computing systems at our disposal. In this paper, a series of simpler mathematical models are presented which either in sequence or in conjunction with each other can be used for obtaining implementable solutions to this complex problem.

The basic model considered in this paper is a Network Model for economically evaluating the large number of feasible alternatives in the design of a water supply and waste disposal system consisting of numerous sources, and demands of various types, treatment plants, storage and conveyance facilities, waste treatment and disposal processes. The network model is a conceptual description of the above system as a network consisting of numerous nodes and arcs through which a minimum-cost flow circulation [2] is to be obtained which corresponds to meeting all water demands from the available sources, if feasible, at minimum cost. This model is solved through a specially designed network analysis computer programme which can accommodate realistic nonlinear cost functions for flow through the arcs of the network representing economies-of-scale in water development, treatment and conveyance processes.

The network model approximates the water transmission process as a simple conveyance activity through a pipeline system for which maximum (and possibly minimum) capacity in terms of flow rate, and a cost function representing the cost of piping and energy spent in overcoming friction losses, are defined. It does not include pressure constraints in the pipeline neither includes the detailed distribution network within a particular zone. Given a water transmission network in terms of available water sources with given heads, several demands of known flow rate and minimum head and a layout of the pipe network, the optimum choice of pipe diameters and lengths and often that of the main pump specifications is by itself a complex problem. A Linear Programming (LP) model for Water Transmission System described later in the paper and based on the well-known Hazen-William's formulae for friction losses in a pipeline, attempts to answer these question. A realistic design of a minimum cost urban water supply system will involve iterations between the network model and the LP model for water transmission.

Similarly, the network model ordinarily would not consider the effect of waste disposal activities on the quality of the receiving waters except that minimum acceptable treatment levels or maximum allowable pollutant concentrations are specified. A water quality model for the receiving waters developed along any of the recognized approaches can be used to study the effects of waste disposal on water quality according to different solutions of the network model. Thus iterations involving the network model and a water quality model can lead to a realistic design of a minimum cost water supply and waste disposal system for an urban area located near a river or an estuary.

The three mathematical models described in this paper if used in an integrated manner as suggested, should lead to optimal allocation of our limited resources in providing implementable solutions to water supply and waste disposal problems in our cities.

Water Supply/Waste Disposal System Description

The basic elements of a Water Supply/Waste Disposal System including the use of renovated wastewater are illustrated in Figure 1. Water demand projections for municipal, industrial and agricultural uses are made available in terms of their geographical locations, quantity and quality specifications for a target year, the year for which the system is being planned. Available water sources are specified as to the maximum quantity available, quality and annualised development and operating costs as a function of their capacity. Water treatment processes are defined to match the quality of treated water to the demand quality requirements. Alternative ways of meeting the demands from various sources of treated water are proposed and the ones considered infeasible from engineering and economic judgment are deleted from further consideration.

Allowing for various losses in use, the municipal wastewater can be considered as another source of water and at least a part of it can be recycled into the system after adequate treatment and purification subject to its acceptance by public health standards and aesthetic considerations. If renovated wastewater is considered unacceptable for municipal use (due to possible danger to public health and traditional and psychological resistance by people) or process use in industry (due to build-up of soluble salts), it could at least be used for various agricultural needs including irrigation of parks and industrial cooling purposes. Various combinations of processes are considered, primary, secondary, activated sludge and trickling filter treatment; chlorination; lime treatment and ammonia stripping; dual media filtration and electro-dialysis. Alternative treatment paths for waste water could be considered each consisting of some of the above processes in sequence, and designed to produce water quality to match given demand quality requirements. The possible alternative locations of the treatment plants are decided from engineering-economic judgment and geographical considerations.

A conveyance network could be developed to include all feasible alternative ways of meeting each demand from sources producing water of matching quality. The actual allocation of sources to demand points is obtained as a solution of the network model described later. At the problem formulation stage, engineering judgment must be used to obtain a realistic network design and eliminate obviously uneconomic or infeasible conveyance routes from the conveyance network showing all alternatives. Finally some storage and pumping facilities must be included at various points in the system to accommodate seasonal and diurnal fluctuations of demand and to provide adequate discharge head at distribution nodes.

The water supply/waste disposal system consisting of the multiple sources of water and demands at distribution nodes, alternative ways of supplying these demands from the given sources, alternative process combinations for treating fresh and wastewater and adequate storage and conveyance facilities as its elements is represented by a conceptual network in the network model.

The description of the waste disposal system will however remain incomplete unless it is known how the wastewater after treatment, if any, is disposed off. Usually cities or towns are located on river banks or near lakes and oceans due to historical reasons for

for navigational purpose and also due to the fact the water is one of the most essential ingredients of human life. Thus any stream, river, lake or ocean may become the receiving water for treated or untreated waste-water as they have some capacity for waste assimilation specially of bio-degradable type which forms most of the domestic waste. It is possible to analyse the interaction of waste-water in the receiving waters specially in the presence of dissolved oxygen and study the resulting concentration of various pollutants in the water body. As due to health, aesthetic and possibly legal reasons the pollutant concentrations cannot be allowed to increase beyond certain specified limits, this might limit the amount of waste-water that can be discharged in a water body and the locations at which these could be discharged.

Network Model of Water Supply/Waste Disposal System

The problem of determining the least-cost design of the Water Supply/ Waste Disposal System is formulated as a problem of finding the minimum-cost flow pattern in a conceptual network. The conceptual network model is developed by incorporating the various activities, their limiting capacities and unit costs on the physical layout of sources, treatment plants and demands connected by conveyance lines. Network flow theory /2/, a very useful tool of operations research applied in the optimization of systems described by physical or conceptual networks, is applied as a solution technique.

A network as used in this paper consists of a collection of elements called nodes or vertices, some pairs of which are connected by directed branches or arcs. The nodes may be divided into three categories -- "sources" at which flow is generated, "sinks" at which flow is consumed and "intermediate" nodes at which flow is conserved. The arcs are usually associated with nonnegative minimum and maximum capacities of flow in one direction of some commodity (fluid, electricity, consumer goods, money, etc) per unit time. If flow is possible in both directions between two nodes, two arcs directed in opposite directions are provided connecting the two nodes. One of the common problems connected with flows in network is to determine the maximum flow from sources to sinks through a network with specified arc capacities.

It is plausible to assume a cost for flow in each arc of the network. The costs may be linear or nonlinear functions of the flow. An interesting problem with application to the water supply/waste disposal system is to find the distribution of flow that minimizes the total cost of transporting a given flow value from sources to sinks through the network, if feasible. In constructing the conceptual network to represent the water supply/wastewater disposal system, an "arc" is assigned (Figure 2) for each function or activity in the system, e.g. drawing water from a fresh water source, treatment processes, conveyance routes, desalination, etc. Thus, for waste-water the arcs WP, PS and ST represent various levels of treatment which make the water suitable for irrigational, industrial and municipal use. An arc has two "nodes" as its terminals and an arrow head specifying the direction of flow. All sources for water become "source" nodes and all demands for water at distribution points become "sink" nodes in the network.

A hypothetical node called as "supersource" is added to the network and this node is connected to all "source" nodes by arcs which represent drawing water from these sources at appropriate costs. Similarly all "sink" nodes are connected to "supersink" and these arcs represent water consumption. Maximum available flow at sources, specified demands and capacity limitations of processes are handled by assigning each arc a minimum and a maximum capacity for flow. If any arc must carry a specified amount of flow, the minimum and maximum capacities for that arc are equal and has the value of the required flow. Cost functions are defined for each arc to describe the cost of the activity represented as a function of flow through that arc.

The mathematical formulation of the minimum cost network flow problem for a network is described in [2].

There are a large number of feasible flow patterns in the network to satisfy the specified demands from the available sources. A powerful computational method for solving minimum cost network flow problems with constant unit costs (representing linear cost functions) is the Ford-Fulkerson "out-of-kilter" algorithm [2] which minimizes the total cost in the network while circulating a specified amount of flow through the network.

The out-of-kilter network algorithm starting with an arbitrary set of flow values, either feasible or infeasible, and unit arc costs, builds flow demands at minimum total cost. The programme simultaneously checks the feasibility and optimality criteria of a flow solution. It checks the feasibility criteria by ascertaining that all flow demands are met and no capacity constraints are violated. The optimality criteria for an arc is determined by the relative values of specially defined node prices at the extremities of the arc, the arc cost and the flow value in relation to lower and upper bounds on the arc.

Any arc for which the feasibility and/or optimality criteria is violated is said to be "out-of-kilter". The flows in these arcs are changed by using a flow circulation technique and if that is not possible the node prices are changed which may now allow flow changes. The algorithm brings out-of-kilter arcs into kilter while maintaining "in-kilter" arcs in kilter. Eventually, all arcs are put in kilter, or it is learned that certain arcs cannot be in kilter simultaneously which indicates that the problem is infeasible. If integer values are used for all the variables, costs and flow bounds, it is easily proved that the algorithm terminates in a finite number of steps [2].

A network Analysis Computer Model, which uses the "out-of-kilter" algorithm described above in an iterative manner, has been developed to determine the minimum cost flow pattern in the urban water supply network with nonlinear arc costs. As the basic network algorithm can only accommodate linear arc cost functions, special techniques are necessary to handle the nonlinear functions described later in this section.

The generalized flow diagram and other details of the Network Analysis Computer Model developed for this application is described in Ref. [7]. It basically consists of three subroutines controlled through an executive computer programme. Because of the nonlinearity of most of the arc cost functions, the optimum flows determined by an out-of-kilter network programme using average unit arc costs will, **in general, not be the least-cost solution. To accommodate the nonlinear cost functions and to formulate the input data in the** correct format an iterative procedure was developed.

The unit arc cost can be computed as a function of flow rate for each activity in a network from basic information regarding the process costs, and physical parameters such as pipe dimaters, distance, elevation, type of land etc. determining the conveyance costs. A cost subroutine consisting of a series of equations is used to convert the system cost information into unit arc costs for a given arc flow expressed in nearest integers in small monetary units, to be subsequently used in the optimisation procedure of the network subroutine. The flow rates used for computing the linearized arc costs are initially assumed values in the input data for the network. These initial flow values are replaced in successive iterations by flow values generated as optimal solution by the linearized network sub-routine, resulting in new unit arc costs. As this routine is repeated, the computed optimal flow rates at successive iterations converge and an optimum flow solution is determined.

The executive programme uses an iterative device to correct the error introduced by linearizing the problem and to ensure convergence. It takes the least-cost solution from the network subroutine and reassigns flows in the input data for the cost subroutine programme. The whole computational process is then repeated with the cost sub-routine computing new unit arc costs for the newly assigned flows and the network sub-routine finding a new least-cost flow solution. The total system cost is computed after each iteration and compared to that at the last iteration. If the percent change in total costs is greater than fixed error value (1 to 2%), then the iterative technique is continued.

Otherwise the computation is terminated and the final solution printed as the optimal solution. Accuracy of the model is increased by decreasing the fixed error value or by using the error test on the cost of flow in individual arcs, though this may increase the number of iterations if the problem does not converge quickly to a solution. In all the cases where the programme was used, convergence to a solution was obtained within a few (4-5) iterations of the network programme [7], and the same flow solution was repeated in two successive iteration.

The nonlinear cost functions included in the Network Programming Model if non-convex in nature will warrant special attention as in specific situations, there may be possibilities of obtaining locally optimum solutions. Specific measures must be included in the programme to provide sufficient assurance that the solution converges to the real optimum. This is an inherent difficulty frequently encountered when cost functions represent economies-of-scale. To obtain sufficient assurance that a globally optimum solution has been obtained, the problem can be solved with several random initial solutions. If the same optimum solution is obtained in all cases then one can state with some confidence that the real least-cost solution has been obtained. Using a slightly different computational scheme as described by the author in [8] it is also possible to obtain a lower bound to the globally optimum solution cost at each iteration and thus ascertain whether a solution sufficiently close to the globally optimum has been obtained or the iteration process has converged to a locally optimal solution possibly far away from the global optimal value.

Since it is desired to design a least-cost system to satisfy all water demands (if feasible) by the application of the network model, the total system cost is the measure of effectiveness. Unit cost functions are developed for each activity connected with water supply - water sources development, treatment, storage and transmission. The capital costs are amortized with appropriate interest rate and added to annual operating costs. The cost functions could then be expressed in suitable units for flow rate such as million gallons per day (MGD) or million litres per day (MLD).

Linaweaver [5] reports a study of the factors influencing the cost of water transmission facilities and a method for minimizing this cost. The cost function for water source development would probably be of the fixed charge type with one or more discrete jumps connected by linear segments. The cost functions for water treatment and conveyance are nonlinear in general basically due to economies-of-scale involved in most of the processes.

It is observed that unit costs (per thousand gallons) for sea water distillation, water and waste-water treatment, storage and conveyance when plotted against flow on a log-log paper result in linear or piece-wise linear curves. The basic equation representing the unit cost functions is thus derived by determining values at two points on the same linear segment of the graph and is represented by:

$$C = aq^{-b} \qquad (1)$$

where

$$b = \frac{\log(c_1/c_2)}{(q_2/q_1)} ;$$

$$a = \frac{c_1}{q_1^{-b}} ,$$

C = unit cost (paise/1000 gal.),

q = flow rate (mgd),

c_1 = unit cost (paise/1000 gal.) corresponding to point 1 on the curve,

c_2 = unit cost (paise/1000 gal.) corresponding to point 2 on the curve,

q_1 = flow rate (mgd) corresponding to point 1 on the curve, and

q_2 = flow rate (mgd) corresponding to point 2 on the curve.

In cases where the cost function cannot be plotted on a log-log graph as a single straight line, the curves are closely approximated by two or more straight lines and break-points in the piece-wise linear curve is used for computing a set of values for the constants a and b for each linear section of the cruve.

In most of the Indian cities, there may not be too many sources of fresh water left and possibly it may be limited to only one as is the case for Ahmedabad where the author is presently studying the water supply problem. Under these circumstances, the applicability of the network model might be questioned. But most of the cities are also augmenting their water supply by tapping ground water sources, where available, and often mining water stored in acquifers for thousands of years. Since the cost of ground water is usually higher than the cost of surface water due to the energy spent in pumping which again increases as the depth of the wells increases due to overpumping conditions prevalent in many cities, finding a least-cost mix of surface and ground water is a meaningful exercise. Similarly the costs of the necessary treatment of waste-water before it could be safely discharged into the receiving waters without creating undue pollution hazards is also increasing with increasing load of waste-water and stringent pollution laws. Under the existing situation of acute shortage of new sources for fresh water the treated waster-water may be too good and too costly to be thrown away. The network model provides a framework in which the economics of recycling treated waste-water in the water supply system or recharging it into the ground water could be analysed and compared with surface and ground water sources to obtain a least-cost water supply system.

Linear Programming Model for Water Transmission System

The network model as described in the preceding section provides an optimal choice among the sources and an optimal layout for the water transmission network leading from the sources to distribution points in various zones in the urban region, along with optimal choice among treatment processes and storage facilities. But it does not provide a detailed design of the transmission network in terms of optimal pipe diameters , lengths and optimal pumping heads. The cost functions used in the network model, however, provide for optimal design in the sense that these cost functions are based on optimal choice of pipe diameters and pumping head as a function of flow in the pipeline [5]. Once the flows along the arcs of the network model corresponding to conveyance facilities are optimized the determination of optimal physical parameters could proceed as the second stage of the optimization process without much apprehension regarding sub-optimization. If increased accuracy is warranted the arc cost functions could be recomputed once the physical parameters of the water transmission system are known and the network model solution could be modified. However, generally this kind of iteration with the network model will not be required.

Schaake, et al [10] and recently Shamir [12] reviews most of the past work related to water transmission and distribution systems analysis and optimal design of water transmission and distribution systems. We would like to distinguish between water transmission and distribution networks and concentrate our efforts on branched water transmission networks for which a linear programming approach will be presented. Much of the past work have been on analysing the distribution system with many loops by solving the non-linear equations describing its hydraulic behaviour. The problem usually has been one of solving for other unknowns such as pressure or flow at nodes given the pipe diameters and the friction coefficients of the pipes. The most significant work in this area has been due to Shamir and Howard [11] who have used the Newton-Raphson method to solve networks. Shamir [12] has recently extended this approach to an optimizing approach for optimal design and operation of water distribution systems.

Schaake, et al [10] presents a linear programming model for water system transmission networks where he uses the pipeline or tunnel diameter as a continuous variable. The linear programming formulation is obtained by linearizing a nonlinear formulation. They provide an algorithm to be used for computing the coefficients for the linear programming model but this algorithm is only applicable in the case of non-looping or tree-like network with a single source of supply. They also present a Dynamic Programming-Capacity Expansion model for determining the best time sequence of possible capacity expansions to meet growing water demands. It would be possible to solve a sequence of network models, with the near-horizon models telescoped into ones further away, to solve the time-sequence problem.

Karmeli, et al [4] provided a linear programming formulation for the optimal design problem for a simple branched network without any closed loops in which discrete diameter sizes could be considered for the pipes and their lengths are determined as the solution of the linear programme along with pumping head at the sources. Gupta, et al [3] have used the same formulation but with given head at a single source and extended this to multiple sources using an electrical network analogy for the water transmission system. We use their formulation here.

The linear Programming Model, as any of the other approaches for the solution of this problem is based on the well-known Hazen-Williams formula for determining the frictional loss in a pipeline as given below.

$$V = 13.8\ R^{0.63}\ S^{0.54} \qquad (2)$$

where

S = loss of head due to friction per foot-length of pipe

V = average velocity in feet per second, and

R = mean hydraulic radius (defined as D/4, where D is the internal diameter of pipe).

If Q denotes the discharge in cubic feet per second, the Hazen-Williams formula may be written as

$$S = 0.887 \times 10^{-3} . \frac{Q^{1.852}}{D^{4.870}} \qquad (3)$$

The relation (5) shows that for given rate of discharge rate and pipe diameter, the head loss H in a pipe is linearly proportional to the length L of the pipe, i.e.

$$H = S' . L \qquad (4)$$

A branched pipe network without closed loops may be considered consisting of a number of "open loops" where an open loop is defined as an imaginary pipe line from a water source to a demand centre. The main constraint in this formulation is that a minimum desired head H_d must be maintained at each demand point to meet the distribution needs. Given H_p as the pump head at the source, the maximum allowable pressure loss due to friction H_k at each of the p open loops is $H_p - H_d$.

The linear programming model is then described as follows, for a single source:

$$\text{Minimize } Z = \sum_{j=1}^{n} \sum_{i=1}^{m} C_j L_{ij} \qquad (5)$$

$$\text{subject to} \quad \sum_{i} \sum_{j} S_{ij} L_{ij} \leq H_k ; \quad k = 1,2, \ldots, p \qquad (6)$$

where $S_{ij} = 0$ if the line i is not included in K^{th} open loop, and

$$\sum_{j=1}^{n} L_{ij} = L_i \qquad i = 1,2, \ldots, m \quad L_{ij} \geq 0 \qquad (7)$$

where

n = number of pipe diameter sizes available

m = total number of lines (i.e. arcs) in the water transmission system

p = number of open loops in the system, one for each demand point.

S_{ij} = friction head loss per foot length of pipe in line i, diameter j where line i may be composed of segments of lengths L_{ij} of diameter j.

C_j = cost per foot of pipe of diameter j including the cost of laying the pipe

L_i = total length of line (arc) i.

Case and White [1] have extended the above formulation to multiple sources systems and minimized the present worth cost of the water supply system considering both the installed cost of pipe and the cost due to flow losses due to friction throughout the life of the system. This involves balancing the trade off relationship between a larger diameter pipe having higher capital cost, but a lower cost due to frictional losses and a small diameter pipe having lower capital cost, but a higher cost due to friction losses.

We have applied both these approaches to determine the optimal pipe sizes for a water transmission network being designed to supply water available from Dharoi reservoir to the demand points on the west side of the river Sabarmati at Ahmedabad. This branched network consisted of 13 demand points and 24 lines and the L.P. solution took approximately 2 minutes on the IBM-360/44 Computing System. The optimal solution as described in [9] had smaller diameters as compared to an earlier design for this network and showed substantial savings in cost.

The linear programming approach does not work in the case of networks with closed loops and in such cases a more complicated approach involving non-linear programming techniques, as described by Shamir [12] is needed. As vast amounts of capital resources are needed in constructing water transmission networks and expenditure of public funds are involved often requiring foreign exchange also, the need for optimizing their design is obvious and methodology is now available to assist the decision making process of the practising engineers involved in these projects.

Dispersion Model of Waste Transport in Estuarine Waters

The design of an urban water supply and waste disposal system will remain incomplete without a system for safe disposal of the resulting waste water or effluent from water consumption. These consist of domestic sewage, industrial waste-water and drainage water, if any, from agricultural uses along with storm water polluted by dirt and dust from the streets. These effluents are usually discharged in any available water body - rivers, lakes or estuaries, usually after some treatment to remove suspended matter and organic wastes and often without any treatment. Waste discharges into a reciving water body beyond its normal assimilative capacity creates water pollution and may be hazardous to human being and to marine life. Decomposition of biodegradable wastes by bacteria reduces the amount of dissolved oxygen in water often exhausting it completely/causing death to the fish population whereas conservative pollutants accumulate in the water increasing its concentration levels unless they are transported away by the receiving waters.

According to the network model any amount of treated waste-water which is not recycled into the system must be discharged into the receiving water, if required after adequate treatment. Thus maximum allowable amounts of waste-water that can be safely discharged into the receiving waters after specified treatment levels must be given as input information for the network model or for any other scheme of designing a water supply/waste disposal system. In this section, we describe a water quality model which relates waste discharges to water quality in the receiving waters and can provide this information. Mathematical modeling of water quality can also aid in the determination of optimal levels of waste treatment and optimal investment in pollution abatement facilities to deal with present and future waste loads.

A predictive water quality model is described here which is applicable to a river system terminating in an estuary. The whole river basin including the estuary and the waste-producing and waste-abatement activities can be considered to be in one "system". The problem of maintaining the water basin at a prescribed level of quality at minimum pollution abatement costs for the region has been formulated as a linear programme by the author [6]. The linear programming model will indicate the optimal levels of waste treatment and optimal investment in pollution abatement facilities. If accurate figures for social costs and benefits are available, they could be accommodated in the model for overall optimization. In this section, we describe the application of the predictive water quality model in conjunction with the network model in an iterative manner.

Most of the past work in this field was confined to considering only one criterion of water quality, i.e. dissolved oxygen concentration. Since discharge from most of the industrial plants contains various toxic and stable chemical compounds, many of which are hazardous to health and marine life, a multi-component model is described here. The water quality model is based on a one-dimensional description of the dispersion process which includes both longitudinal dispersion and advection in an estuary. As a result, this model is well suited for a well mixed tidal estuary or a fast flowing river, where advection plays the major role in the transport of pollutants.

The one-dimensional non-steady-state diffusion equation is used as the basis of the model and is expressed as

$$\frac{\partial}{\partial x} AE \frac{\partial c}{\partial x} = \frac{\partial}{\partial x} (Avc) = S + A \frac{\partial c}{\partial t} \qquad (8)$$

where x and t are the longitudinal distance (from an upstream origin) and the time co-ordinate; A is the cross sectional area; E, a coefficient of longitudinal dispersion; v, the advective velocity; c, the concentration of a water-quality constituent; and S, the net sum of sources and sinks of the constituent in the estuary per unit length of the estuary. A, E, v, c, and S represent averages taken over both a period of time equal to at least one complete tidal cycle and the entire estuarine cross-section. This equation is derived from mass continuity considerations of a constituent.

Equation (8) can be integrated but there is, as yet, no satisfactory way of evaluating E analytically, though various methods have been suggested. The coefficient E may be estimated at discrete cross sections of the estuary from the concentration of a common constituent such as chloride ions obtained from sampling studies in the estuary.

These coefficients are used in difference equations obtained from Equation (8) by modification and/or discretization and they represent the relationships in the physical system.

Integrating Equation (8) seaward from the landward termination (origin) of the system gives

$$AE\,\frac{\partial \bar{c}}{\partial x} = Avc - \int_0^x S\,dy + \int_0^x A\,\frac{\partial \bar{c}}{dt}\,dy \qquad (10)$$

With steady state assumptions the diffusion equation has the form

$$AE\,\frac{dc}{dx} = Avc - \int_0^x (S - k\,AC)\,dy \qquad (10)$$

for a non-conservative substance, and

$$AE\,\frac{dc}{dx} = Avc - \int_0^x S\,dy \qquad (11)$$

for a conservative substance. In Equation (10), k is the first order decay rate constant of the degradable constituent in question, and S now excludes the decaying process by biochemical reaction which is now expressed by the term kAc.

The estuary is divided into n segments, denoted as reaches, by locating n + 1 sampling stations at sections numbering from 0th to nth section. A_i, E_i, v_i and c_i denote the cross-sectional area, dispersion coefficient, advective velocity, and constituent concentration, respectively, at the ith sampling station, whereas S_i denotes the constituent discharge rate per unit time within the ith reach of the estuary.

In solving for conservative constituents under steady state assumptions, Equation (11) is the most convenient form to use. This is normalized by adding the term F denoting the transport of the constituent across the boundary at the origin. Equation (12) is obtained by modifying Equation (11) in terms of central finite differences on the space axis, by substituting

$$(dc/dx)i = (c_{i+1} - c_{i-1})/(x_{i+1} + x_i) \text{ and } c_i = (c_{i+1} + c_{i-1})/2$$

$$c_{i+1} = d_i c_{i-1} - g_i \sum_{k=1}^{i} S_k - g_i F; \; i = 1,2,\ldots, n-1 \tag{12}$$

where $d_i = g_i f_i$ and coefficients g_i and f_i are functions of the physical and hydrological parameters of the estuary. As Equation (12) does not involve S_n, an additional equation is needed. Applying Equation (11) at the nth section with the approximation

$$(dc/dx)n = (c_n - c_{n-1})/x_n,$$

Equation (13) is obtained

$$c_n = d_n c_{n-1} - g_n \sum_{i=1}^{n} S_i - g_n F \tag{13}$$

The above system of equations may be used to predict water quality expressed as concentration of a particular pollutant when the physical parameters of the estuary and waste discharge data are available.

The network model is used in conjunction with water quality models to determine least-cost system design for water supply, wastewater treatment, conveyance and disposal that will satisfy present and projected water quality criteria at future target year. The predictive water quality models for conservative pollutants, bio-chemical oxygen demand (BOD) and dissolved oxygen (DO) or other pollutants simulates water quality expressed as concentration profiles for these constituents. The network model analysed the "on-shore" facilities and made comparative economic evaluation of alternative feasible system designs under different boundary conditions expressed as water quality criteria.

The water quality models provide information regarding the maximum allowable quantities of pollutants that can be safely discharged at the existing or proposed effluent outfalls and still achieve water the desired quality in the receiving waters. In the network model the effluent outfalls are represented as sink nodes. The maximum allowable discharge quantities (kg/day) at these outfalls are converted into maximum allowable flow rates (MLD/day) based on the initial waste loading and the level of treatment received and these flow rates are set as the maximum flow capacity of discharge arcs leading from the treatment process to the outfall (sink). With the above strategy the discharge arcs leading to an effluent outfall following different degrees of treatment have different maximum flow bounds. Thus a discharge arc after primary and secondary treatment may have 100 MLD/day as maximum flow rate to meet the same water quality criteria which act as boundary conditions for the network model. The solution of the network model would choose the appropriate treatment level to satisfy boundary conditions at least annual cost.

Conclusions

The preceding sections described a flexible approach for optimal design of urban water supply/waste disposal system based on the conjunctive application of three different models which could be suited for the decision problem facing an urban area. If the expansion of the water supply system to meet growing demands is of major concern before the city, the network model and the linear (or non-linear) programming model of water transmission system play the major role and the water quality model could be used to provide the boundary condition and to ensure that waste discharges do not create pollution hazard. But if the design of waste treatment and disposal system and water quality control in the receiving waters are of prime interest, the network model coupled with a water quality model or an integrated model of the type discussed by the author in [6] would be applicable. However, the close interrelation between the water supply and waste disposal system has been made clear and it is advisable to analyse these systems simultaneously specifically to investigate the economics of recycling treated waste-water in the water supply system or using it to recharge the ground water acquifers artificially.

Various modifications and improvements of the models discussed in this paper is conceived and these should be subjects of future research. The static network model could be modified to include sequencing decisions in water supply projects to meet growing demand over a longer planning horizon. A study similar to the one described in Lineaweaver [5] is in order to develop realistic cost functions for water supply and treatment projects based on Indian cost data possibly showing regional variations. The network model could be modified to represent blending of water from different sources to improve water quality, e.g., the concentration dissolved solids in ground water could be reduced by blending it with surface water.

Better computational methods are needed for optimising water transmission networks with closed loops resulting from interconnection between different pipelines to improve reliability of water supply. Here again the need for better cost data based on India experience is obvious.

One-dimensional models of water quality may not be always satisfactory for all water bodies and it is possible to develop two-dimensional linear programming or network models of water quality prediction and control. Discharge limits on several pollutants could be simultaneously handled by the network model by the application of a network programme with "gains". The same approach can handle leakage and losses in the water system and a formulation involving a continuous flow of water from sources to consumers and then through waste treatment plants to the receiving waters is possible.

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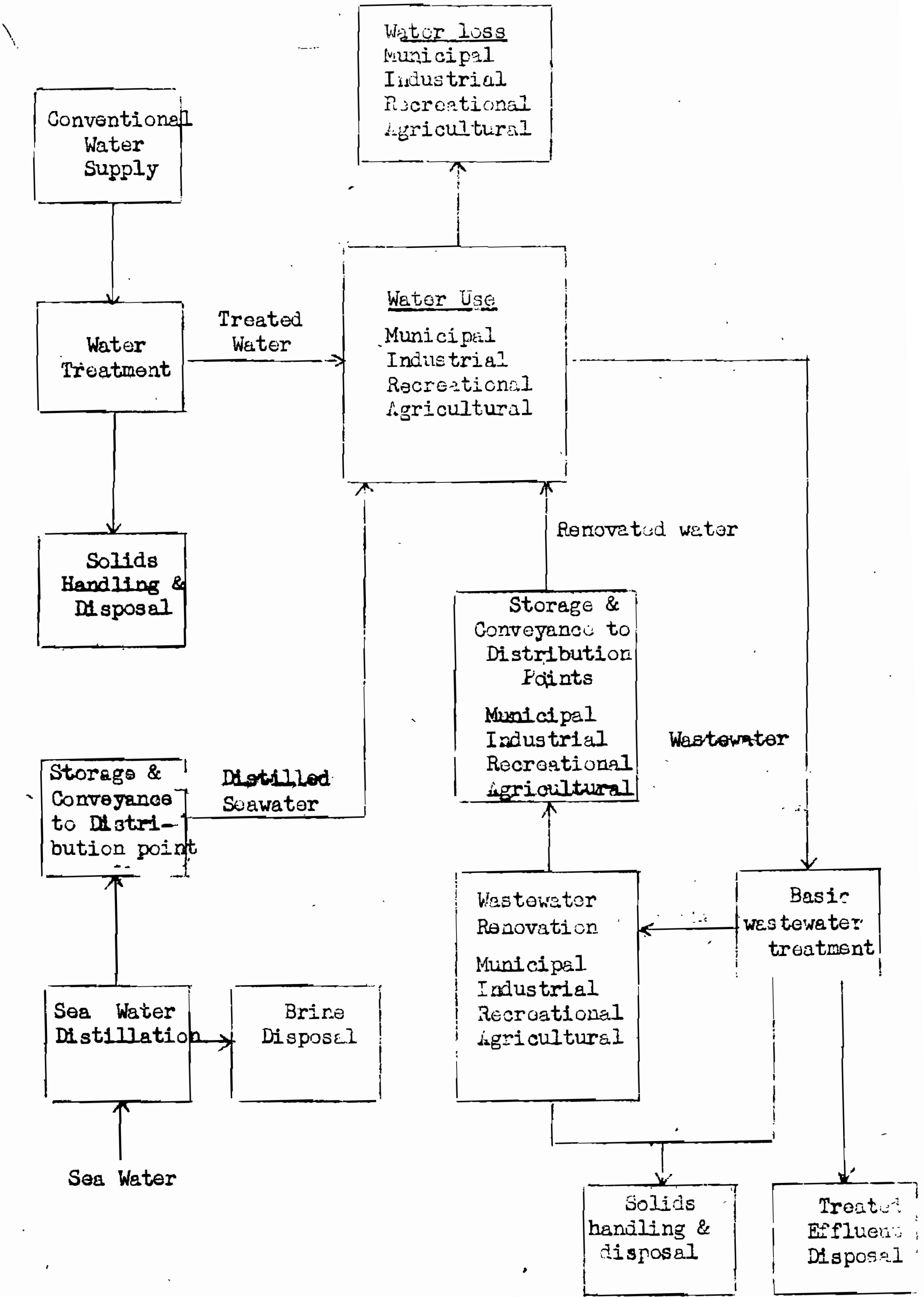

Figure 1: Basic Elements in Water Supply and Wastewater Disposal Systems

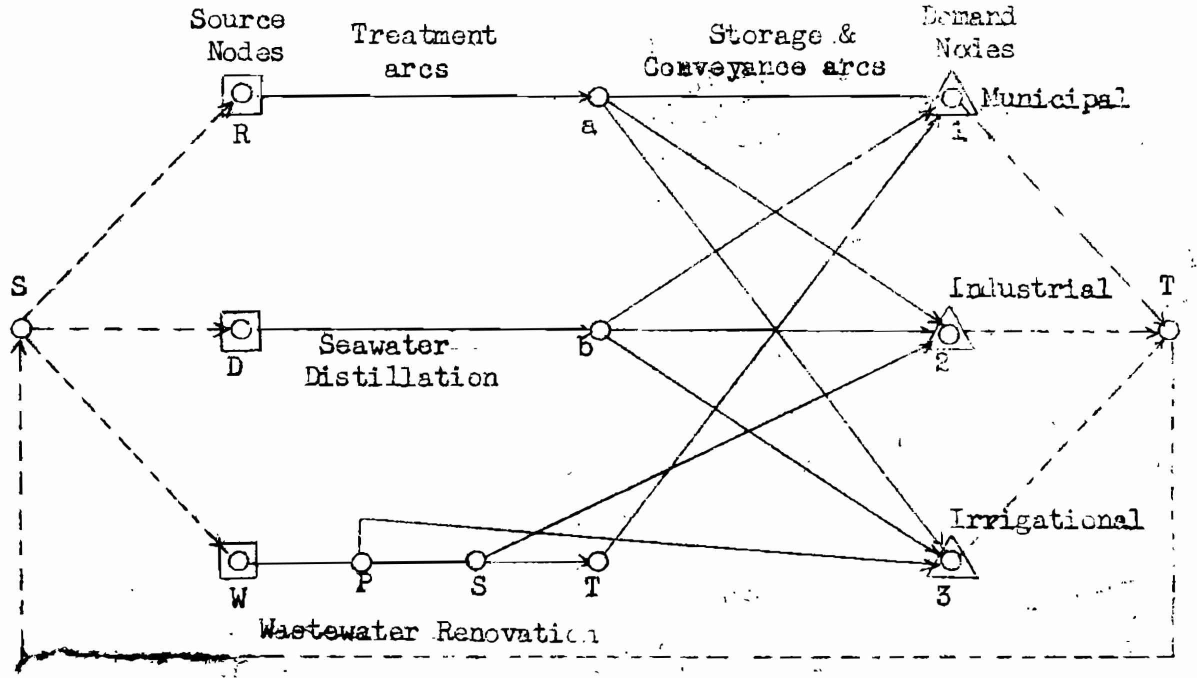

Figure 2: Sample Network to Illustrate Arcs and Nodes

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