THE REVELATION PRINCIPLE FOR GENERAL PRINCIPAL - AGENTS PROBLEMS WITH INCOMPLETE INFORMATION

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Abstract

In this paper we propose a model of a general principal - agents problem (with many agents) in which players choose actions and signals as part of their strategy. Subsequently we show that any principal - agents problem admits a Bayesian - Stackelberg equilibrium if and only if there exists another principal - agents problem where truthful revelation of types by the agents is possible at an equilibrium.

1. Introduction :

The purpose of this paper is to study an incentive property of a general principal agent problem with many agents. Our model is sufficiently general to allow for a simultaneous treatment of moral hazard problems as well as adverse selection problems inherent in many incomplete information monopoly situations (e.g. Chan and Thakor (1937)). It can also be used to extend to the incomplete information case, the work of Holmstrom (1979), Shavell (1979), Grossmann and Hart (1983), Rogerson (1985), Page (1987) on the complete information principal agent problem as also to extend to a more general setting some of the results of Myerson (1982) on the incomplete information principal - agent problem. However, our point of departure is the body of literature which includes the work of Demski and Sappington (1984), Green and Stokey (1983), Holmstrom (1982), Lazear and Rosen (1981), Mookherjee (1984), and Nalebuff and Stiglitz (1983).

The basic idea in our paper is that each agent will choose an action and a signal (containing pertinent information about his personal characteristics) in the light of the incentive scheme proposed by the principal. A problem that now arises is that of specifying the way in which agents choose their actions and signals given their strategic interdependence. This problem is resolved in this paper by adopting the Bayesian equilibrium solution concept (see Myerson (1983)). However, unlike a traditional Bayesian game, the incentive scheme is now manned by a rational decision maker (see Lahiri (1990 a, b)) - the principal. To characterize Bayesian equilibria of such games we establish a revelation principle, more on which has been discussed in Repullo (1986) and Lahiri (1990 b).

2. <u>The Model</u> : The model of principal - agent problems we now introduce is an amalgam of the model of an arbitration game introduced in Lahiri (1990 a,b) and of the Stackelberg incentive compatibility problem introduced by Page (1989).

Suppose there are n <u>agents</u> numbered $i = 1,\ldots,n$. We let T^i denote the set of possible <u>types</u> of agent i, and let $T = T^1 \times \ldots \times T^n$. We assume for simplicity of exposition, that each T^i is a finite set. Let D be the set of possible <u>actions</u> available to the principal, and let Y^i be the set of possible <u>actions</u> available to agent $i = 1,\ldots,n$. Each agent has a Von Nenmann - Morgenstern utility function, $u^i(d,y,t)$, which denotes the payoff to i if d is the principal's action, $y = (y^1,\ldots,y^n) \in Y^1 \times \ldots \times Y^n = Y$ is the vector of actions taken by the agents and if $t = (t_1,\ldots,t_n)$ is the vector of agents' types. Each agent also has a probability distribution $p^i(t_{-i} | t_i)$ where $t_{-i} \in T_{-i} \equiv T^1 \times \ldots \times T^{i-1} \times T^{i+1} \times \ldots \times T^n$, which denotes the subjective probability that player i would assign to the event t_{-i} if i's actual type were t_i.

Each agent is equipped with a message space through which he communicates pertinent information about himself to the principal. The set M^i is the set of possible <u>messages</u> agent i can use and $M = M^1 \times \ldots \times M^n$ is called the <u>language</u>. The principal on his part has a Von - Neumann - Morgenstern utility function $W : D \times Y \times T \to \mathbb{R}$ which gives the payoff he receives for each realization of $d \in D$, $y \in Y$ and $t \in T$. Further, like a Bayesian Statistician, based on the message $m \in M$, the principal forms posterior beliefs about $t \in T$, which is assumed to be summarized by a conditional probability mass function $f(t|m)$ available to the principal. In this paper we assume that the tuple $[T,D,Y,M,W,f,u^1,p^1,\ldots,u^n,p^n]$ is common knowledge and is referred to as a principal - agents problem.

An action plan for the principal is a function

$$g : M \to D$$

Given an action plan 'g', agents choose messages m^i and actions y^i as a function of their types and their common knowledge. We call a pair of mappings $\langle \beta^i, \gamma^i \rangle$ ($\beta^i : T^i \to M^i, \gamma^i : T^i \to Y^i$) a <u>strategy</u> for i. Under such circumstances,

$$\sum_{t_{-i} \in T_{-i}} u^i(g(\beta^1(t_1), \ldots, \beta^n(t_n)), \gamma^1(t_1), \ldots, \gamma^n(t_n), t) p^i(t_{-i} | t_i)$$

represents the expected utility payoff to i if i's type is t^i and if $\langle \beta, \gamma \rangle = (\langle \beta^1, \gamma^1 \rangle, \ldots, \langle \beta^n, \gamma^n \rangle)$ is the vector of strategies used by the agents. A Bayesian equilibrium of g in $\alpha = [T, D, Y, M, W, f, u^1, p^1, \ldots, u^n, p^n]$ is a strategy n-tuple $(\beta^*, \gamma^*) = (\langle \beta^{*1}, \gamma^{*1} \rangle, \ldots, \langle \beta^{*n}, \gamma^{*n} \rangle)$ such that for each player i,

$\beta^{*i} : T^i \to M^i$, $\gamma^{*i} : T^i \to M^i$ satisfies

$$\sum_{t_{-i} \in T_{-i}} u^i(g(\beta^*(t)), \gamma^*(t), t) p^i(t_{-i} | t_i) \geq \max_{\substack{m^i \in M^i \\ y^i \in Y^i}} \sum_{t_{-i} \in T_{-i}} u^i(g(\beta^*(t)/m^i), \gamma^*(t)/y^i, t) \, p^i(t_{-i} | t_i) \qquad (1_i)$$

where

$$\langle \beta^*(t)/m^i \rangle = \langle \beta^{*1}(t_1), \ldots, \beta^{*(i-1)}(t_{i-1}), m^i, \beta^{*(i+1)}(t_{i+1}), \ldots, \beta^{*n}(t_n) \rangle$$

$$\text{and } \langle \gamma^*(t)/y^i \rangle = \langle \gamma^{*1}(t_1), \ldots, \gamma^{*(i-1)}(t_{i-1}), y^i, \gamma^{*(i+1)}(t_{i+1}), \ldots, \gamma^{*n}(t_n) \rangle$$

In view of the above, the problem faced by the principal is given by the following :

$$\max_{\substack{g(m) \in D \\ \gamma_i^* : T^i \to Y^i}} \sum_{t \in T} W(g(m), \gamma^*(t), t) f(t | m) \ \forall \ m \in M \qquad (P)$$

subject to $\beta^*(t) = m$

and $\langle \beta^*_i(.), \gamma^*_i(.) \rangle$ satisfies (1_i) for $i = 1, \ldots, n$.

Provided that $\langle \beta_i^*(.), \gamma_i^*(.) \rangle$ satisfies (1_i) for $i=1,\ldots,n$, the principal can request the agents to select actions, which in addition maximizes the objective function. The triplet $\langle \beta^*(.), \gamma^*(.), g(.) \rangle$ which solves problem (P) will be called a Bayesian Stackelberg equilibrium for the principal - agents problem α.

Given a principal - agents problem , consider another principle agents problem $\bar{\alpha}$ where $M^i = T^i \; \forall \; i = 1,\ldots,n$ and $h(t|t') = f(t|\beta^*(t'))$ defines the posterior beliefs of the principal that 't' is the true type of the agents when t' is the announced type. In the following section we prove the main theorem of this paper.

3. **Main Theorem :**

Theorem : (The Revelation Principle for Principal - Agents Problems with Incomplete Information) : Given $\beta^{*i} : T^i \to M^i$, $\gamma^{*i} : T^i \to Y^i$, such that $\beta^{*i}(T^i) = M^i$ and $\gamma^{*i}(T^i) = Y^i$:

A. $\langle \beta^*, \gamma^*, g^* \rangle$ is a Bayesian - Stackelberg equilibrium for α if and only if

B. $\langle \bar{\beta}, \gamma^*, g^* \circ \beta^* \rangle$ is a Bayesian - Stackelberg equilibrium for $\bar{\alpha}$ where $\bar{\beta}^i(t_i) = t_i \ \forall t_i \in T^i$ and $i = 1,\dots,n.$;

Proof : (A)$\Rightarrow$(B). Let $\langle \beta^*, \gamma^*, g^* \rangle$ be a Bayesian - Stackelberg equilibrium for α. Hence,

$$\sum_{t \in T} W(g^*(\beta^*(t), \gamma^*(t), t) f(t| \beta^*(t) \geq \sum_{t \in T} W(d, \gamma(t), t) f(t| \beta^*(t))$$

$$\forall \quad d \in D, \gamma : T \to Y$$

subject to,

$$\sum_{t_{-i} \in T_{-i}} u^i(g^*(\beta^*(t)), \gamma(t), t) p^i(t_{-i}| t_i) \geq \sum_{t_{-i} \in T_{-i}} u^i(g^*(\beta^*(t)/m^i), \gamma(t)/y^i, t) p^i(t_{-i}| t_i)$$

$\forall\ m^i \in M^i, y^i \in Y^i$ and $t_i \in T^i$, $i = 1,\dots,n.$

Therefore,

$$\sum_{t \in T} W(g^*(\beta^*(t')), \gamma^*(t), t) h(t|t') \geq \sum_{t \in T} W(d, \gamma(t), t) h(t|t') \quad \forall\ t' \in T,$$

subject to

$$\sum_{t_{-i} \in T_{-i}} u^i(g^*(\beta^*(t)), \gamma(t), t) p^i(t_{-i}| t_i) \geq \sum_{t_{-i} \in T_{-i}} u^i(g^*(\beta^*(t/t_i')), \gamma(t|t_i'), t) p^i(t_{-i}| t_i)$$

$\forall\ t_i' \in T^i$ and $i = 1,\dots,n.$

which implies $\langle \bar{\beta}, \gamma^*, g^* \circ \beta^* \rangle$ is a Bayesian - Stackelberg equilibrium for $\bar{\alpha}$.

(B)$\Rightarrow$(A) suppose $\langle \bar{\beta}, \gamma^{*}, g^{*}\circ\beta^{*} \rangle$ is a Bayesian - Stackelberg equilibrium for $\bar{\alpha}$. Hence,

$$\sum_{t\in T}' W(g^{*}\circ\beta^{*}(t), \gamma^{*}(t), t)h(t|t') \geq \sum_{t\in T}' W(d, \gamma(t), t)h(t|t') \qquad \forall\ t'\in T$$

subject to

$$\sum_{t_{-i}\in T_{-i}} u^{i}(g^{*}\circ\beta^{*}(t), \gamma(t), t)p^{i}(t_{-i}|t_{i}) \geq \sum_{t_{-i}\in T_{-i}} u^{i}(g^{*}\circ\beta^{*}(t/t_{i}'), \gamma(t|t_{i}'), t)p^{i}(t_{-i}|t_{i})$$

$$\forall\ t_{i}'\in T^{i} \text{ and } i = 1,\dots,n.$$

Therefore,

$$\sum_{t\in T}' W(g^{*}\circ\beta^{*}(t)) \gamma^{*}(t), t)f(t|\beta^{*}(t)) \geq \sum_{t\in T}' W(d, \gamma(t), t)f(t|\beta^{*}(t))$$

subject to

$$\sum_{t_{-i}\in T_{-i}}' u^{i}(g^{*}\circ\beta^{*}(t), \gamma(t), t)p^{i}(t_{-i}|t_{i}) \geq \sum_{t_{-i}\in T_{-i}}' u^{i}(g^{*}(\beta^{*}(t)/m^{i}), \gamma(t)/y^{i}, t)p^{i}(t_{-i}|t_{i})$$

$$m^{i}\in M^{i},\ y^{i}\in Y^{i} \text{ and } t_{i}\in T^{i},\ i = 1,\dots,n,$$

since $\beta^{*i}(T^{i}) = M^{i}$ and $\gamma^{*i}(T^{i}) = Y^{i}$.

This shows that $\langle \beta^{*}, \gamma^{*}, g^{*} \rangle$ is a Bayesian - Stackelberg equilibrium for α.

<u>Q.E.D.</u>

4. <u>Conclusion</u> : In this paper we have proposed a model of a general principal - agents problem in which agents choose actions and signals as part of their strategy. Subsequently we show that any principal - agents problem admits a Bayesian - Stackelberg equilibrium if and only if there exists another principal - agents problem where truthful revelation of types by the agents is possible in a Bayesian - Stackelberg equilibrium. Such solutions are called incentive compatible.

The revelation principle for Bayesian Collective Choice Problems exists in the literature (see Repullo (1980)). Our contribution is to show that the revelation principle is valid for a strict subclass of such problems known as the principal - agents problems.

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