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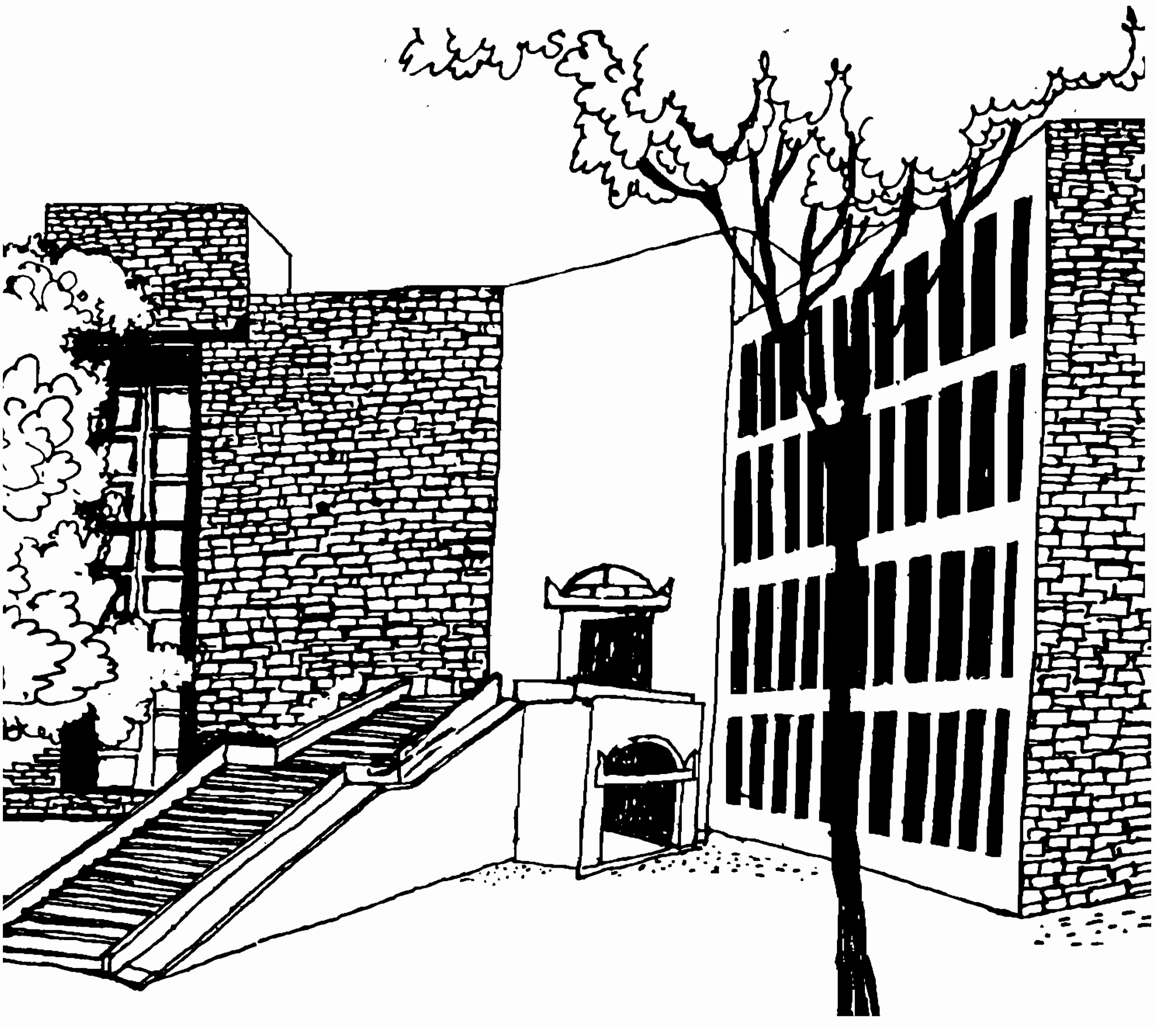

OPTIMAL ORDERING AND INSPECTION POLICIES WHEN THE INCOMING LOTS ARE SUBJECTED TO SAMPLING INSPECTION

By

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ABSTRACT

In this paper a mathematical model has been developed in which the interactions of Economic Order Quantities with quality control policies have been considered. The incoming lots of an inventory system are assumed to be subjected to single sample inspection plan. Expressions for optimal order quantities and inspections policies have been developed assuming the probability distribution function of fraction defective in an incoming lot is uniformly distribued. A numerical example has been provided to illustrate the model.

INTRODUCTION

Although an enoromous amount of research is done on the application of mathematical techniques for modelling inventory systems, very few works have been reported on inventory control of defective items. The presence of defective items in the incoming lots leads to a difference in the quantity ordered and quantity received. Silver (1976) has considered the determination of Economic order Quantity of a product for which the quantity ordered may not be equivalent to the quantity received and presented the analyses for two different cases Viz., i) the standard deviation of the quantity received is independant of the quantity ordered (ii) the standard deviation of the quantity received is dependant on the quantity ordered. Shih (1980) has considered the determination of Economic Order Quantity of a product of imperfect quality, wherein he assumed 100% inspection of the incoming lots. Kalro and Gohil(1982) developed a lot size model when the amount received is uncertain with complete and partial backlogging. Lee and Rosenblatt (1984) have proposed two inspection policies viz the Blind Purchase policy and Selective Purchase policy and studied their affects on economic orer quantity.

With an exception for the model considered by Shih all the other models considered by earlier researchers doesn't consider the affect of quality control procedures generally followed in the industry. Even the 100% inspection considered by Shih is not feasible in many practical situations, particularly when the inspection is of destructive type. In all such situations the buyer will be constrained to go in for statistical sampling techniques to decide upon the acceptance of the incoming lot. Recently, Poornachandra Rao(1988) developed an Economic Order Quantity model when the incoming lots are subjected to single sample inspection. However, this model doesn't consider the quality related costs and doesn't give optimal inspection policies. The present paper aims at determining the optimal order quantity and the inspection policies of an item subjected to single sample inspection plan to decide upon the acceptance of the incoming lot. When the incoming lots are subjected to single sample inspection plan, the buyer inspects a certain predetermined number of items(n) and accepts the lot if the number of defectives in the inspected sample are less than the predetermined acceptance number 'C'. Hence there is a certain probability (P_a) associated with the acceptance of the incoming lot and a certain probability (P_r) of rejecting the lot. If the lot is accepted, all the items including the defectives will be taken into the inventory system. But as the stock is depleted due to demand, the defective items are gradually traced out and removed from the system. Thus the stock is depleted not only because of the demand but also due to the removal of defective items identified during the depletion cycle. Alternatively, if the incoming lot is rejected due to poor quality, stockouts occur until a second lot of acceptable quality is received. It may be pointed out, that the stockout in this case is different from the traditional definition. By traditional definition, stockout (or shortages) occurs when demand exceeds the amount of inventory on hand. In the present situation the stockout is triggered, due to the rejection of an incoming lot of poor quality.

ASSUMPTIONS AND NOTATIONS

The following assumptions will apply throughout our discussion:

i) Demand is constant and deterministic

ii) The planning horizon is infinite

iii) All the incoming lots are subjected to single sample inspection plan.

iv) The average out going quality (α) of the incoming lots is a random variable.

v) Stockouts occur only when the incoming lots are rejected because of poor quality.

vi) If an incoming lot is rejected due to poor quality, the demands occuring until a lot of acceptable quality is received, is backordered.

vii) The demand during the replenishment lead time is known and constant.

Notations:

D	Demand rate in units per unit time
Q	Quantity ordered in units.
C_1	Unit cost in Rs. per unit
C_2	Cost of inspecting one unit
I	Inventory carrying cost, in Re. per Re. per unit time.
A	Fixed ordering cost per order in Rs. per order
p	Random variable representing the process average fraction defective of incoming lots.
P_a	Probability of acceptance of the incoming lot
P_r	Probability of rejecting an incoming lot
α_1	Expected value of probability of acceptance
α_2	Expected value of probability of rejection
n	Size of the sample inspected
α	Average outgoing quality of the product
$g(p)$	Probability density function of 'p'
M_α	Expected value of the average outgoing quality of the product
S	Fixed demand during the replenishment lead time, if an incoming lot is rejected.
P	Fixed penalty cost per unit demand lost in Rs/unit
P_1	Shortage cost per unit backordered per unit time in Re/unit/unit time.

THE MODEL

The average out going quality (α) of the product subjected to single sample inspection plan which is a measure of the fraction defective is given by (Duncans,1986):

$$\alpha = P_a p(Q - n)/Q \qquad \ldots (1)$$

If the ordered quantity is sufficientyly large, compared to the sample inspected, can be approximated as,

$$\alpha = P_a p \qquad \ldots (2)$$

Assuming Bernoulli's process, during the process of inspection, we have,

$$P_a = \sum_{i=0}^{c} {}^{n}C_i \, p^i (1 - p)^{n-i} \qquad \ldots (3)$$

$$\therefore \alpha = \sum_{i=0}^{c} ({}^{n}C_i \, p^i (1 - p)^{n-i}) p \qquad \ldots (4)$$

Since the fraction defective 'p' of the incoming product is a random variable, the expected value of the average out going quality (α) of the product is given by:

$$M_\alpha = \int_0^1 [\sum_{i=0}^{c} {}^{n}C_i \, p^i (1 - p)^{n-i}] p g(p) dp \qquad \ldots (5)$$

Thus the expected number of good pieces in a cycle is given by:

$$\bar{Q} = Q(1 - M_\alpha) \qquad \ldots (6)$$

Since the depletion cycle is associated with a probability of P_a and the backorder cycle is associated with a probability of P_r we have, cycle length

$$T = (\bar{Q} - S P_r) P_a / D + S P_r / D$$

$$= \{(1 - M_\alpha) Q / D\} P_a + (S/D) P_r^2 \qquad \ldots (7)$$

Expected cycle length is given by,

$$E(T) = T_m = Q(1 - M_\alpha)/D \int_0^1 P_h f(p)\, dp + S/D \int_0^1 P_h^2 f(p)\, dp$$

$$= ((1 - M_\alpha)Q/D)\alpha_1 + (S/D)E(P_h^2) \quad \ldots (8)$$

The variability in probability of rejection is given by

$$\sigma^2 = \int_0^1 (P_h - \alpha_2)^2 g(p)\, dp$$

$$= E(P_h^2) - \alpha_2^2$$

$$E(P_h^2) = \sigma^2 + \alpha_2^2 \quad \ldots (9)$$

From equations (8) and (9), we have

$$\therefore T_m = ((1 - M_\alpha)Q\alpha_1)/D + S(\sigma^2 + \alpha_2^2)/D \quad \ldots (10)$$

Since the second term in the above expression is a constant, letting

$$r = S(\sigma^2 + \alpha_2^2)/D \quad \ldots (11)$$

$$T = ((1 - M_\alpha)Q\alpha_1)/D + r \quad \ldots (12)$$

Expected cost per cycle

$$E(C) = \int_0^1 [\, nC_2 + A + ((\bar{Q} - P_h S)^2 / (2D))\, IC_1 P_a\; QP_a pC_1 + (PS + P_1 (S^2/2D))P_h \,]\, g(p)\, dp$$

$$= nC_2 + A + (IC/2D)\int_0^1 [\, \bar{Q}^2 + S^2 P_h^2 - 2\bar{Q}SP_h]\, g(p)\, dp - (IC/2D)\int_0^1 [\, \bar{Q}^2 P_h + S^2 P_h^3 - 2\bar{Q}SP_h^2]\, g(p)\, dp \; QM_\alpha C_1 + (PS + P_1 (S^2/2D))\alpha_2 \quad \ldots (13)$$

Neglecting the third power of P_n, we have,

$$E(C) = nC_2 + A + (IC/2D)[\bar{Q}^2(1-\alpha_2) + S^2(\sigma^2+\alpha_2^2) + 2\bar{Q}S(\sigma^2+\alpha_2^2-\alpha_2)] + QM_\alpha C_1 + \{PS + P_1(S^2/2D)\}\alpha_2 \quad \ldots (14)$$

Expected cost per unit time $K(Q) = E(C)/E(T)$

$$K(Q) = (A + nC_2)D/\{(1-M_\alpha)Q\alpha_1 + Dr\} + IC/2\{(1-M_\alpha)Q\alpha_1 + Dr\} \times [\bar{Q}^2(1-\alpha_2) + (\sigma^2+\alpha_2^2)(S^2 + 2\bar{Q}S) - 2\bar{Q}S\alpha_2] + (\bar{Q}M_\alpha C_1)[D/\{(1-M_\alpha)Q\alpha_1 + Dr\}] + \{PS + P_1(S^2/2D)\}[D\alpha_2/\{(1-M_\alpha)Q\alpha_1 + D r\}] \quad \ldots (15)$$

We can rewrite (15) in terms of T using (12), we have

$$K(T) = (A + nC_2)/T + (IC/2DT)[D^2\{(T-r)^2/\alpha_1^2\}(1-\alpha_2) + (\sigma^2+\alpha_2^2)\{S^2 + 2D(T-r)S/\alpha_1\} - 2D(T-r)S\alpha_2/\alpha_1] + QM_\alpha C_1/T + \{PS + P_1(S^2/2D)\}\alpha_2/T \quad \ldots (16)$$

Differentiating equation (16) with respect to T and equating to zero, we have:

$$T^* = (2\alpha_1/IC\,D)^{1/2}[\,nC_2 + A + IC\,(S^2/2D)(\sigma^2+\alpha_2^2)\{1 + 2\alpha_2/\alpha_1 + (\sigma^2+\alpha_2^2)/\alpha_1 - 2(\sigma^2+\alpha_2^2)/(S\alpha_1)\} - (S/\alpha_1)(\sigma^2+\alpha_2^2)(C_1M_\alpha/(1-M_\alpha)) + \{PS + P_1(S^2/2D)\}\alpha_2] \quad \ldots (17)$$

To determine optimal inspection policies, the following components of costs have been considered: i) cost of inspection, ii) Expected cost of rejected items when the lot is accepted and iii) Expected cost of lost sales per cycle. The total cost function is given by,

$$TC(n,C) = nC_2 + QC_1 M_\alpha/T + [PS + P_1(S^2/2D)](\alpha_2/T) \quad \ldots (18)$$

Since n and C are discrete, the optimal values are determined using the following inequalities.

$$\Delta(C - 1) \leqslant 0 \leqslant \Delta(C) \quad \ldots (19)$$

$$\Delta(n - 1) \leqslant 0 \leqslant \Delta(n) \quad \ldots (20)$$

Assuming that probability density function of 'p' follows uniform distribution, we have

$$g(p) = 1 \quad 0 < p < 1$$
$$= 0 \text{ otherwise}$$

Using the above probability density function, we have,

$$M_\alpha = (C + 1)(C + 2)/\{2(n + 1)(n + 2)\} \quad \ldots (21)$$

$$\alpha_2 = (n - C)/(C + 1) \quad \ldots (22)$$

Using (18), (21) and (22) in (19), we have

$$C(C + 1) < (n + 1)(n + 2)\{PS + P_1(S^2/2D)\}/QC_1 < (C + 1)(C + 2) \quad \ldots (23)$$

Using (18), (21) and (22) in (20), we have

$$M(n) < C_2 < M(n - 1) \quad \ldots (24)$$

where $M(n) = QC_1(C+1)(C+2)/\{(n+1)(n+2)(n+3)\}$

$$- (PS + P_1 S^2/(2D))(C+1)/\{(n+1)(n+2)\} \quad \ldots (25)$$

SPECIAL CASES

<u>Case i</u>: When S = 0, equation (16) can be rewritten as:

$$K(T) = nC_2/T + A/T + IC_1/(2DT)[(D^2T^2/\alpha_1 + (\sigma^2+\alpha_2^2)(2DT/\alpha_1)] + (DC_1/\alpha_1)\{M_\alpha/(1 - M_\alpha)\} \quad \ldots (26)$$

where $I = Q(1 - M_\alpha)\alpha_1/D$ ... (27)

Therefore, the optimal cycle time is given by

$$T^* = \sqrt{2\alpha_1(A + nC_2)/(IC_1 D)} \quad \ldots (28)$$

The optimal values of acceptance number and sample size, given by inequalities (23) and (24) can be rewritten as:

$$C(C + 1) < O < (C + 1)(C + 2) \quad \ldots (29)$$

$$X(n) < C < X(n - 1) \quad \ldots (30)$$

where $X(n) = QC_1(C+1)(C+2)/\{(n+1)(n+2)(n+3)\}$ ... (31)

<u>Case ii</u>: When S = 0, $C_2 = 0$ and $\alpha_1 = 1$ (when no inspection is adopted), equation (17) can be rewritten as:

$$T^* = \sqrt{2A/(IC_1 D)} \quad \ldots (32)$$

SOLUTION PROCEDURE

<u>Step 1</u>: Set $n = 1$ and $C = 0$

<u>Step 2</u>: Determine T^* from equation (17) and the corresponding Q from equation (12).

<u>Step 3</u>: Determine C using inequality (23) and the current value of n.

<u>Step 4</u>: Determine n using inequality (24) and the value of $\bar{C}$ obtained from Step 3 above.

<u>Step 5</u>: Repeat Steps 2,3 and 4 until the solution converges.

EXAMPLE

To illustrate the model, a hypothetical system is considered with the following parameter values:

D = 2000 units per year

A = Rs. 50 per order

I = Rs. 0.2 per unit per year

C_1 = Rs. 5 per unit

P = Rs. 1 per unit

P_1 = Rs. 0.1 per unit

C_2 = Rs. 0.5 per unit

S = 50 units

For this system, using the solution procedure described earlier, we have, T = 0.247 years and the corresponding order quantity is 3174 units. The optimal sample size to be inspected is 664 and the acceptance number is 96. The total cost of operating this system is Rs. 3596/-. Instead of using the optimal T^* developed here, if the cycle time given by the classical economic order quantity model ($T^{*} = \sqrt{2A/(IC_1 D)}$) is used, the cost of operating the inventory system would have been Rs. 3614/-. Hence there is a saving of 0.5% as a result of using the present model. If the demand during lead time, when the incoming lot is rejected is zero, the optimal value of T is 0.0416 years and the corresponding order quantity is 3330 units and the cost of operating the system is Rs. 3552/-. The optimal sample size(n) is 39 and the optimal acceptance number (C) is zero.

CONCLUSIONS

In this paper, a mathematical model has been developed to study the affect of ordering policies when the incoming lots are subjected to single sample inspection plan. Expressions for optimal cycle time, optimal sample size and optimal acceptance number have been determined, assuming that the probability density function of fraction defective follows uniform distribution. An iterative solution procedure has been presented to determine the optimal policies. Table-1 gives the optimal cycle time, optimal order quantity, optimal sample size, optimal acceptance number, cost of operating the system and the cost of using traditional EOQ model for various values of demand lost during replenishment lead time, if an incoming lot is rejected because of poor quality. It is evident that the model is sensitive to the demand lost. It is interesting to note that the sample size increases and approaches the order quantity with the increase in lost sales. This implies that at higher values of lost sales it is better off to go in 100% inspection rather than sample inspection. The savings in cost resulting due to the present model is around 28.5% when the lost sales is 5 units and 68% when the lost sales is 1000 units.

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Table-1

OPTIMAL POLICIES FOR VARIOUS VALUES OF BACKORDERED SALES

S	T	Q	n	C	TC	TC
0	0.0416	3330	39	0	3552	9506
5	0.0902	2591	131	8	2906	4064
10	0.1206	2648	214	18	3008	3539
25	0.1806	2889	410	48	3300	3368
50	0.2470	3174	664	96	3596	3614
75	0.2990	3387	885	144	3796	3957
100	0.3440	3558	1087	192	3948	4321
500	0.7700	4890	3406	934	4814	9836
1000	1.1250	5798	5570	1827	5174	16311