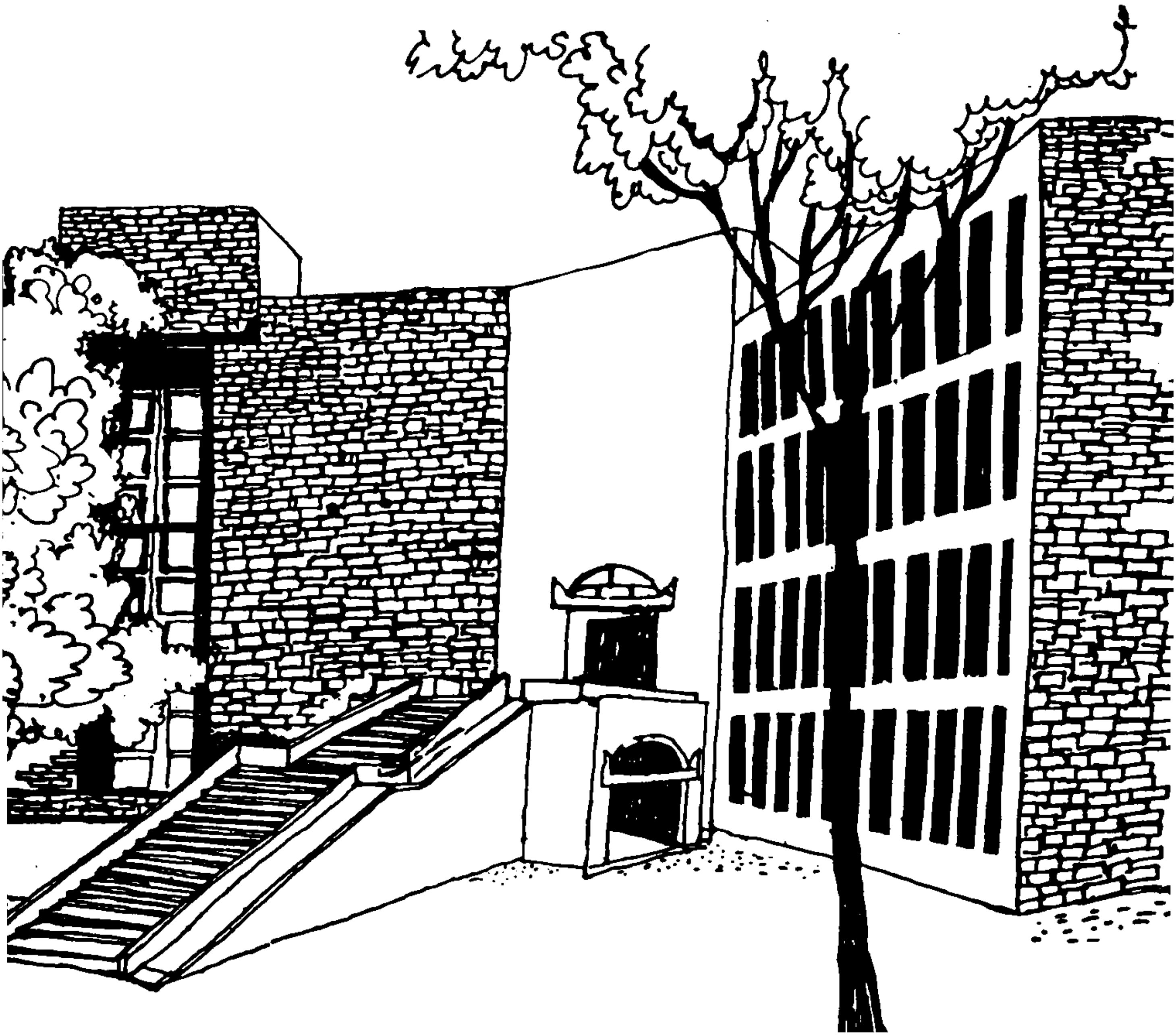
Working Paper



AN IMPROVED PROCEDURE FOR ECONOMIC ORDER QUANTITY WITH ALL UNIT PRICE DISCOUNTS

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AN IMPROVED PROCEDURE FOR ECONOMIC ORDER QUANTITY WITH ALL UNIT PRICE DISCOUNTS

Suppliers often offer price discounts for large purchases. These discounts are generally of two types: all unit discounts and incremental discounts.

This paper presents an improved procedure for determining the economic order quantity under all unit discounting scheme. Upper bounds on the total relevant costs are obtained which are used to discard certain high-price ranges from consideration.

INTRODUCTION

Suppliers often offer price discounts for large purchases. These discounts are generally of two types: all unit discounts and incremrental discounts. In all unit discounting scheme the discounts are offered to every unit purchased whereas in the incremental discounting scheme discounts are offered only to the additional units ordered beyond a specified quantity. Hadley and Whitin [1] provide a procedure for computing economic order quantity with discounting pricing scheme. Sethi [2] has considered the possibility of disposals of excess purchases at some finite cost in order to avail discounts., Chandrasekhar [3] has considered the case when units are charged more for larger quantity orders. This paper deals with all unit discounting and introduces an improved procedure for determining the optimal lot size.

PRELIMINARIES

We will assume that all the assumptions of classical EOQ model hold except for the discounts. Let us use the following notations:

A = Ordering Cost Per Order

i = Inventory Carrying Cost Per Year Per dollar

D = Annual Demand

Q = Order Quantity

C_j = Price Per Unit When Lot Size

$Q \in [Q_j, Q_{j+1})$; $(j = 0,1,2,\ldots,m)$

with $Q_0=0$, and $Q_{m+1}=\infty$.

TC(Q) = Total Annual Relevant Costs(Sum of annual ordering costs, inventory carrying costs, and item costs)

Q^* = Optimal Lot Size

It is assumed that $C_0 > C_1 > \ldots > C_m$.

Hadley and whitin [1] have described following procedure to determine the economic order quantity:

1. Compute EOQ using Wilson's formula, with unit cost = C_m

and if EOQ so obtained $\geq Q_m$ (i.e. EOQ is price-feasible), then stop. Otherwise, set $Q^* = Q_m$, and compute $TC(Q^*)$.

2. Move to the next higher price level, and compute EOQ. If lot size Q is price-feasible, stop. Compute total relevant cost TC(Q). If $TC(Q) < TC(Q^*)$, replace Q^* by Q. [If $TC(Q) = TC(Q^*)$, Q and Q^* are both optimal order quantities.]

   If Q is not price-feasible, replace Q by the lowest price-feasible value of Q, and compute TC(Q). If $TC(Q) < TC(Q^*)$, set $Q^* = Q$.

3. Move to next higher price-level, if any, and go to step 2; otherwise, stop.

With (m+1) price levels, above procedure may require several computations of economic order quantities and corresponding total relevant costs before the optimal lot size is determined. Suggested modified approach develops upper bounds on the total cost function which help in discarding computations at certain higher price-levels.

MODEL ANALYSIS

Suppose after computing EOQ with price level $C=C_m$ the lot size Q is found infeasible, then $Q^* = Q_m$, and

$$TC(Q^*) = DC_m + AD/Q_m + Q_m iC_m/2$$

$TC(Q^*)$, therefore, becomes an upper bound (UB) on the total annual relevant costs TC. Therefore, if we consider procurement at some price level C, the total annual relevant costs have to be at least $DC + \sqrt{2\,ADiC}$ (It will be exactly equal to this quantity if EOQ is price-feasible).

Therefore if purchasing at the price level C is to be considered, it is necessary that the following condition holds true:

$$DC + 2\,ADiC < UB$$

Simplifying, we get,

$$C < \frac{UB + Ai - \sqrt{(Ai)^2 + 2\,AiUB}}{D} = r, \text{ say.}$$

Therefore procuring at price levels $C_j \geq r$ can be ignored from further consideration. We apply this analysis and suggest the following modified procedure.

A MODIFIED PROCEDURE

1. Determine optimal lot size Q with unit cost $C=C_m$ and if found price-feasible, stop. If Q is not price feasible, set $Q^*=Q_m$, compute $TC(Q^*)$. Also set $UB = TC(Q^*)$.

2. Compute ratio r by:

$$r = \frac{UB + Ai - \sqrt{(Ai)^2 + 2AiUB}}{D}$$

3. Ignore price levels where $C_j \geq r$.

4. If there are no price levels left, stop. Otherwise, move to next higher price level and compute EOQ. If lot size Q is price-feasible, stop. Compute TC(Q). If TC(Q) < TC(Q^*), set Q^*= Q. (If TC(Q)= TC(Q^*), Q and Q^* are both optimal lot sizes).

If Q is not price feasible, replace Q by the lowest price-feasible value of Q, and compute TC(Q). If TC(Q) < TC(Q^*), set Q^*=Q, and UB = TC(Q). Recompute ratio r with the updated bound UB. Go to step 3.

A NUMERICAL EXAMPLE

Suppose:

A = $10

D = 1000 units per year

i = 20% per year

Unit costs are as follows:

Q	C
0 - 199	$5.00
199 - 499	$4.75
500 - 999	$4.50
1000 - 1999	$4.20
2000 or more	$4.00

At price level C = $4.00, economic order quantity is

WP 619

$$Q = \sqrt{\frac{2(10)(1000)}{(.20)(4)}} = 158.11$$, which is price-infeasible.

Therefore, $Q^* = 2000$

$TC(Q^*) = \$4(1000) + 10(100/2000) + (2000/2)(.2)(4)$

$= \$4805$

Therefore, UB= \$4805

Now calculate ratio r:

$r = (4805 + 10(.2) - \sqrt{(2)^2 + 2(2)(4805)})/(1000)$

$= 4.668$

Therefore we need not consider price levels \$5.00 and \$4.75 any further.

Next evaluate economic order quantity at price level

C = \$4.20

$$Q = \sqrt{\frac{2(10)(1000)}{(.20)(4.20)}} = 154.30$$ which is also price-infeasible.

The lowest order quantity Q in this price range is 1000 units.

$TC(Q) = \$4630$

Since $TC(Q) < UB$, update Q^* and UB.

Updated Q^* and UB are 1000 units and \$4630, respectively.

Since UB is updated, we update the value of r:

$r = (4630 + 2 - \sqrt{(2)^2 + 2(2)(4630)})/(1000)$

$= 4.496$

Therefore we need not consider procurement at price level of

$4.50 either. Since no more price levels are left for further consideration, we have the optimal solution with economic order quantity of 1000 units at price level of $4.20 per unit.

CONCLUSION

In this paper we have considered an EOQ model with all unit discounting. A modified procedure is suggested for determining the order quantity. Upper bounds are developed on the total annual relevant costs which help reducing computations of economic lot sizes and corresponding total annual relevant costs at various price levels.

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