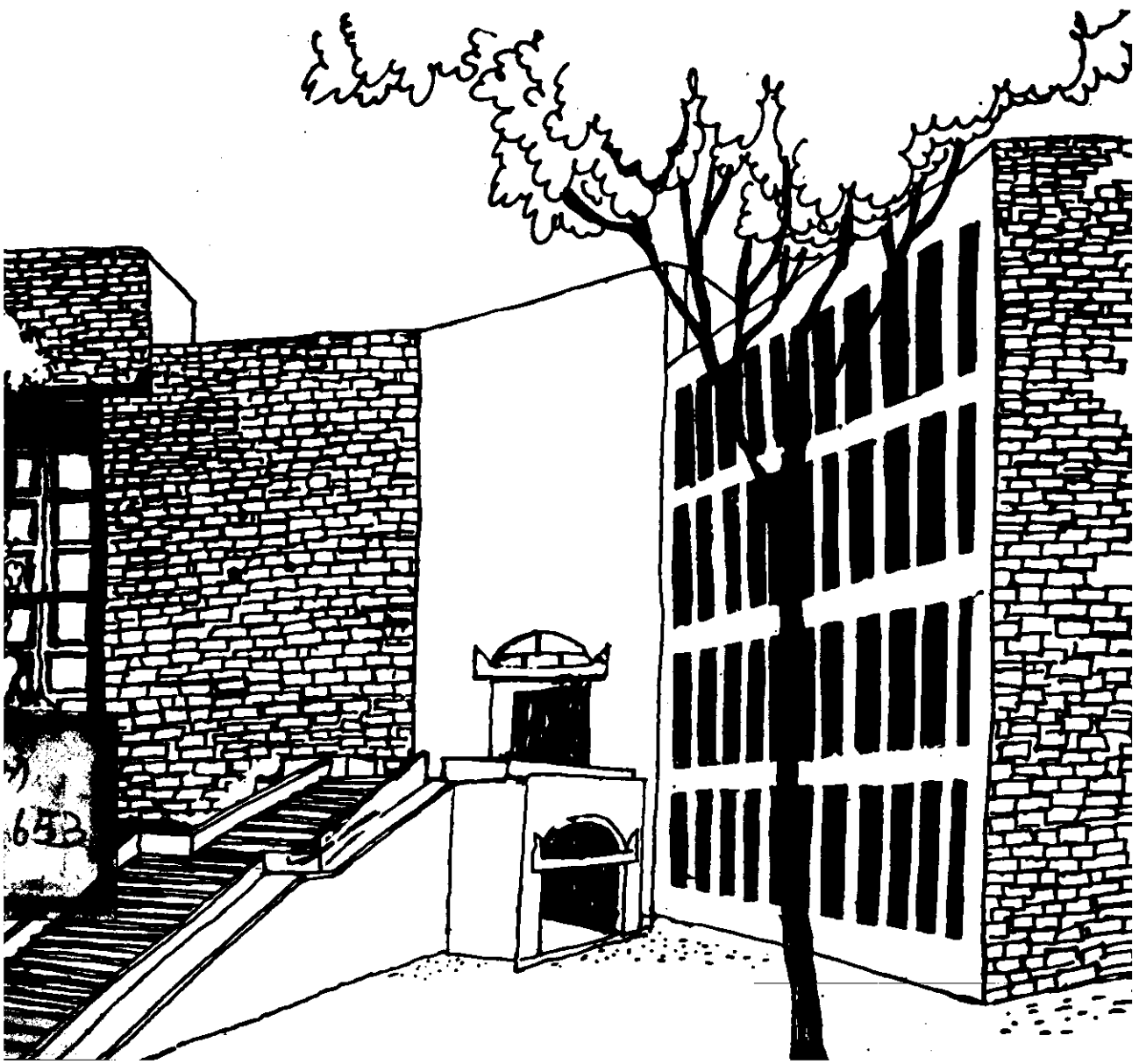
# Working Paper



# A PROJECT IS A COMPOUND - NOT A MIXTURE
# CONCEPTUAL PROBLEMS IN VALUATION

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## CONCEPTUAL PROBLEMS IN VALUATION

## Introduction

The use of Risk Adjusted Discount Rate (RADR) in project evaluation has attracted considerable attention in finance literature. The major issues of concern centre around three areas:

a) The implication of RADR in a multiperiod set-up (Robichek and Myers, 1966; Fama,1977),

b) the applicability or otherwise of RADR for valuing negative cash flows (Lewellen 1977; Beedles 1978; Celec and Pettway 1979; Miles and Choi 1979; Booth 1982), and

c) the question of whether in the project evaluation context, a single RADR should be used to discount the net cash flow stream or separate RADRs should be used to discount each of the different cash flow streams constituting the net cash flow stream separately (Schall, 1974; Hull, 1986; and others).

Robicheck and Myers (1966) first criticised the use of RADR in a multi-period context, where they showed that it implies an assumption of increasing risk over time. Subsequently, the other two issues have attracted the attention of researchers in the context of asset acquisition decisions, wherein each cash

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flow stream is sought to be discounted at a different RADR (Schall, 1974 and others). In this context, some researchers (Lewellen, 1977; Miles and Choi, 1979 and Hull, 1982) maintain that both cash inflows and cash outflows with identical riskiness should be discounted at the same rate and that, the higher the riskiness, the higher should be the discount rate. Celec and Pettway (1979), Booth (1982) and others criticise the above view point on the basis of its counter intuitive content, wherein, ceteris paribus, a project with greater uncertainty in its cash outflows is shown to be more attractive. These researchers favour using a RADR below the riskfree rate for risky cash outflows (when they are negatively correlated with market return), so that the results obtained are consistent with what one would expect using the certainty Equivalent (CE) framework. Under the CE framework, the higher the riskiness of a cash outflow stream, the higher (more negative) will be the certainty equivalent and consequently, the less attractive will be the project, which is intuitively appealing. Thus, Beedles (1978) and Hull (1982) argue in favour of the CE approach over the separate RADRs for the cash inflow and outflow streams. Hull (1982) further argues that even when the RADR approach is used, separate RADRs should be used to discount the cash inflow and outflow streams rather than use a single RADR for the net or aggregate cashflow stream. He maintains that the aggregate cashflow is liable to "require a discount rate which is counter-intuitive".

In this paper we proceed to show in a single period framework, that misleading inferences may result in the project evaluation context under certain conditions, whenever cash inflow and outflow of a project are valued separately and aggregated, using the value additivity principle; whether the valuation of cash inflows and outflows is done by using appropriate RADRs or by using the standard CE framework being of little consequence. Consequently we show that it is erroneous to value cash inflows and outflows separately to arrive at the total value of the project and that the total value of the project can be estimated either by using a single RADR to discount the net cash flow or by discounting the certainty equivalent of the net cash flow by risk free rate of return.

## Valuation of Negative Cash flows : Discussion

### When systematic variability alone is relevant to Valuation:

It seems to be widely held in the literature (Lewellen,1977, 1979; Celec and Pettway, 1979; Booth, 1982, etc.) that whenever cash outflows are positively correlated with market returns, and thus have a positive $\beta$ , they are to be valued just as cash inflows are valued. And whenever cash outflows are negatively correlated with market returns, and thus have a negative $\beta$ , the expected outflow stream is to be discounted at a rate less than

the risk free return. Such a position appears to steer clear of the counter-intuitive results discussed earlier. Thus, just as positive cash flows with negative beta are considered desirable in the portfolio context[1], similarly negative cash flows with positive beta are also considered desirable in a portfolio. The mathematical symmetry of the logic appears to be inviolable.

Notwithstanding the mathematical logic of the portfolio theory, the fact remains however that a prospect with negative expected value is never tradeable independently in the financial market and hence not valued in itself. Implicit in the value additivity principle is the truism that the value of a prospect with positive expected value does not change with how the prospect is distributed among the stake-holders. In other words value additivity holds only in the non synergistic context, where two or more assets are traded independently. Thus it is possible to arrive at the value of the firm, by aggregating the values of its shares and bonds, which is much like finding out the properties of a simple mixture by finding out the properties of its individual components. On the other hand, the process of valuing the cash inflows and outflows separately to value the project is like attempting to estimate the properties of a compound through the properties of its elements. One may well

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[1] For that matter, it is difficult to conceive of the independent existence of a security with a time invariant negative $\beta$ and positive expected cash flows in a rational market, for no rational investor would ever invest in a risky security yielding a return less than the risk free rate of return for any length of time. Negative $\beta$s can at best be tolerated as temporary phenomenon in a non-stationary $\beta$ world.

conjecture that water is inflammable since hydrogen burns and oxygen aids burning and water is but two parts hydrogen and one part oxygen! Little surprise then, that, when non-tradeable prospect such as a cash outflow is valued separately and aggregated with the value of cash inflow, the result is often counter-intuitive.

The key to the resolution of the controversy in the literature lies in recognising the fact that it is the net cash flow which is in fact traded and therefore valued in the market place. Notwithstanding the sign of its $\beta$, a prospect with negative expected net cash outflow would be summarily rejected in the market.[2]

When total variability is relevant to valuation:

We shall now show through an example that valuing cash inflows and outflows separately whether using appropriate RADRs or by using CE approach can lead to erroneous results in evaluating projects. We shall consider three alternative single period scenarios of projects A and B and discuss the relative attractiveness of one over the other. In the first scenario, the cash inflows are certain but the outflows are uncertain; in the second scenario, the

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2 See Foot-note 1.

cash inflows are uncertain but the outflows are certain; and in the third scenario, both inflows and outflows are uncertain, such that the net cash flows are certain.

Scenario 1

Table 1

| State of Nature (S) | Probability S will prevail (P) | Project A | | | Project B | | |
|---|---|---|---|---|---|---|---|
| | | Inflow ($I_A$) | Outflow ($O_A$) | Net flow ($N_A$) | Inflow ($I_B$) | Outflow ($O_B$) | Net flow ($N_B$) |
| 1 | 1/3 | 600 | 700 | (100) | 600 | 800 | (200) |
| 2 | 1/3 | 600 | 500 | 100 | 600 | 500 | 100 |
| 3 | 1/3 | 600 | 300 | 300 | 600 | 200 | 400 |

Table 1 completely depicts scenario 1, wherein Project B has an outflow stream which has the same expected value as that of A, but a higher risk. It is clear that given a mean preserving increase in risk, the certainty equivalent will decrease (Beedles, 1978) so that the CE of B's outflow will be less (more negative) than that of A's outflow. If in this case, the inflows and the CE of outflows are discounted at the risk free rate and the resulting values aggregated, project A will be shown to be more attractive than B. An identical indication

would be obtained, even if the CE of the net cash flow for the two projects were discounted at the risk free rate and the resulting values compared. The result would again be unaltered if suitable RADRs were used to discount the two net cash flows, since given a mean preserving increase in risk, the net expected cash flow of project B would be discounted at a higher rate. On the other hand, if the expected values of cash outflows for the two projects are discounted at separate RADRs and resulting values of the outflows aggregated with the values of their inflows (discounted at risk-free rate), the result would be counter intuitive, since project B would be indicated to be superior to A, as the expected value of A's outflow would be discounted at a smaller rate as compared to the expected outflow of B. Thus under this scenario, using separate RADRs for valuing the inflows and outflows separately and then aggregating them to compare the projects, leads to a counter intuitive result.

Scenario 2

The above arguments may be extended to Scenario 2 (Table 2) to show that using separate RADRs for inflows and outflows again result in counter-intuitive indication. In this scenario, the outflow for the two projects are certain while the inflow is uncertain. Once again using suitable RADRs for discounting the expected values of net cash flows of the two projects turns out to be consistent

with the CE framework, whether applied to the inflows and outflows separately or to the aggregated cashflows as a whole. This is because in both of the above scenarios, the variance of the cash outflow/inflow is fully transferred to the net cash flow. However, this consistency breakdown in Scenario 3.

Table 2

| S | p | Project A | | | Project B | | |
|---|---|---|---|---|---|---|---|
| | | $I_A$ | $O_A$ | $N_A$ | $I_B$ | $O_B$ | $N_B$ |
| 1 | 1/3 | 800 | 500 | 300 | 900 | 500 | 400 |
| 2 | 1/3 | 700 | 500 | 200 | 700 | 500 | 200 |
| 3 | 1/3 | 600 | 500 | 100 | 500 | 500 | 0 |

Scenario 3

Table 3 depicts the scenario wherein both inflow and outflow are risky but are perfectly correlated so that the net cash flow is certain. Here, the expected values of inflows and outflows for the two projects

Table 3

| A | p | Project A | | | Project B | | |
|---|---|---|---|---|---|---|---|
| | | $I_A$ | $O_A$ | $N_A$ | $I_B$ | $O_B$ | $N_B$ |
| 1 | 1/3 | 1100 | 1000 | 100 | 1000 | 900 | 100 |
| 2 | 1/3 | 600 | 500 | 100 | 600 | 500 | 100 |
| 3 | 1/3 | 100 | 0 | 100 | 200 | 100 | 100 |

are the same, but the variance of both inflow as well as the outflow for B is lesser than the respective variances of A. It is clear therefore that the CE of A's inflow must be lesser than that of B's inflow. Also the CE of A's outflow must be lesser (more negative) than that of B's outflow. Thus if the inflows and outflows of the two projects are valued separately using the CE framework and then aggregated, clearly project B is indicated to be superior to project A. But a look at the net cash flows of the two projects reveals that the market must view the two projects to be identical and must value them equally by discounting the net cash flows at the risk free rate. It may be noted that under this scenario, using separate RADRs for the inflows and outflows of the two projects and aggregating them together also leads to the misleading result. However, when the net cash flows of the two projects are valued in aggregate, whether by using appropriate RADRs or by using the CE framework (in this case though, the two approaches coincide since the net cash flows are certain), the correct indication is obtained.

From the discussion hitherto, it turns out that the correct valuation approach in the project evaluation context lies in valuing the aggregate cash flows either by using a suitable RADR or by using the CE framework and not by valuing the inflows and outflows separately. This is because when the RADR or CE framework is applied to negative cash flows, the resulting value is meaningless

since the market attaches no value whatever to negative cash flows. In other words, a prospect with negative expected cash flow is rejected outright in the market, except when such a prospect is inextricably linked with a larger prospect, such that the two prospects together have a positive net expected cash flow. In such a case, however, the "two prospects" are synergistically linked so that together they constitute a single composite prospect, which must be valued in its entirety. It would be erroneous to view the selection of a project as a decision to "take a position" in the market -- one being a "long" portfolio commitment to project inflows, and the other implicitly being a "short" sale of project outflows (Lewellen, 1977). This is because the "short" sale of project outflows have no independent existence outside of the associated project inflows. In reiteration then, the smallest unit which is traded and therefore valued in the market is a composite project - complete with its inflows and outflows, and not its dismembered components viz., inflows and outflows separately.

Disregarding this basic premise of an asset tradeability leads to other confusions as well. For example, Miles and Choi (1979) maintain that on account of arbitrage possibilities, certainty equivalents for inflows and outflows must be symmetric. This can

happen only when the certainty equivalent for an uncertain inflow or an identical outflow equals the expected value of the inflow or outflow (see Appendix I). This would obviously violate the well known forms of risk averse utility functions.

## Conclusion

We may sum up our discussion as follows :

a. Much of the confusion surrounding the valuation of negative cash flows stems from the fact that a prospect with net negative cash flows is not tradeable in the market. Thus a symmetric transference of mathematical logic of portfolio theory *from* positive net cash flows to negative net cash flows is meaningless.

b. Value additivity principle holds only when the prospects being added are not synergistically related. Since in a given project the cash inflows and outflows are not independent of each other, valuing the project using the *value* additivity principle can be misleading.

c. Hence a project must be valued by *discounting* its net cash flow alone, whether by using an appropriate RADR or the CE framework.

## Appendix I

Miles and Choi (1979) assume a firm A which has to make an uncertain payment of $\tilde{X}$ dollars to another firm B, at the end of the current period. If firm A uses $\alpha_A > 1$ as the CE factor to value this negative benefit, then the value of this cash outflow for A will be

$$V_A(\tilde{X}) = \frac{\alpha_A \bar{X}}{1 + R_F}$$

Where $R_F$ is the risk-free rate

$\bar{X}$ is the expected cash outflow.

For Firm B, however, the CE factor ($\alpha_B$) should be less than one, so that

$$V_B(\tilde{X}) = \frac{\alpha_B \bar{X}}{1 + R_F}$$

Since $\alpha_A > \alpha_B$, we have $V_A(\tilde{X}) > V_B(\tilde{X})$.

Now, according to Miles and Choi, a rational investor can offer to firm A to make the payment of $\tilde{X}$ to firm B for a price of $V_A(\tilde{X})$. Simultaneously, he can offer firm B, $V_B(\tilde{X})$ in exchange for the promised payment of $\tilde{X}$. The market

participants will compete for the arbitrage profit $V_A(\tilde{x}) - V_B(\tilde{x})$ till $V_A(\tilde{x})$ or $\alpha_A = \alpha_B = 1$. This is counter-intuitive!

The origin of this counter-intuitive result is easy to follow. The CE co-efficient assumed are for evaluating the negative and positive cash flows in themselves, without regard to the overall impact of these cash flows in their respective net cash flows. The inconsistency resulting from assigning CE to uncertain inflows and outflows independently has been highlighted in our example.

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