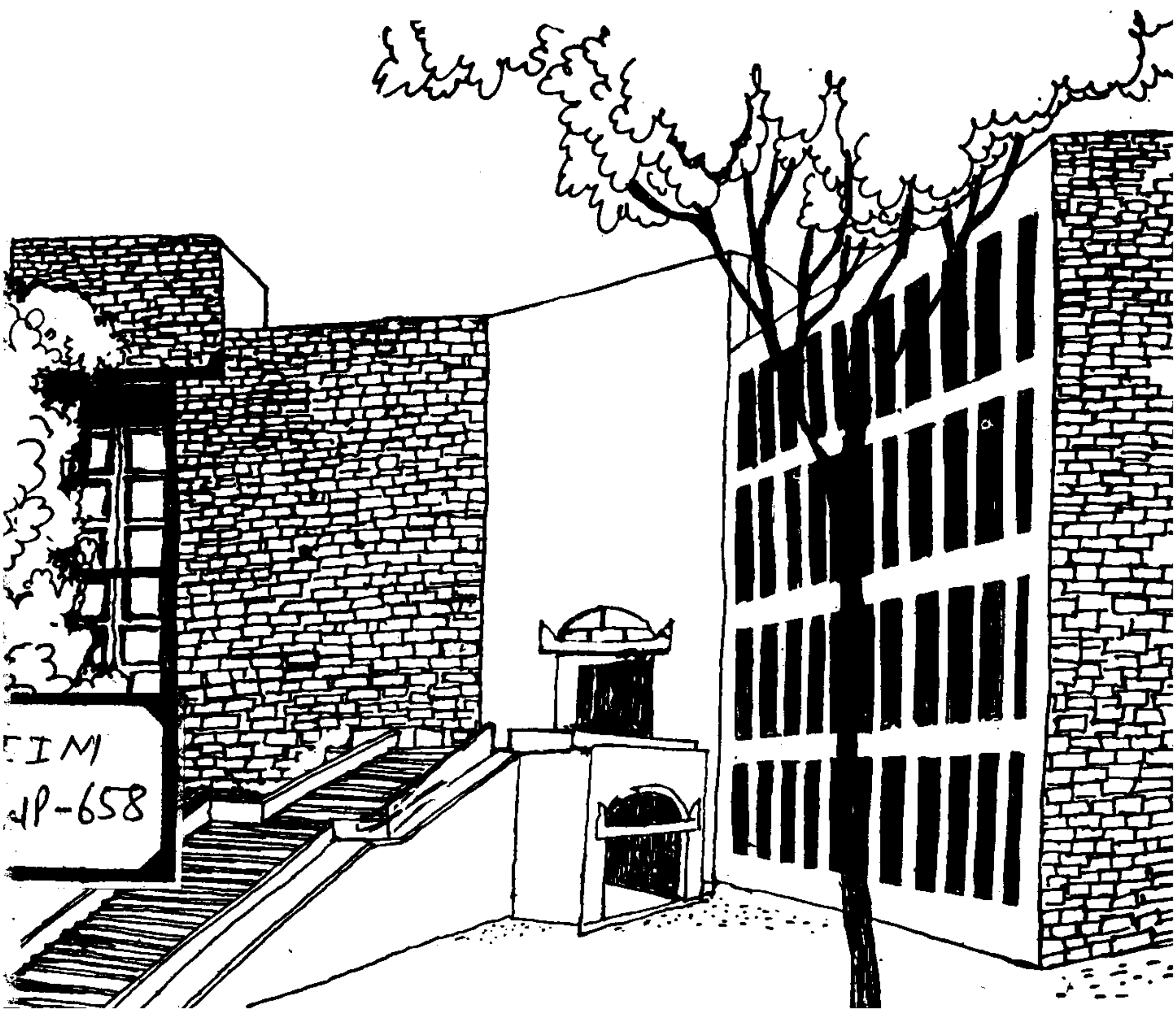
विद्याविनियोगाद्विकासः

IIM
AHMEDABAD

W.P. 658

Working Paper



INVESTIGATION OF DECISION CRITERIA
FOR INVESTMENT IN RISKY ASSETS

By

S.K. Barua
&
G.Srinivasan

W P No. 658
January 1987

The main objective of the working paper series of the IIMA is to help faculty members to test out their research findings at the pre-publication stage.

INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD-380015
INDIA

INVESTIGATION OF DECISION CRITERIA FOR INVESTMENT IN RISKY ASSETS

This paper examines the empirical validity of stochastic dominance rules and the mean-variance framework by analysing data generated through an experiment on individual investment decisions under uncertainty . The analyses indicated that none of the two approaches provided adequate explanation for the observed pattern of choice . An alternate framework , based on preference for skewness in addition to mean and variance , was examined . This framework provided a significantly better explanation compared to the two parameter framework . The preference for skewness was significant at higher levels of borrowing and at all levels of wealth .

INVESTIGATION OF DECISION CRITERIA
FOR INVESTMENT IN RISKY ASSETS

INTRODUCTION

The pioneering work on individual investment decision making was done by Markowitz [11,12] , who suggested a method of efficient allocation of funds to a set of risky assets using a mean-variance framework. Subsequently , others [15,10,13] developed a market equilibrium model for a set of risky assets and one riskless asset Mossin [13] showed that the results obtained by using mean-variance approach are identical to those obtained under the assumption that investors use quadratic utility function for deciding the composition of their portfolios.

A more general aporoach to explain individual investment decisions under uncertainty would be through use of stochastic dominance rules [14,3-9] . Unlike the mean-variance framework,the derivation of stochastic dominance rules does not need any restrictive assumptions on the nature of distributions. It also needs very little information on investor preferences. However, the rules are difficult to apply in practice because, it is not always easy to arrive at the distribution of a mix of random variables, and also because inclusion of a riskless asset does not yield the neat results obtained under the mean -variance framework.

There is a lack of empirical evidence on whether the investor behaviour conforms to the frameworks mentioned above. While some experiments have been conducted to validate the mean-variance framework , no experimental support for conformity with stochastic dominance rules is available.

An experimental investigation of mean-variance framework by Gordon et. al. [2] was conducted on a group of students to conjecture about their utility function by use of an investment game. Each student , starting with certain initial endowment, made a series of single period consumption and investment decisions. The data generated from the experiment were then examined to find out whether their investment decisions corresponded to some well known forms (quadratic , logarithmic or power) of utility functions. The results suggested that the data did not support the hypothesis that investors use any of the above mentioned utility functions.

In Cooley's experiment [1] on the determinants of the risk perception of investors , each investor was required to rank a set of return distributions in order of preference. These distributions had different second, third and fourth moments ,while the first moment was the same for all distributions. Analysing the preference data thus generated, he concluded that all the higher moments influenced the risk perception of investors. Thus , the assumption under the mean-variance framework that risk is completely defined by the variance of return of an asset may not be entirely correct.

These two experiments threw considerable light on the actual behaviour of investors. The experiments , however , suffered from certain limitations. In the experiment used by Gordon,the distributions of return on risky assets were symmetrical and hence the impact of the third moment on the process of choice could not be investigated. By assuming zero interest rates for both lending and borrowing they introduced an avoidable 'unreality' in the experiment. Cooley's experiment suffered from the fact that the investors were not required to form a portfolio; this would introduce aberrations in the data generated. The facility to form a portfolio of riskless and risky assets gives an unlimited flexibility to an investor to achieve any desired return-risk combination. The response without this facility could be very different from what it would be otherwise.

The experiment described in this paper attempted to overcome the drawbacks mentioned above by using asymmetric distributions of return on risky assets and also permitting the inclusion of riskless asset in the portfolio. Through the experiment, the following issues were investigated :

a. Whether the choice of risky asset for a portfolio was according to stochastic dominance criteria developed by Levy and Kroll [9]

b. Whether such choice was influenced by moments higher than the second moment of return distributions , and if so

c. Whether wealth level and the scale of investment (the ratio between the amount invested in risky asset and the amount available) affected the criteria for choice.

THE EXPERIMENT

The experiment was administered to a set of fifteen post-graduate students in business management who had a fair exposure to portfolio theory. Each participant in the experiment was given an initial wealth and a constant periodic income received at the beginning of a period . He could augment his resources (subject to some conditions described later) through borrowing. In each period he had to decide on the following :

a. Outlay on consumption

b. Outlay on investment

c. Amount to be borrowed

It is obvious that in every period the sum of the outlays on consumption and investment would equal the sum of initial wealth, the periodic income and the borrowing . Starvation and bankruptcy were not allowed and the participants had to observe the following constraints on consumption and borrowing.

a. They had to maintain a minimum consumption level of Rs.1000 in each period.

b. They could invest in no more than one investment opportunity out of six alternatives (described in Table 1) available each period. They could also lend funds at the rate of 5% .

c. They were allowed to borrow at a rate of 10% subject to a ceiling determined by the amount which could be repaid even if the outcome of the investment opportunity chosen turned out to be adverse .

```
+---------------+
|   Table  1    |
+---------------+
```

The participants did not know in advance the number of periods for which the experiment was to be conducted . During the experiment they maintained two distinct records : (i) a main record giving a continuous account of their decisions and outcomes , and (ii) a separate record of their decisions for each period which was collected from them before announcing the outcome of the investment opportunities for the period . The outcomes of these opportunities for each period were generated according to the probability distributions specified for the opportunities. Based on the outcomes, they computed their final wealth, net of loan and interest payable on the loan. This, together with the periodic income, became the starting resource for the next period. The data provided on the separate record were used to verify the computations done on the main record . The experiment was conducted for fifteen periods .

STOCHASTIC DOMINANCE RULE

Levy and Kroll [9] developed a stochastic dominance rule, denoted by SSDR, for portfolio decisions with inclusion of a riskless asset. The rule , used to establish the dominance of a distribution F over another distribution G , is summarized below .

Let F and G be the cumulative distributions of two options with quantiles $Q_F(p)$ and $Q_G(p)$, respectively.

Let
$$Z(p) = \frac{\int_0^p Q_G(t)dt - rp}{\int_0^p Q_F(t)dt - rp}$$

$$X = \underset{0 \leq p < p_0}{\text{Minimum}} \quad Z(p)$$

$$Y = \underset{p_0 < p \leq 1}{\text{Maximum}} \quad Z(p)$$

where r is the riskless interest rate and

p_0 is the value which solves the following equation :

$$rp_0 = \int_0^{p_0} Q_F(t)\, dt$$

Then F dominates G, if and only if : $X \geq Y$

This rule was used to identify the dominant opportunities from the six opportunities available in the experiment . Opportunity six was an unfair gamble , and hence was clearly dominated by the other opportunities .The necessary pairwise comparisons for providing the entire picture on dominance are presented in Table 2 .

Table 2

The results displayed in Table 2 clearly imply that under SSDR , the choice should be restricted only to opportunities 2 , 3 and 4 . The observed pattern of choice is contained in Table 3 .

+-----------------+
| Table 3 |
+-----------------+

Opportunities 2 , 3 and 4 were chosen on 185 occasions . This appeared to support the inference that SSDR might be able to explain the choice of investors . However , on closer examination it was necessary to modify this inference. A comparison of opportunities 2 and 3 revealed that 2 dominated 3 if the riskless rate was 1.10 . Thus,if an individual borrowed to invest in a risky asset , then he would prefer 2 to 3 under SSDR . The data revealed that of the 98 cases when opportunity 3 was chosen , SSDR was violated in 57 cases . Thus , the number of choices that conformed strictly to SSDR were only 128.This was about 57% of the total number of observations , not high enough to support that investors' choices were according to SSDR .

MEAN - VARIANCE FRAMEWORK

The next question was whether the pattern of choices could be explained by using the mean-variance framework . The mean and the variance of the return distributions are presented in Table 4 .

+-----------------+
| Table 4 |
+-----------------+

The return on investment opportunities 1 , 3 and 5 had identical expected values and the variance associated with the fifth opportunity was the lowest. Thus, opportunity 5 dominated 1 and 3 under the mean-variance framework . However, as opportunity 2 had a higher expected return and a lower variance than 5, it dominated opportunities 1, 3 and 5. Opportunity 6, being an unfair gamble, was dominated by all the others. The choice set under the mean-variance framework thus reduced to opportunities 2 and 4.

An expected return of 1.2 or less was obtainable by investing a part of the resources in either of the two opportunities and lending the remaining part. The proportion of lending needed to achieve the same expected return can be obtained from the following expression.

$$E(R_4) - a_4 [E(R_4) - 1.05] = E(R_2) - a_2 [E(R_2) - 1.05] \quad \ldots (1)$$

Where R_i is the return on opportunity i

a_i is the proportion of lending in a portfolio with opportunity

Substituting the expected returns, expression (1) reduced to

$$a_2 = 4/3\ a_4 - 1/3 \quad \ldots (2$$

To achieve a return of 1.2 or less, a_2 should be positive which implied that $a_4 > 0.25$.

The variance of return on the two portfolios would be :

$$V_4 = (1 - a_4)^2 v_4 \quad \ldots (3)$$

$$V_2 = (1 - a_2)^2 v_2 \quad \ldots (4)$$

where V_i is the variance of return on portfolio with asset i

v_i is the variance of return on asset i

Substituting the values of variances of the two opportunities the portfolio variances would be :

$$V_4 = (1 - a_4)^2 \times .0845 \qquad \ldots \quad \ldots \quad (5)$$

$$V_2 = (1 - a_2)^2 \times .045 \qquad \ldots \quad \ldots \quad (6)$$

To maintain identical expected returns (2) should hold ; then for portfolios with the same expected returns ,

$$V_2 = [4/3 (1 - a_4)]^2 \times .045 \qquad \ldots \quad \ldots \quad (7)$$

Thus a comparison of (5) and (7) showed that for expected return of 1.2 or less opportunity 2 dominated 4 , as for a given expected return , portfolio with 2 had a lower variance . In a similar manner , it can be shown that for expected returns of 1.25 or higher ,which can be achieved through borrowing , opportunity 4 dominated 2 .

An expected return above 1.20 but below 1.25 can be achieved either by borrowing and investing in opportunity 2 or by lending and investing in 4 . To achieve the same expected return the following relationship should hold true.

$$E(R_4) - a_4 [E(R_4) - 1.05] = (1 + b_2)E(R_2) - 1.1 b_2 \qquad \ldots \quad (8)$$

Where b_2 is the proportion of wealth borrowed to augment resources for investing in opportunity 2 .

Substituting the expected returns, expression (8) reduced to

$$a_4 = 0.25 - 0.5 b_2 \qquad \ldots \quad \ldots \quad (9$$

The variance of return on the two portfolios would be,

$$V_2 = (1 + b_2)^2 \times .045 \qquad \text{... ... (10)}$$

$$V_4 = (1 - a_4)^2 \times .0845 \qquad \text{... ... (11)}$$

Substituting the value of a_4 in expression (11),

$$V_4 = (0.75 + 0.5\ b_2)^2 \times 0.0845 \qquad \text{... ... (12)}$$

A comparison of expressions (10) and (12) indicated that the dominance of any one portfolio over the other depended on the value of b_2, needed to achieve the specified expected return . It can be easily shown that for expected return of 1.2088 or less, portfolio with opportunity 2 dominated portfolio with opportunity 4 . Beyond an expected return of 1.2088 , opportunity 4 was superior .

In summary, under the mean- variance framework, the choice should be restricted to opportunities 2 and 4 . Upto an expected return of 1.2088, opportunity 2 should be chosen to form a portfolio while to achieve a return beyond this value, 4 should be chosen. That is, if 2 is chosen , the scale of investment should be below 1.088, while if 4 is chosen, it should not be below 0.794 .

It can be observed from Table 3 that there were 138 cases when opportunities other than 2 and 4 were chosen . These were a clear violation of the mean-variance framework . Further, when the cases when 2 and 4 were chosen , were examined in relation to the scale of investment, another 31 choices of the remaining 87, violated the mean-variance framework . The final score in favour of the framework was only 56 out of the total of 225 . Thus , the mean-variance framework was even less adequate compared to the SSDR in explaining the pattern of choice .

SEARCH FOR AN ALTERNATE CRITERION

The pattern of choice clearly showed a marked preference for opportunity 3 . An examination of the skewness of return distributions , presented in Table 5 , revealed that opportunity 3 was the only opportunity with a positive skewness . Could preference for skewness explain the pattern of choice ?

Table 5

If the mean-variance framework is modified to mean-variance-skewness framework ,the following statements can be made about the process of choice :

a. If two portfolios have identical mean and variance , then the one with a higher skewness would be preferred .

b. If two portfolios have identical mean and skewness , then the one with a lower variance would be preferred .

c. If two portfolios have identical variance and skewness , then the one with a higher mean would be preferred .

Under this framework , opportunity 2 dominated 5 and opportunity 3 dominated 1 . The choice set reduced to opportunities 2 , 3 and 4 .

Between opportunities 2 and 4 , it has already been established that choice of opportunity 2 for an expected return higher than 1.2088 violated the mean-variance framework . However, if skewness is also a factor affecting preferences , then the choice of opportunity 2 for returns higher than 1.2088 could be justified . For a given expected return, the value of skewness for a portfolio with opportunity 2 would always be higher than for a portfolio with opportunity 4 . This implied that the choice of opportunity 4 for a return of 1.2088 or below (that is a scale of investment of 0.794 or below) would be a violation of the mean-variance-skewness framework.

On the basis of the above discussions, the 169 choices violating the mean-variance framework were re-examined to see whether preference for positive skewness could expalin the choices. It was observed that 122 out of the 169 choices were in conformity with this new criterion. This clearly established that skewness was a dominant factor in determining the choice in a large number of cases. To gain further insight into the skewness seeking behaviour of individuals, the impact of wealth level and the scale of investment on the preference for skewness were examined .

The 122 observations which indicated preference for skewness were classified according to wealth level and scale of investment. This two way classification is presented in Table 6 .

Table 6

The figure in the top right corner of each cell is the total number of observations in the cell and the other number is the number of observations consistent with skewness preference . It is quite clear from the table that there was a marked preference for skewness at higher scales of investment. The impact of wealth and scale of investment were examined for statistical significance by testing the following hypotheses.

a. Null Hypothesis (H_0) : Skewness-preference is independent of wealth

Wealth W (Rs. '000)	No. consistent with skewness-preference	No. not consistent with skewness-preference	Total
$0 < W \le 50$	41	43	84
$50 < W \le 100$	28	16	44
$100 < W \le 200$	29	23	52
$W > 200$	24	21	45
Total	122	103	225

The computed value of $\chi^2 = 2.63$

The hypothesis could not be rejected at a significance level of 5% .

Null Hypothesis (H_0) : Skewness-preference is independent of scale of investment

Scale S	No. consistent with skewness-preference	No. not consistent with skewness-preference	Total
$0 < S \leq 1$	42	61	103
$50 < S \leq 2$	36	35	71
$100 < S \leq 3$	16	5	21
$S > 3$	28	2	30
Total	122	103	225

The observed value of χ^2 = 30.41

The hypothesis is rejected at a significance level of 5% .

SUMMARY

The analyses of data clearly demonstrated that the preference pattern of individuals while investing in risky assets cannot be explained adequately either by stochastic dominance rules or by the less general mean-variance framework . An extension of the mean-variance framework , with inclusion of skewness provided a significantly better explanation as compared to the two parameter framework . Investors showed a distinct preference for positive skewness . Futher analyses revealed that this preference increases with increase in the scale of investment . Since higher scales of investment imply larger borrowing , preference for skewness possibly results from an attempt to minimize the maximum loss . It reduces the chance of bankruptcy . This preference was exhibited at all levels of wealth .

The inferences drawn in the paper reinforce the view that theoretical frameworks available are still quite inadequate in explaining the investment decisions of individuals under uncertainty. There could be two reasons for this observed inadequacy : (i) the dominance of one option over another cannot be easily noticed by investors , unaided by expert advice or computing facilities , (ii) there are other facets in the decision making process of individuals, which are not captured by rules based on sound economic reasoning . While the former can be tackled by developing suitable decision support systems , more experiments under varying conditions are needed to understand the latter .

TABLE 1

Investment Opportunities

ppurtunity No.	Outcomes			
	Return(r_1)	Probability(p_1)	Return(r_2)	Probability(p_2)
1	1.50	0.50	0.80	0.50
2	1.35	0.67	0.90	0.33
3	1.72	0.25	0.96	0.75
4	1.38	0.83	0.60	0.17
5	1.28	0.75	0.76	0.25
6	90.00	0.01	0.00	0.99

TABLE 2

Exploration of Stochastic Dominance

F	G	r	p_0	X	Y	Remarks
2	1	1.05	1/2	5/3	2/5	2 dominates 1
		1.10	3/5	3/2	1/2	2 dominates 1
2	3	1.05	1/2	3/5	2/3	2 doesnot dominate 3
		1.10	3/5	7/10	1/2	2 dominates 3
2	4	1.05	1/2	$\to -\infty$	$\to +\infty$	2 doesnot dominate 4
		1.10	3/5	$\to -\infty$	$\to +\infty$	2 doesnot dominate 4
2	5	1.05	1/2	16/15	2/3	2 dominates 5
		1.10	3/5	21/20	1/2	2 dominates 5
3	2	1.05	57/67	$\to -\infty$	$\to +\infty$	3 doesnot dominate 2
		1.10	57/62	$\to -\infty$	$\to +\infty$	3 doesnot dominate 2
3	4	1.05	57/67	$\to -\infty$	$\to +\infty$	3 doesnot dominate 4
		1.10	57/62	$\to -\infty$	$\to +\infty$	3 doesnot dominate 4
4	2	1.05	13/33	1/3	3/4	4 doesnot dominate 2
		1.10	13/28	2/5	2/3	4 doesnot dominate 2
4	3	1.05	13/33	1/5	1/2	4 doesnot dominate 3
		1.10	13/28	7/25	1/3	4 doesnot dominate 3

TABLE 3

Pattern of Choice

Opportunity	Freqency of Choice
1	9
2	50
3	98
4	37
5	17
6	14

TABLE 4

Mean and Variance of Distributions

Opportunity	Mean	Variance
1	1.15	0.1225
2	1.20	0.0450
3	1.15	0.1083
4	1.25	0.0845
5	1.15	0.0507
6	0.90	79.3881

TABLE 5

Skewness of Distributions

Opportunity	Skewness
1	0.0000
2	-0.0068
3	0.0412
4	-0.0439
5	-0.0132

TABLE 6

Preference for Skewness

Wealth (W)	Scale of Investment (S) $0 < S \leq 1$	$1 < S \leq 2$	$2 < S \leq 3$	$S > 3$
$0 < W \leq 50$	7 [31]	11 [24]	6 [10]	17 [19]
$50 < W \leq 100$	10 [18]	11 [19]	2 [2]	5 [5]
$100 < W \leq 200$	13 [28]	8 [15]	3 [4]	5 [5]
$W > 200$	12 [26]	6 [13]	5 [5]	1 [1]

REFERENCES

1. COOLEY PL (1976) A multidimensional analysis of institutional investor perception of risk. J. Finance. 32(1), 67-78.

2. GORDON MJ et. al. (1972) Experimental evidence on alternative portfolio decision rules. Am. Econ. Rev. 62(1), 107-118.

3. HADAR J & RUSSEL WR (1969) Rules of ordering uncertain prospects. Am. Econ. Rev. 59(1), 25-34.

4. ----- & ----- (1971) Stochastic dominance and diversification. J. Econ. Theory. 3(3), 283-305.

5. HANOCH G & LEVY H (1969) The efficiency analysis of choices involving risk. Rev. Econ. Studies. 36(3), 335-346.

6. ----- & ----- (1970) Efficient portfolio selection with quadratic and cubic utility. J. Business. 43(2), 181-189.

7. LEVY H (1973) The demand for assets under conditions of risk. J. Finance. 28(1), 79-96.

8. LEVY H & KROLL Y (1976) Stochastic dominance with riskless assests. J. Financial and Quantitative Anal. 11(5), 743-777.

9. ----- & ----- (1978) Ordering uncertain options with borrowing and lending. J. Finance. 33(2), 553-574.

10. LINTNER J (1965) Security prices risk and maximal gains from diversification. J. Finance. 20(4), 587-616.

11. MARKOWITZ H (1952) Portfolio selection. J. Finance. 7(1), 77-91.

12. ----- (1959) Portfolio selection : efficient diversification of investment. Cowles Foundation Monograph. No.16, New York.

13. MOSSIN J (1966) Equilibrium in capital asset market. Econometrica. 34(4), 768-783.

14. QUIRK JD & SAPOSNIK R (1962) Admissibility and measurable utility functions. Rev. Econ. Studies. 29(1), 140-146.

15. SHARP WF (1964) Capital asset prices : a theory of market equilibrium under conditions of risk. J. Finance. 19(3), 425-442.