

CORRELATED EQUILIBRIA UNDER BOUNDED AND  
UNBOUNDED RATIONALITY

By

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## ABSTRACT

In this paper we establish an isomorphism between the set of correlated equilibria of a game on the one hand and the set of ordered pairs of coordination mechanisms and equilibrium decision rules for the same game on the other, in the case of bounded and unbounded rationality. The paper develops a systematic theory establishing an injection from the set of ordered pairs of coordination mechanisms and equilibrium decision rules to the set of correlated equilibria. The converses follow easily from the methods of the proofs. As an intermediate step, we introduce the concept of a conditionally correlated equilibrium under bounded rationality.

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## CORRELATED EQUILIBRIA UNDER BOUNDED AND UNBOUNDED RATIONALITY

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### ABSTRACT

In this paper we establish an isomorphism between the set of correlated equilibria of a game on the one hand and the set of ordered pairs of coordination mechanisms and equilibrium decision rules for the same game on the other, in the case of bounded and unbounded rationality. The paper develops a systematic theory establishing an injection from the set of ordered pairs of coordination mechanisms and equilibrium decision rules to the set of correlated equilibria. The converses follow easily from the methods of the proofs. As an intermediate step, we introduce the concept of a conditionally correlated equilibrium under bounded rationality.

### 1. INTRODUCTION

A kind of incentive constraint that limits people's ability to reach mutually beneficial agreements in social and economic affairs is when a person controls some private decision variable that others cannot control or monitor and so he cannot be directed to choose any particular decision or action unless he is given the incentive to do so. As observed by Myerson [1986], a social contract or coordination system may not be feasible if it gives people incentives to cheat in their actions. Such is the problem of moral hazard. In this paper we propose to study conditions under it is possible to expect people to choose a particular decision i.e. to be obedient, without violating incentive constraints.

The basic object of analysis here is a game with complete information. In the notation here, we suppose that there are  $n$  players in the game, and that they are numbered  $1, 2, \dots, n$ . For each  $i$  in  $\{1, 2, \dots, n\}$ , we let  $D_i$  denote the set of possible actions or strategic decisions available to player  $i$  in the game. Let  $D$  denote the possible combinations of decisions available to the  $n$  players, so that

$$D = D_1 \times \dots \times D_n \quad (1)$$

We let  $u_i(d)$  denote utility pay off (measured in some von Neuman-Morgenstern utility scale) that player  $i$  would get if  $d = (d_1, \dots, d_n)$  were the combination of decisions chosen by the  $n$ -players. Thus,

$$\Gamma = (D_1, \dots, D_n, u_1, \dots, u_n) \quad (2)$$

is a game with complete information if for each  $i$ ,  $D_i$  is a nonempty set. Following Aumann [1976], we assume that  $\Gamma$  is common knowledge. To simplify our analysis, we will henceforth assume that the decision sets  $D_i$  are all finite sets.

Let us suppose that the players communicate with the help of a mediator, who recommends a strategic action to each player. The mediator's recommendation which may be deterministic or random is summarized by a co-ordination mechanism,  $\mu: D \rightarrow [0, 1]$  which is just a probability distribution over  $D$ , satisfying

$$\sum_{e \in D} \mu(e) = 1 \text{ and } \mu(d) \geq 0, \quad \forall d \in D. \quad (3)$$

We must allow, each player to disobey the mediator's recommendations. Hence, each selection of an action  $d_i$  in  $D_i$  can ultimately be controlled only by player  $i$ . Thus, the coordination mechanism  $\mu$  induces a game  $\hat{\Gamma}_\mu$  in which each player must select his plan for choosing an action in  $D_i$  as a function of the mediator's recommendation. Formally,  $\hat{\Gamma}_\mu$  is a game with complete information, of the form

$$\hat{\Gamma}_\mu = (\hat{D}_1, \dots, \hat{D}_n, \hat{u}_1, \dots, \hat{u}_n) \quad (4)$$

where

$$\hat{D}_i = \{\delta_i / \delta_i : D_i \rightarrow D_i\}, \text{ and}$$

$$\hat{u}_i(\delta_1, \dots, \delta_n) = \sum_{d \in D} \mu(d) u_i(\delta_1(d_1), \dots, \delta_n(d_n)) \quad (5)$$

A strategy  $\delta_i$  in  $\hat{D}_i$  represents a plan by player  $i$  to choose his action in  $D_i$  as a function of the mediator's recommendation according to  $\delta_i$ , so that he would do  $\delta_i(d_i)$  if the mediator recommended  $d_i$ . We assume that each player communicates with the mediator separately and confidentially, so that player  $i$ 's action cannot depend on the recommendations to the other players.

An equilibrium for  $\hat{\Gamma}_\mu$  is an  $n$ -tuple  $(\delta_1^*, \dots, \delta_n^*) \in \hat{D}_1 \times \dots \times \hat{D}_n$  such that for all  $i \in \{1, \dots, n\}$  and  $\forall \delta_i \in \hat{D}_i$ ,

$$\hat{u}_i(\delta_1^*, \dots, \delta_{i-1}^*, \delta_i, \delta_{i+1}^*, \dots, \delta_n^*) \geq \hat{u}_i(\delta_1^*, \dots, \delta_{i-1}^*, \delta_i^*, \delta_{i+1}^*, \dots, \delta_n^*). \quad (6)$$

Such equilibria are self-enforceable.

Since obedience is a virtue, following Aumann [1974, 1987], we say that  $\mu$  is a correlated equilibrium for  $\Gamma$  if there exists  $(\delta_1^*, \dots, \delta_n^*)$  which is an equilibrium for the associated game  $\hat{\Gamma}_\mu$  and  $\forall i \in \{1, \dots, n\}$ ,  $\delta_i^*(d_i) = d_i \forall d_i \in D_i$ .

This is the condition of incentive compatibility. In this paper we intend to study conditions which guarantee incentive compatibility under conditions of bounded and unbounded rationality. Under unbounded rationality an agent behaves like an expected utility maximizer. This is the traditional approach to game theory. Faced with bounded rationality and limited computing facilities, a player may take the help of a statistician who assists him in estimating the unknown parameters of the model before he arrives at a decision. The statistician is assumed to be Bayesian in the sense that he draws his inferences on the basis of the posterior distribution of the unknowns conditional on the current observations. Hence the player must communicate to the statistician

the entire posterior distribution of his beliefs about the unknowns conditional on the current data. The statistician renders his services free of charge. The beliefs that the players form may be subjective; however in most of our analysis the basis for such beliefs are objective and based on the coordination mechanism used by the mediator.

2. THE SITUATION UNDER UNBOUNDED RATIONALITY

In this section we propose some results which characterize correlated equilibria under unbounded rationality.

Lemma 1.  $\delta^*$  is an equilibrium for  $\hat{\Gamma}_\mu$  if and only if  $\forall i \in \{1, \dots, n\}$  and

$$\forall d_i \in D_i,$$

$$\sum_{d_{-i} \in D_{-i}} \mu(d) u_i(\delta_1^*(d_1), \dots, \delta_i^*(d_i), \dots, \delta_n^*(d_n)) > \sum_{d_{-i} \in D_{-i}} \mu(d) u_i(\delta_1^*(d_1), \dots, \delta_{i-1}^*(d_{i-1}), e_i, \delta_{i+1}^*(d_{i+1}), \dots, \delta_n^*(d_n))$$

$$\forall e_i \in D_i. \text{ Here, } D_{-i} = D_1 \times \dots \times D_{i-1} \times D_{i+1} \times \dots \times D_n.$$

Proof: Let  $\delta^*$  satisfy the above condition and let  $\delta_i : D_i \rightarrow D_i$ , be any function.

Then  $\forall d_i \in D_i$ , if we put  $\delta_i(d_i)$  in place of  $e_i$  and sum over  $d_{-i}$ , we get that

$\delta^*$  is an equilibrium for  $\hat{\Gamma}_\mu$ .

Conversely, suppose that  $\delta^*$  is an equilibrium for  $\hat{\Gamma}_\mu$  and towards a contradiction assume that for some  $i \in \{1, \dots, n\}$  and for some  $d_i \in D_i$ ,

$$\sum_{d_{-i} \in D_{-i}} \mu(d) u_i(\delta^*(d)) < \sum_{d_{-i} \in D_{-i}} \mu(d) u_i(\delta_1^*(d_1), \dots, \delta_{i-1}^*(d_{i-1}), e_i, \delta_{i+1}^*(d_{i+1}), \dots, \delta_n^*(d_n))$$

for some  $e_i \in D_i$ .

Define  $\delta_i : D_i \rightarrow D_i$  as follows:

$$\begin{aligned} \delta_i(d'_i) &= \delta_i^*(d'_i) \quad \forall d'_i \neq d_i \\ &= e_i \quad \forall d'_i = d_i. \end{aligned}$$

Therefore,

$$\sum_{d'_i \in D_i} \sum_{d_{-i} \in D_{-i}} \mu(d_{-i}, d'_i) u_i(\delta_{-i}^*(d_{-i}), \delta_i^*(d'_i))$$

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$$< \sum_{d'_i \in D_i} \sum_{d_{-i} \in D_{-i}} \mu(d_{-i}, d'_i) u_i(\delta_{-i}^*(d_{-i}), \delta_i(d'_i))$$

where  $d_{-i} \in D_{-i}$  and  $\delta_{-i}^* = (\delta_1^*, \dots, \delta_{i-1}^*, \delta_{i+1}^*, \dots, \delta_n^*)$ . This contradicts that  $\delta^*$  is an equilibrium. Q.E.D.

We now state and prove a main theorem of this paper.

Theorem 1. If  $\delta$  is an equilibrium for  $\hat{\Gamma}_\mu$  where  $\mu$  is a coordination mechanism, then there exists another coordination mechanism  $\mu^*$  which is a correlated equilibrium for  $\Gamma$ .

Proof: Let  $\delta$  be an equilibrium for  $\hat{\Gamma}_\mu$ . By Lemma 1,

$$\sum_{d_{-i} \in D_{-i}} \mu(d) u_i(\delta(d)) \geq \sum_{d_{-i} \in D_{-i}} \mu(d) u_i(\delta_{-i}(d_{-i}), e_i)$$

$$\forall i \in \{1, \dots, n\}, \forall d_i \in D_i, \forall e_i \in D_i.$$

Define  $\mu^* : D \rightarrow [0, 1]$  as follows:

$$\mu^*(d) = \sum_{d' \in \hat{c}^{-1}(d)} \mu(d').$$

where if  $\hat{c}^{-1}(d) = \emptyset$ ,  $\mu^*(d) = 0$ .

$$\begin{aligned} \text{Hence } \sum_{d_{-i} \in D_{-i}} \mu^*(d) u_i(d) &= \sum_{d_{-i} \in D_{-i}} \sum_{d' \in \hat{c}^{-1}(d_{-i})} \mu(d') u_i(d) = \sum_{d_{-i} \in D_{-i}} \sum_{d' \in \hat{c}^{-1}(d_{-i})} \mu(d') u_i(\hat{c}(d')) \\ &= \sum_{d' \in D_{-i}} \mu(d') u_i(\hat{c}(d')) \end{aligned}$$

where  $d'_i \in \hat{c}^{-1}(d_{-i})$ .

$$\text{Similarly, } \sum_{d_{-i} \in D_{-i}} \mu^*(d) u_i(d_{-i}, e_i) = \sum_{d' \in D_{-i}} \mu(d') u_i(\hat{c}_{-i}(d'_{-i}), e_i)$$

Hence from the inequality mentioned in the beginning of the proof it follows that

$$\sum_{d_{-i} \in D_{-i}} \mu^*(d) u_i(d_{-i}, e_i) \leq \sum_{d_{-i} \in D_{-i}} \mu^*(d) u_i(d) \quad \forall i \in \{1, \dots, n\}, \forall d_i \in D_i, \forall e_i \in D_i.$$

Hence  $\mu^*$  is a correlated equilibrium for  $\Gamma$ . Q.E.D.

### 3. THE SITUATION UNDER BOUNDED RATIONALITY

It is interesting to see what would happen if the players were subject to bounded rationality. Such behaviour in the context of arbitration games has been studied in Lahiri [1990]. In the present context we motivate further discussion by the following lemma.

Lemma 2. If  $\mu^*$  is a correlated equilibrium for  $\Gamma$ , then there exists function

$\mu_i^*(\cdot | d_i) : D_{-i} \rightarrow [0,1], \forall d_i \in D_i, \forall i \in \{1, \dots, n\}$  such that

$$(i) \quad \sum_{d_{-i} \in D_{-i}} \mu_i^*(d_{-i} | d_i) = 1, \mu_i^*(d_{-i} | d_i) \geq 0 \quad \forall d_{-i} \in D_{-i}, \forall d_i \in D_i.$$

$$(ii) \quad \forall i \in \{1, \dots, n\}, \quad \forall d_i \in D_i$$

$$\sum_{d_{-i} \in D_{-i}} \mu_i^*(d_{-i} | d_i) u_i(d) \geq \sum_{d_{-i} \in D_{-i}} \mu_i^*(d_{-i} | d_i) u_i(d_{-i}, e_i) \quad \forall e_i \in D_i.$$

Proof: Define  $\mu_i^*(d_{-i} | d_i) = \frac{\mu^*(d)}{\sum_{d_{-i} \in D_{-i}} \mu^*(d)}$ . These  $\mu_i^*$ 's are the required functions. Q.E.D.

We say that an n-tuple of functions  $(\mu_1^*, \dots, \mu_n^*)$  forms a Conditionally Correlated equilibrium for a game  $\Gamma$  if it satisfies conditions (i) and (ii) of Lemma 2. Condition (ii) of Lemma 2 displays on either side of the inequality player i's conditionally expected utility from using  $d_i$  and  $e_i$ , respectively, given that the mediator recommended  $d_i$ . This will form our point of departure for the subsequent analysis.

Suppose that the player i being recommended  $d_i$  by the mediator confronted a statistician with his posterior beliefs  $\mu_i(\cdot | d_i) : D_{-i} \rightarrow [0,1]$ . (It is possible that the player i is himself a statistician in which case he need not consult another statistician). The statistician would then use an estimator  $T_i^{\mu_i} : D_i \rightarrow D_{-i}$  to convey to player i an estimate of the actions that the players other than i would adopt. Equipped with this estimate  $T_i^{\mu_i}(d_i)$ , player i would now choose an action  $\delta_i(d_i) \in D_i$  such that  $u_i(\delta_i(d_i), T_i^{\mu_i}(d_i)) \geq u_i(e_i, T_i^{\mu_i}(d_i)) \quad \forall e_i \in D_i$ . This is the behaviour expected of player i under bounded rationality. We assume

that the family of estimators  $\{(T_1^{\mu_1}, \dots, T_n^{\mu_n})\}$  indexed by  $(\mu_1, \dots, \mu_n)$  is common knowledge. This family is denoted  $(T_1, \dots, T_n)$ . Let

$$\bar{\Gamma} = (D_1, \dots, D_n, u_1, \dots, u_n, T_1, \dots, T_n) \quad (7)$$

and

$$(\bar{\Gamma}, \mu_1, \dots, \mu_n) = (D_1, \dots, D_n, u_1, \dots, u_n, T_1, \dots, T_n, \mu_1, \dots, \mu_n) \quad (8)$$

An n-tuple of function  $\delta = (\delta_1, \dots, \delta_n)$  is said to be an equilibrium for  $(\bar{\Gamma}, \mu_1, \dots, \mu_n)$  under bounded rationality if  $\forall i \in \{1, \dots, n\}, \forall d_i \in D_i,$

$$u_i(\delta_i(d_i), \delta_{-i}(T_i^{\mu_i}(d))) \geq u_i(e_i, \delta_{-i}(T_i^{\mu_i}(d))) \quad \forall e_i \in D_i.$$

$(\mu_1^*, \dots, \mu_n^*)$  is said to be a conditionally correlated equilibrium under bounded rationality for  $\bar{\Gamma}$  if  $\delta = (\delta_1^*, \dots, \delta_n^*)$  is an equilibrium for  $(\bar{\Gamma}, \mu_1^*, \dots, \mu_n^*)$  under bounded rationality where  $\delta_i^*(d_i) \equiv d_i \quad \forall d_i \in D_i \quad \forall i \in \{1, \dots, n\}.$

The two definitions above are analogous to the definitions in section 2, and given our framework are quite self explanatory. Once again a conditionally correlated equilibrium under bounded rationality requires obedience at an equilibrium.

Our subsequent analysis will focus on the situation when  $(T_1, \dots, T_n)$  is a family of generalized maximum likelihood estimators (see Berger [1985]).

Assumption 1:  $\forall i \in \{1, \dots, n\}, \forall d_i \in D_i,$

$$T_i^{\mu_i}(d_i) = \arg \max_{d_{-i} \in D_{-i}} u_i(d_{-i} | d_i).$$

For reasons which are technical and without which our analysis would fail to proceed, we need to make the following assumption:

Assumption 2: If  $\delta = (\delta_1, \dots, \delta_n)$  is an equilibrium for  $(\bar{\Gamma}, \mu_1, \dots, \mu_n)$  under bounded rationality then  $\delta_i : D_i \rightarrow D_i$  is a one-to-one function for all  $i \in \{1, \dots, n\}.$

Equipped with these two assumptions we can establish the following theorem.

**Theorem 2.** If  $\delta$  is an equilibrium for  $(\bar{\Gamma}, \mu_1, \dots, \mu_n)$  under bounded rationality then there exists an n-tuple  $(\mu_1^*, \dots, \mu_n^*)$  which is a conditionally correlated equilibrium for  $\bar{\Gamma}$  under bounded rationality.

Proof: Let  $\delta$  be an equilibrium for  $(\bar{\Gamma}, \mu_1, \dots, \mu_n)$ .

Hence  $\forall i \in \{1, \dots, n\}$  and  $\forall d_i \in D_i$ ,

$$\mu_i(\delta_i(d_i), \delta_{-i}(T_i^{\mu_i}(d_i))) \geq u_i(e_i, \delta_{-i}(T_i^{\mu_i}(d_i))) \quad \forall e_i \in D_i.$$

Define,  $\mu_i^*(d_{-i} | d_i) = \mu_i(\delta_{-i}^{-1}(d_{-i}) | \delta_i^{-1}(d_i)) \quad \forall d_i \in D_i$  and  $\forall i \in \{1, \dots, n\}$ .

Therefore,

$$\begin{aligned} \mu_i^*(\delta_{-i}(T_i^{\mu_i}(\delta_i^{-1}(d_i))) | d_i) &= \mu_i(T_i^{\mu_i}(\delta_i^{-1}(d_i)) | \delta_i^{-1}(d_i)) \\ &\geq \mu_i(\delta_{-i}^{-1}(d_{-i}) | \delta_i^{-1}(d_i)) = \mu_i^*(d_{-i} | d_i) \quad \forall d_{-i} \in D_{-i}. \end{aligned}$$

Therefore

$$T_i^{\mu_i^*}(d_i) = \delta_{-i}(T_i^{\mu_i}(\delta_i^{-1}(d_i))) \quad \forall d_i \in D_i.$$

$$\begin{aligned} u_i(d_i, T_i^{\mu_i^*}(d_i)) &= u_i(d_i, \delta_{-i}(T_i^{\mu_i}(\delta_i^{-1}(d_i)))) \\ &= u_i(\delta_i(\delta_i^{-1}(d_i)), \delta_{-i}(T_i^{\mu_i}(\delta_i^{-1}(d_i)))) \\ &\geq u_i(e_i, \delta_{-i}(T_i^{\mu_i}(\delta_i^{-1}(d_i)))) \\ &= u_i(e_i, T_i^{\mu_i}(d_i)) \quad \forall e_i \in D_i, \quad \forall d_i \in D_i, \quad \forall i \in \{1, \dots, n\}. \end{aligned}$$

Hence  $(\mu_1^*, \dots, \mu_n^*)$  is a conditionally correlated equilibrium for  $\bar{\Gamma}$  under bounded rationality.

Q.E.D.

It is interesting to note that if  $(\mu_1, \dots, \mu_n)$  are consistent in the sense that there exists a function  $\mu : D \rightarrow [0, 1]$  such that  $\sum_{d \in D} \mu(d) = 1$  and  $\forall i \in \{1, \dots, n\}, \forall d_i \in D_i, \forall d_{-i} \in D_{-i}$ ,

$$\mu_i(d_{-i} | d_i) = \frac{\mu(d_i, d_{-i})}{\sum_{d'_{-i} \in D_{-i}} \mu(d_i, d'_{-i})},$$

then the conditionally correlated equilibrium under bounded rationality

$(\mu_1^*, \dots, \mu_n^*)$  obtained above is also consistent in the same sense, i.e. there exists a function  $\mu^* : D \rightarrow [0, 1]$  such that  $\sum_{d \in D} \mu^*(d) = 1$  and  $\forall i \in \{1, \dots, n\}, \forall d_i \in D_i, \forall d_{-i} \in D_{-i}$ ,

$$\mu_i^*(d_i, d_{-i}) = \frac{\mu^*(d_i, d_{-i})}{\sum_{d'_{-i} \in D_{-i}} \mu^*(d_i, d'_{-i})}$$

where  $\mu^*(d_i, d_{-i}) = \mu(\delta_i^{-1}(d_i), \delta_{-i}^{-1}(d_{-i})) \forall (d_i, d_{-i}) \in D$ . In keeping with accepted terminology such a  $\mu^*$  may be called a correlated equilibrium under bounded rationality. Hence under Assumptions 1 and 2, we may state the following corollary.

Corollary 1. If  $\delta$  is an equilibrium for  $(\bar{\Gamma}, \mu_1, \dots, \mu_n)$  under bounded rationality and if  $(\mu_1, \dots, \mu_n)$  are consistent in the above sense, then there exists a correlated equilibrium under bounded rationality,  $\mu^*$ , for  $\bar{\Gamma}$ .

Proof: Theorem 2 along with what has been mentioned above is sufficient to prove the corollary.

Q.E.D.

#### 4. Existence of Correlated Equilibria under bounded rationality:

In this section we show that under some conditions there exists a correlated equilibrium under bounded rationality, which is moreover the uniform distribution on the strategy space. The condition we have in mind is the following:

Assumption 3:  $\bar{\Gamma} = (u_1, \dots, u_n, D_1, \dots, D_n)$  satisfies the following condition:

$$\forall i \in \{1, \dots, n\}, \forall d_i \in D_i, \exists d_{-i} \in D_{-i} \text{ such that } u_i(d_i, d_{-i}) \geq u_i(e_i, d_{-i}) \forall e_i \in D_i$$

what assumption 3 says is that for each player every strategy qualifies as a best reply against some strategy combination of the other players. We may now state and prove the following theorem.

Theorem 3 : Under assumptions 1 and 3, there exists a correlated equilibrium under bounded rationality for  $\bar{\mu}$ , which moreover is the uniform distribution on  $D$ .

Proof : Define  $\mu^* : D \rightarrow [0,1]$  as follows :

$$\mu^*(d) = \frac{1}{|D|} \quad \forall d \in D$$

$$\text{Then } \mu_{-i}^*(d_{-i} | d_i) = \frac{1}{|D_{-i}|} \quad \forall d_{-i} \in D_{-i} \text{ and } d_i \in D_i$$

By assumption 3,  $\forall d_i \in D_i$ ,  $\exists$  a strategy which may be denoted  $\tau_i^*(d_i) \in D_{-i}$  such that

$$u_i(d_i, \tau_i^*(d_i)) \geq u_i(e_i, \tau_i^*(d_i)) \quad \forall e_i \in D_i$$

Clearly,  $\mu_{-i}^*(\tau_i^*(d_i) | d_i) \geq \mu_{-i}^*(d_{-i} | d_i) \quad \forall d_{-i} \in D_{-i}$

Hence  $\mu^*$  is a correlated equilibrium under bounded rationality.

u.f.v.

It is instructive to note that if Assumption 3 is violated then the uniform distribution ceases to qualify as a correlated equilibrium under bounded rationality, because then whatever strategy the other players must choose, there exists a "disobedient" strategy for some player which does better than at least one recommendation by the arbitrator.

## 5. CONCLUSION

In this paper we have established conditions for incentive compatibility under unbounded and bounded rationality. Our notion of incentive compatibility under unbounded rationality is standard. Our concept of human behaviour and incentive compatibility under bounded rationality deserves a second thought.

In so far as human behaviour goes, almost anything is possible, including what we consider to be an expression of human behaviour under bounded rationality. So the question that really confronts us, is, whether

such behaviour could give rise to a decision theory analogous to the decision theory that obtains under unbounded rationality?

A substantial body of conventional decision theory could be rewritten along the lines we suggest in our analysis of bounded rationality, provided suitable assumptions are made (as for instance our Assumption 2). This is expected since under bounded rationality we are merely using approximations suggested to us by statistical decision theory. Assumption 1 is justifiable on grounds that if any player unilaterally deviated from the behaviour recommended by this assumption, then he could land up with an outcome which gives him an utility which is less than or equal to the maximum attainable and that too under circumstances which are less probable than the predicted event. Hence as a characterization of human behaviour, what we have outlined in this paper is reasonable.

As far as the results in our paper are considered, what we have achieved are general statements concerning 'obedient' behaviour, which turns out to be useful in any real world situations. A major portion of Section 2 is an adaptation to the present context of general results established in Myerson (1982). They have been provided here in detail to motivate the ensuing discussion.

## REFERENCES

- (1) Aumann, R.J. (1974): "Subjectivity and Correlation in Randomized Strategies". *Journal of Mathematical Economics*, 1 : 67-96.
- (2) Aumann, R.J. (1976): "Agreeing to Disagree". *Annals of Statistics*, 4 : 1206-1239.
- (3) Aumann, R.J. (1987): "Correlated Equilibrium as an Expression of Bayesian Rationality," *Econometrica*, 55, 1-16.
- (4) Berger, J.D. (1985): "Statistical Decision Theory and Bayesian Analysis", Springer Verlag, New York.
- (5) Lariri, S. (1990): "Arbitration Games and Approximate Equilibria". I.I.M.A. Working Paper no. 862.
- (6) Myerson, R.B. (1982): "Optimal Coordination Mechanisms in Generalized Principal - Agent Problems." *Journal of Mathematical Economics*, 16 : 67-81.
- (7) Myerson, R.B. (1986): "Bayesian equilibrium and incentive-compatibility: an introduction", in L. Hurwicz, D. Schmeidler and H. Sonnenschein (eds.) *Social Goals and Social Organization : Essays in Memory of Elisha A. Pazner* Cambridge : Cambridge University Press.

