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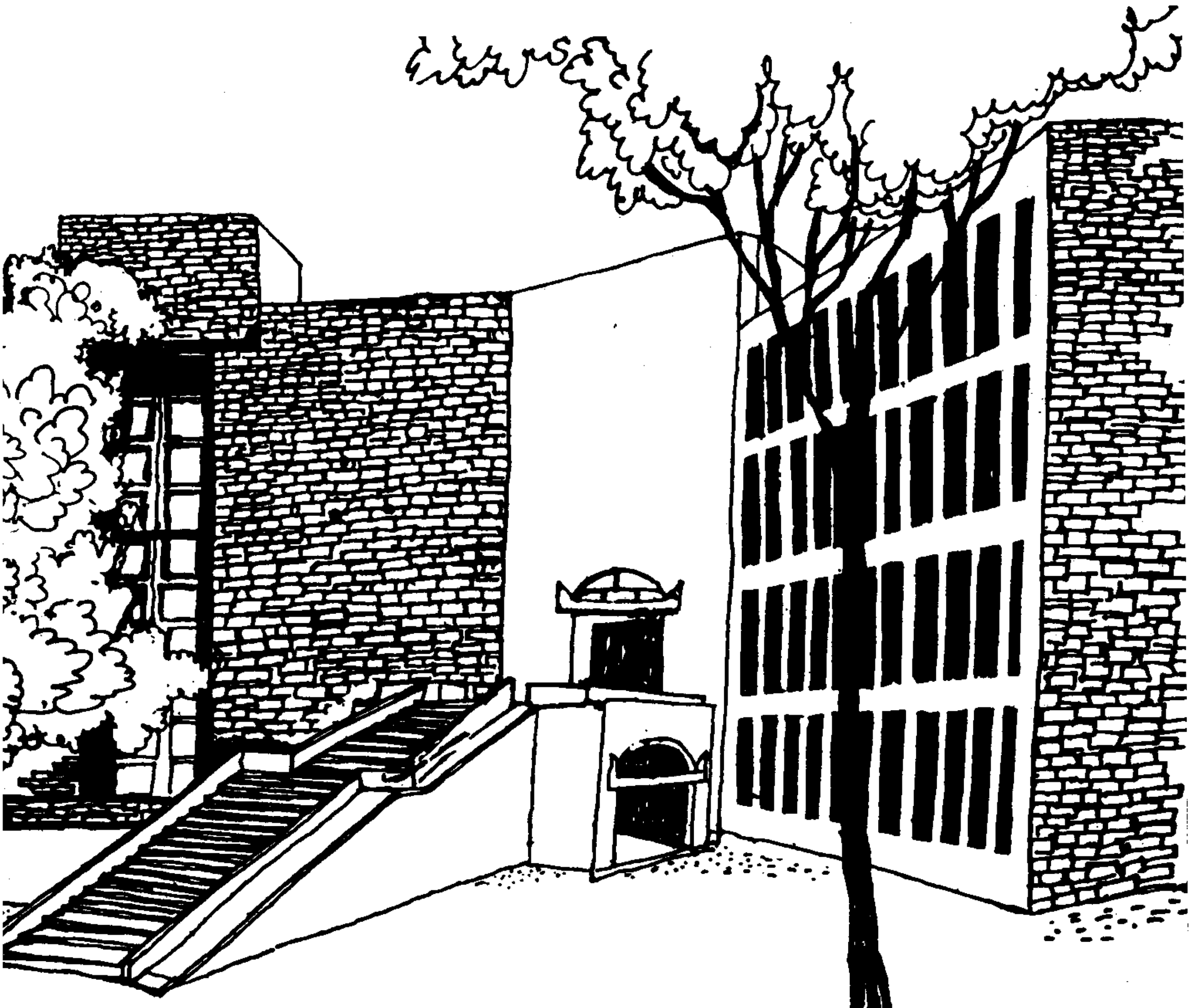
Working Paper



MEDIA SELECTION MODELS

DIRECTIONS FOR FUTURE RESEARCH

by

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WORKING PAPER No.344
January,81

The main objective of the working paper series of IIMA is to help faculty members to test out their research findings at the pre publication stage.

INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD

MEDIA SELECTION MODELS
DIRECTIONS FOR FUTURE RESEARCH

Abstract

The work done in media selection both in India and abroad is reviewed. Two approaches that can be adopted immediately for media selection so as to maximise response are discussed and the results of empirical work using the data of a large advertiser are shown. The directions for future research in this field are discussed.

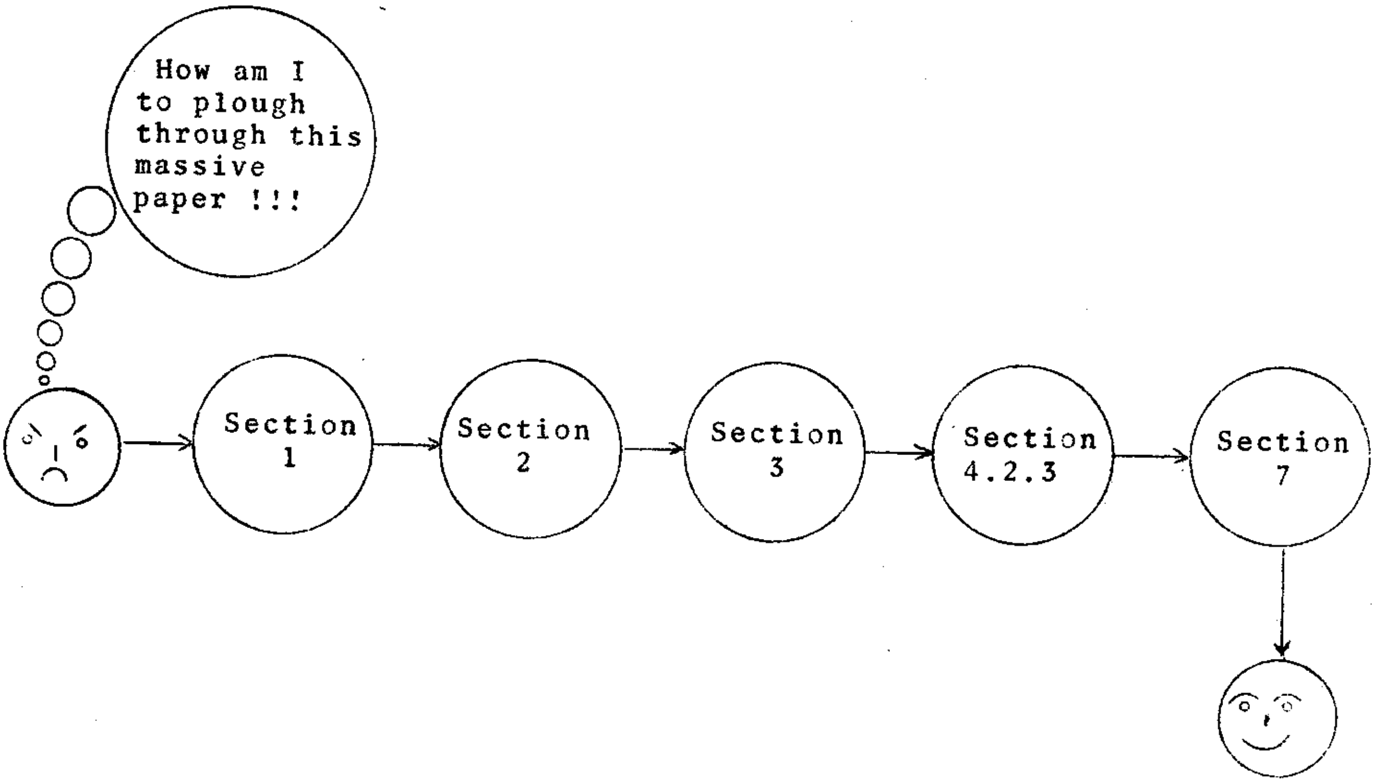
SUGGESTED PLAN OF READING FOR THE BUSY AD EXECUTIVE

Objective Function : i) Minimise Reading Time

ii) Maximise understanding

START WITH SECTION I, FOLLOW THE

"CPM" CHART



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MEDIA SELECTION MODELS : DIRECTIONS FOR FUTURE RESEARCH

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1.0. Introduction

Almost a decade ago, it was perceived that the advertising function in India would have to be placed on a more systematic foundation than ever before. The markets for many consumer products have been changing from sellers' to buyers', necessitating ever increasing advertising efforts. Simultaneously, the cost of advertising has been increasing by leaps and bounds. These two changes in the Indian environment have made it imperative that all advertising decisions be subjected to a systematic scrutiny.

Defining advertising as non-personal forms of communication, conducted through paid media, under clear sponsorship, Kotler[1] lists four major choice problems that an advertiser has to resolve. These are :

1.1. Choice of the message regarding the product qualities that is to be conveyed to the target audience.

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1 Philip Kotler, Marketing Management : Analysis, Planning and Control (New Delhi, Prentice Hall of India, 1979) p.347.

1.2. Choice concerning the manner in which the desired message is to be structured and the pictures, symbols, colours and tones that are to be used to facilitate the communication of the message.

1.3. Choice of the media class[2] and the media vehicles[2] within a media class that could possibly be used for conveying the message, and

1.4. Choice of the frequency with which each of the potential vehicles is to be used during the period of the campaign.

In the language of advertising, choice problems 1.1 and 1.2 are referred to as copy problems or creative problems. Choice problems 1.3 and 1.4 are referred to as media selection problems.

It has been claimed that in India, considerable attention has been given to the resolution of the creative problems.[3] In the media selection problems,

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2 Little and Lodish (24) define the terms media class and media vehicle as "A Media Class will be a general means of communication such as television, magazines or newspapers. A media vehicle will be a cohesive group of advertising opportunities within a class, such as a particular television show, magazine or newspaper." Thus within the class of press medium, Readers Digest, Illustrated Weekly, Times of India etc., constitute the media vehicles.

3 Some of the American Scholars, working in the advertising field, have pointed out that the advertising agencies create too few advertisement alternatives for their clients. The advertiser does not get a very good advertisement, but only the best of the few that have been created. See Irwin Gross, "An Analytical Approach to the Creative Aspects of Advertising Operations," (unpublished Ph.D. Dissertation, Cleveland, Case Institute of Technology, Nov, 1967).

only the choice of media has received some attention. It is only since 1970, that the problems of choosing media vehicles within a media class and the problem of the frequency with which each vehicle is to be used have received the attention that the decisions warrant. Even in United States, the search for better ways of making the choice, started only since 1964[4]. Engel and Warshaw(12) were among the pioneers who explored better methods of media selection using linear programming. Realising the need for better methods of media selection, some of the Indian advertising agencies, in 1970, started work on development of formal methods to aid media selection. The earlier work was devoted to the development of an approach for selection of media vehicles in the press medium. For details of this approach, the reader is referred to Clarion McCann Advertising Services Limited (8) and Jain (16). The aim of these approaches was to provide aids to the advertiser in the selection of publications, for achieving the advertising objectives

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4 In 1964 the first application of linear programming was attempted to decide which media vehicles to use, to achieve the advertising objectives at minimum cost.

at a minimum cost.[5]

Encouraged by the results obtained from the use of formal methods in the selection of media vehicles in the press medium, the experts in the field of advertising have shown interest in expanding the scope of application of such formal methods. It was desired that the methods developed should be refined and extended to other media as well. Considering this interest shown by the practising world, in the search for better methods of media selection, we came to the conclusion that time was now opportune for identifying the directions in which the selection methods developed should be improved. The objective of this paper is to list the possible paths that research in this area could follow.

2.0. Approach to the problem

The first step in the process of identifying the future directions for research, is to report the work that has already ~~been~~ done in this area. After all, the frontiers of what is known and what is not known should be clearly demarcated, before we chalk out the course of

5 Most of the work that is being done in this field in India is by advertising agencies. The agencies treat the work as confidential, and do not report to the outside world. It is therefore impossible to acknowledge all the work that is being done in this field.

action for the future. Part of this paper is therefore devoted to a survey of some of the important work done in this field. After completion of the survey, the paper then addresses itself to the task of identifying the directions for future research.

3.0. Overview of the working paper

To facilitate reading of this paper, we are giving a brief overview of the sections to follow.

Section 4 describes the earlier work, which followed one of the following two approaches :

a. To make the selection of vehicles in order to maximize the number of exposures created in the target audience subject to the constraints given by the advertiser.

b. To make the selection of vehicles so as to achieve the desired coverage and exposure[6] at a minimum cost.

In these approaches, the concepts of mathematical programming are extensively used.

Section 5 sketches the models that deal with the problem of deciding the optimum allocation of funds for advertising. These models do not directly consider the problem of selecting advertising vehicles.

Section 6 deals with the selection approach which selects the vehicles so as to maximise either the average (expected) sales or expected net contribution that will be

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6 The terms coverage and exposure are more precisely defined in Section 4.2.

obtained from the selection of media vehicles. Specifically the MEDIAC model developed by Little and Lodish (24) the ADMOD model of Aaker (2) are described in some detail.

Section 7 presents two possible approaches that can be adopted for future research. The data that may have to be generated so that the proposed methods can be used in practice are also discussed in this section.

4.0. Application of Mathematical Programming to the Resolution of Media Selection

4.1. Application of Linear Programming

In the earlier approaches, the media selection problem was formulated as a linear programming problem. In its simpler form, the problem was formulated as that of maximising the exposure values subject to the constraints on budget, maximum number of insertions that will be taken and the minimum number of insertions that must be taken, if so desired by the advertiser. The simplest formulation of such a problem and some of its variations are given in Appendix II. For a more detailed description the reader is advised to refer to Day (11), Buzzel, (7), Engel and Warshaw (12) Bass and Lonsdale (6), Wilson (41), and Stasch (39).

4.1.1. Data Required from Advertisers

In the simplest formulation, the data that the advertiser had to supply was i) a list of potential media vehicles that he was willing to consider, ii) the value to him of the exposures resulting from each of the potential media vehicles, if selected, iii) the cost per insertion in each of these vehicles, iv) the maximum amount that he was willing to spend for advertising, v) the vehicles which must be included in the plan (for reasons other than economic) and vi) the minimum number of insertions in such vehicles. With these data, the model produced a media plan showing the vehicles that should be selected and the number of insertions to be taken in the selected vehicles.

4.1.2. The Strengths and Weaknesses of this Approach

The major strength of this approach is its simplicity. The data required, except for the exposure value, was extremely simple to obtain. There are standard library programmes available, for solving the linear programming problem and these packages are very inexpensive to use.

The simplicity of this approach was achieved at a cost. The approach had three major drawbacks. The first was that it did not differentiate between two types of plans, one of which would result in ten exposures to one person and another one which would result in one exposure to ten persons. The second drawback was that it maximised exposures in aggregate. In many situations, the advertiser wishes to communicate the message to different segments of the audience and the importance of each of these segments may not be the same. The third drawback was that the solution to the linear programming problem would not be guaranteed to be in integers. To have a solution which requires the advertiser to take fractional insertions is operationally meaningless.

In our view, the first drawback is the major one. The second drawback could be corrected by slightly different formulations. In case, as pointed out in the third drawback, the solution turns out to be fractional, then it can be rounded off to the nearest integer. Certainly, this will lead to suboptimal solutions. But the error is unlikely to be large, if the cost per insertion is low.

In can be argued that the first drawback is not as severe as it sounds, because of the fact that upper limits have been placed on the number of insertions to be taken in each of the vehicles. It is therefore unlikely that only a few vehicles would be included in the final plan. When a large number of vehicles are included, the possibility of the plan resulting in large number of exposures only to a few people is unlikely to arise.

This argument is only partially correct. In case the upper bounds on the media vehicles are to be used as a mechanism to ensure that large number of exposures are not given to few people, then one must have a method of knowing how the upper bounds should be placed. In the absence of such a method, the formulation will be arbitrary.

4.2. Reach and Frequency Models

Realising the limitations of the use of linear programming, attempts have been made both in United States and in India, to overcome some of these limitations. Before we describe the features that are sought to be incorporated in the media selection model, we digress to define the terms reach and frequency.

The reach of a media schedule is defined as the fraction of people who are in the audience of at least one vehicle of the schedule. Frequency is defined as the average number of times a person is in the audience of a schedule, given that he is in the audience at least once.

The desirable features that are sought to be included in this model are listed below :

a. Different Segments of Interest to the Advertiser :

The selection method must recognise the fact that the advertiser may be interested in multiple target groups.

b. Interest to the Advertiser in Coverage and Frequency

In each of the target groups of interest, the advertiser would want to ensure that he reaches at least a certain proportion of the target group and that the number of times he reaches them on an average, does not fall below a minimum number. This interest is based on the premise that advertising leads the individual in the target audience to the desired action by taking him through four stages. The first stage is that of making him aware of the product or service offered. The second stage is that of enabling him to comprehend the characteristics and attributes of the product or service offered. The third stage is that of changing his attitude towards the product. The final stage is that of leading him to take the desired action, if the attitude is favourable. It is believed that a certain minimum number of exposures must be given to the individual in the target group, to accomplish the movement through each of the stages. Hence the need for minimum frequency. The need for reach arises from the fact that the larger the number of people exposed, greater will be the number of people who might take the desired course of action.

c. <u>Interest to the Advertiser of Minimising Cost Subject to the Constraints of Reach and Frequency</u>

The selection procedure should, as far as possible, ensure that the selected plan achieves the objectives of coverage and frequency at a minimum cost.

In the other parts of the world, work in this direction was done by Zangwill (43), Lee (22) and Chance, et al (10).

In India, work in similar direction was done by Clarion McCann Advertising Services Limited, Jain and Search Consultants Private Limited.

4.2.1. Approach Developed by Zangwill

The main features of the approach developed by Zangwill are sketched below :

a. The advertiser has decided upon the selections that he is willing to consider. For instance, the selections could be Life Magazine, Look and San Francisco Chronicle etc.,

b. Within each selection, the advertiser has identified a list of mutually exclusive advertising options. To illustrate, within the major selection Life, the advertising options are, to use four issues of Life or six issues of Life or eight issues of Life.

c. The advertiser may not see a particular selection, but if he decides to use it, then only one of the mutually exclusive options within that selection must be selected. For example, Life magazine may not be used at all. But if used, then only one of the three options illustrated in (b) must be selected.

d. The problem posed is to select the advertising options within each selection so that the total rated effectiveness units are maximised subject to the following constraints :

i) A selection may or may not be chosen. But if a selection is chosen, then only one of the mutually exclusive options within that selection must be chosen.

ii) The cost incurred in scheduling the options should not exceed the budget allocated and

iii) A minimum number of effective exposures must be created, if so specified.

e. The total rated effectiveness for any selection of options is computed in the manner indicated below :

For each option within a selection, a rated effectiveness value is assigned. For instance, the option - 4 issues of Life was assigned a rated effectiveness of 6042. The rated effectiveness is assigned after considering the composition of the audience of the option and the importance of each category of the audience to the advertiser. The rated effectiveness units of all the options included in the plan are totalled. From this total rated effectiveness units, the loss in rated effectiveness because of duplication within two options is deducted. Thus for example, if the plan included two options of using four issues of Life and six issues of Look, then a total of 1760 rated effectiveness units will be deducted from the total.

4.2.1.1. Critique of Zangwill's Approach

The major limitation of the paper is the lack of clarity of the approach. While the formulations of the constraints is clear, the reasons for maximising the rated effectiveness units are not at all clear. The author gives us no insights into the reasoning that has lead him to maximise the rated effectiveness. In the absence of any reasoning provided by the author, we have tried to develop a reasoning that would lead to the formulation of an objective function similar to the one the author has used.

It appears that the approach the author has adopted is that of maximising the expected value of a response function, similar to what has been used by Little and Lodish (24) in the model called MEDIAC. While Little and Lodish (24) use an exponential response function, Zangwill uses a quadratic response function. We briefly outline the reasoning that would lead to the formulation of the objective function that the author has used. The detailed mathematical reasoning is presented in Appendix III.

There are three steps in the reasoning. The first step is to segment the target audience according to some criteria. The second step is to asses the rated effectiveness of the advertisement in the different segments. The third step is to set up a relationship between the response to a media plan and the rated effectiveness level created by the plan.

The author has segmented the target audience on the basis of age, sex, income and region which, are to use his words, "important sales factors which had to be considered in the effectiveness calculation". Four age groups, five income groups and three regions have been used. The target audience is therefore classified into 120 segments (4 x 2 x 5 x 3 = 120).

The author then measures the rated effectiveness of the advertisement in a segment by the sales potential of a person in that segment. Let us denote the sales potential weight (value) of a person in the age group a, sex s, income group I and region r by v(a, s, I, r). For convenience, we will use the index t for (a, s, I, r). Thus v(t) will stand for the sales potential weight of a person in the segment t, namely (a, s, I, r).

A person in the t^{th} segment may nor may not see the advertisements in the media plan. The number of advertisements that he will see will depend on i) the choices made in each of the selections, and ii) whether or not he is in the audiences of the selections (media optinns) made by the advertiser. Since he may not be a member of the audience of all the selections, the rated effectiveness level created by the media plan, in respect of this person will <u>not</u> be equal to v(t) x Total number of insertions taken,. Let y(t) denote the rated effectiveness level of a person in the t^{th} segment, after taking into account the fact that he may or may not be in the audience of some of the choices that we make. Let us denote by R(t) the response that will be generated by the media plan, from a person in the t^{th} segment. The author assumes that R(t) is given by

$$R(t) = y(t) - b[v(t)-K_t]^2$$

where b is a constant and $K_t = E\{y(t)\}$

A quadratic response function is used because the response will be a diminishing function of the rated effectiveness, after a certain level of effectiveness is reached. The total response to the plan from the t^{th} segment is therefore N_r R(t), where N_r stands for the number of persons in the region r to which the person in t belongs.

The total response R' is obtained by summing N_r R(t) over all the segments. R' by itself cannot be maximised because each R(t) is an uncertain quantity, depending on the uncertain event that a person in the t^{th} segment is in the audience of the choices made. The author therefore chooses to maximise the expected value R of R'. Notice that R' also depends on the choices made. The choices are therefore to be made so as to maximise the average value of R'.

There are several limitations in this approach. The first one is the manner of computing total response. In arriving at the total expected response from a segment the expected response from a person in each segment should be multiplied by the number of persons in that particular segment and not by the total number of persons in the region to which the person belongs.

The second limitation is the manner in which the response function is formulated. The response function is not based on the currently accepted theory which explains how advertising works. It is now generally accepted that advertising increases the chances of a person taking the desired action, by taking him through the different stages of awareness, comprehension, conviction and action. In constructing the response function, it is desirable to see how the probabilities of a person moving from one stage to another are altered, by the exposures he receives from the advertisements. The quadratic response function, as constructed, needs more justification.

The third limitation is the assumption that a reader of a publication reads all the issues of the publication.

There are a number of assumptions made regarding form of the function v(t). These assumptions have not been stated clearly. The absence of the reasoning that [illegible] led the author to use the special structures, for v(t) is therefore not surprising.

4.2.2. <u>Approach developed by Lee</u>

The main features of the approach developed by Lee are sketched below :

4.2.2.1. <u>Main Features of the Model</u>

1. The model is confined to the selection of vehicles within the medium of press.

2. The decision problem that the model is set up to solve is that of deciding the number of insertions to be taken in each of the publications under consideration, and the size of the advertisement in each of the insertions.

3. There are four assumptions made in the model. The first one is concerning the relationship between the size of the advertisement and the chance of it being seen by readers of the publication in which the

advertisement has been placed. More specifically, if the size of the advertisement is expressed as a proportion of the whole page it occupies and is denoted by p, then the chance that it will be seen by the readers of the publication will be $\sqrt{p}$

The second assumption is regarding the chance that a person in the target audience will be a reader of a group of publications. It is assumed that the chance that a person is a reader of m publications, is the product of the chances of his being a reader of the individual publications. Thus if p_i (i = 1, 2, ... m) is the chance that he is a reader of the ith publication, then the chance that he is a reader of all m publications is $p_1 p_2 \ldots p_m$. This relation is assumed to be true for all values of m.

The third assumption states that all the readers of a publication read all the issues of the publication.

The fourth assumption is concerning the number of target groups that are of interest to the advertiser. It is assumed that only one group amongst the audience of the publication is of interest to the advertiser.

4. With these assumptions, Lee proposes three approaches for solving the problem stated in (2). In each of the approaches simple decision rules are obtained to determine the optimal number of insertions to be taken in each of the publications and to decide the optimal size of the advertisement in the respective insertions.

5. The first approach that is proposed for resolving the choice problem is that of maximising impact (average frequency per person in the target audience) subject to the condition that the cost of advertising does not exceed the budget that is allotted. The second approach is that of maximising coverage (reach) subject to the condition that the cost of advertising does not exceed the allocated budget. The third approach is that of maximising the impact subject to achieving a minimum specified coverage (reach) and ensuring that the cost of advertising does not exceed the limit set on the advertising expenditure.

Lee shows that, when an upper limit is fixed for the expenditure on advertising, in general, it is not possible to maximise both the reach and the frequency simultaneously.

4.2.2.2. Strengths and weaknesses of the model

There are two major strengths of the paper. The first one is that it formally links the creative aspects of the advertising problem with the problem of media selection. This link is forged through the assumption that the chance of a reader of a publication seeing an advertisement in that publication is equal to the square root of the size of the advertisement. The second strength of the paper is the reasoning it provides for using the criteria of maximising coverage in resolving the selection problem stated in paragraph 2, page 17. Lee points out that if the response function has a particular structure, then given his assumptions, maximizing coverage will lead to the maximisation of response. The structure that the response function of the campaign should have is

$$R = \sum_{r=0}^{M} (1-p^r) I_r$$

In the above equation, the notations have the meanings stated below :

R denotes the response to the advertising campaign

M denotes the total number of insertions taken

r denotes the number of times the advertisement is seen by a member of the target audience

P is a constant and

I_r is the probability that a member of the target audience sees the advertisement exactly r times.

The main weakness of the paper is the restrictive assumptions made in the development of the model. The assumption that the chance of a person being a reader of m publications is the product of the probabilities of his being a reader of the individual publications is far too restrictive. This assumption is certainly not valid in India.

The approach of maximising impact (frequency) without any control on coverage will have the problem of not being able to differentiate between a plan that gives ten exposures to one person and a plan that gives one exposure to ten persons.

4.2.3. Media Selection Models in India

4.2.3.1. Press Media Planning

In India, work towards the development of formal methods for media selection was started in the early seventies. The earlier work in the field was due to Clarion McCann Advertising Services Limited (8) and Jain (16) The essential features of the early models were :

a. The media selection models were developed only for the press medium. The data pertaining to the readership of publications used in the selection models was obtained from National Readership Survey I (18).

b. The models adopted two different formulations for the resolution of the selection problem One formulation was to select the publications so as to achieve desired minimum reach and frequency in the target segments of interest, at a minimum cost. The other formulation was to select the publications in such a way that a minimum specified reach and frequency were obtained in the specified target segments. The total cost of the selection was within the budget allocated and the overall frequency was maximised.

In order to compute the reach of a media plan, common readership between 2, 3.. publications was required. Since common readership between three or more publications was not available, efforts were made to find the upper and lower bounds of the reach of a plan.

The earlier methods used Kwerel's (21) method to obtain the lower bounds on reach. Jain improved the lower bound developed by Kwerel. In both Clarion, Mote and Jain models, the media planning problem was formulated as a 0-1 integer programming problem, but efficient heuristic methods were developed to solve the problem.

The data used in the earlier models for selection of the publication was the readership of an average issue of a publication as reported in NRSI. It was assumed that the reach achieved by taking more than one insertion in a publication (by advertising in more than one issue of a publication) will be the same as that achieved by taking one insertion in the publication and that this will be equal to the readership of the publication, as reported in the NRS. This would be so if all the readers of a publication read all the issues.[7] This assumption is not valid particularly in the case of magazines where many of the readers do not read

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[7]The phenomenon that all the readers of a publication do not read all the issues of a publication is referred to as the phenomenon of casual readership. The increase in reach achieved by taking more insertions in a publication because of the presence of casual readership, is referred to as the phenomenon of accumulation of audience.

all the issues. Hence the reach achieved by a publication does not remain constant as the number of insertions taken in the publication are increased. It can be shown that the reach increases as the number of insertions are increased but increases at a decreasing rate and finally reaches a maximum.

In the subsequent developments of the models, this deficiency has been removed. A method has been found to compute the reach of a publication when the publication has casual readers. In order to get over the difficulty of non-availability of data on common audience of three or more publications, the method suggested by Metheringham (26) has been adopted.

4.2.3.2. Cinema media Planning in India

Cinema, with its audio visual effects, offers many advantages to the advertiser. Also, for reaching the majority of the Indian population which is unlettered, cinema is one of the most useful media. Considering these factors, the Indian experts came to the conclusion that the next priority for development of selection models should be given to the the selection problem in the mediam of cinema.

The main difficulty in developing a selection model for the cinema medium was the lack of data on the characteristics of the people who visit the different theatres in a town and the frequency with which they visit the respective theatres. The only data that NRS II reports is the data on the frequency with which people in different income strata, different age groups, in different professions visit a theatre in the course of one year. It does not report data on the type of theatres visited by these persons. In case the number of theatres were small this data could be obtained by carefully planned research studies. Unfortunately this is not the case. There are approximately 3000 theatres spread over the length and breadth of the subcontinent.

The first task in the development of these models was to generate the possible patterns of visits that a member of the target audience could make to different theatres in a town. These patterns were to be developed on the basis of the data on movie going habits, as reported in NRS II or from the research data obtained by the advertiser.

There are three approaches for generating these patterns. These approaches were developed keeping in view the fact that the minimum purchasing unit in the cinema medium is a week in a theatre. Thus for a theatre, an advertiser, has 52 options (weeks) for screening the advertisement. If the list of his potential theatres in town is m, then he has 52 m potential options.

Given that a person has made i visits to theatres, during the course of a year, "how would these be distributed amongst the 52 m potential weeks"? was the question to be answered. This question is answered under different sets of assumptions.

A brief outline of the three approaches is given below :

Model I: In this approach two assumptions were made. These are i) a person goes to any theatre at most once in a week and ii) all the combinations obtained by selecting i theatre weeks from the 52 m possibilities have equal probability of being chosen. With these assumptions, the model evaluates a given cinema plan in terms of reach and frequency and also helps the advertiser in preparing a plan that will achieve the desired reach and frequency at a minimum cost.

Model II: This model differs from Model I in its basic assumption that a person visits a movie and not a theatre. The model converts the number of potential theatres in a town into the number of potential movies where the advertisements can be screened. Two assumptions are made in this regard : i) all candidate theatres screen distinct movies and ii) a person has no preference for any particular movie. The number of visits to a theatre by an individual in the target audience is interpreted as the number of films seen by the individual. Hence the interest of the model is in the average life of a film. The point of interest is, "Given that the advertisement is screened for a specified number of weeks with each distinct movie, what is the reach and frequency achieved by the plan?" The model uses the principles of probability to evaluate a given plan in terms of reach and frequency and also to select a minimum cost cinema plan in order to obtain specified reach and frequency.

Model III: This model relaxes the assumption of Model I that all the theatre week combinations are equally likely. All the candidate theatres are divided into two classes such that within a class, all the threatre week combinations have the same probability of being selected. The chances of selecting theatre week combinations belonging to the different classes however are not the same. Other assumptions of the model are the same as those of Model I. With these assumptions, the model proceeds to evaluate a given plan in terms of its reach and frequency and also develops a minimum cost plan.

5.0. Models pertaining to the Optimum Allocation of funds for advertising

In this section, we present a brief review of the models that are concerned with the problem of deciding the optimum allocation of funds for advertising. Some of the important papers on this topic are by Gould (32) Nerlove and Arrow (30) Parsons (31) Rasmussen (34) and Sasieni (37).

5.1. Models by Parsons

The central theme of the paper is the study of the behaviour of advertising elasticities, during the different stages of the product life cycle. The elasticity of advertising is defined as the ratio of percentage change in sales, associated with a percentage change in advertising. The stages in the life cycle considered are introduction, growth, maturity and decline.

The paper attempts to test whether or not the demand elasticity of advertising changes, as the product goes through the different stages in its life cycle. This is sought to be done in two steps. Firstly, two models are proposed, to establish a relationship between sales achieved in a period, with the advertising expenditure incurred in that period and the sales achieved in the immediate past period. In the second step, empirical data for a period of 30 years is used to estimate the coefficients in the proposed relationship. Specifically the two models proposed are :

$$\ln.S_t = \alpha_1 + \alpha_2 \ln.(A_t/P_t) + \alpha_3 \ln.S_{t-1} \quad \ldots (5.1.1)$$

$$\ln.S_t = \beta_1 e^{-\beta_2 t} \ln.(A_t/P_t) + \beta_3 (1-e^{-\beta_2 t}) \ln.S_{t-1} \quad \ldots (5.1.2)$$

where

S_t = Sales at time period t

S_{t-1} = Sales at time period (t-1)

P_t = General price index at time period t

A_t = Advertising expenditure at time period t, and

$\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3$ are constants to be estimated

In both the cases, the advertising expenditure is deflated by the general price index. It can easily be verified that the advertising elasticity is measured by α_2 in equation (5.1.1) and by $\beta_1 e^{-\beta_2 t}$ in equation (5.1.2). In the first case, the elasticities are assumed to be constants, while in the second, the elasticities change over a period of time. Further mathematical details are furnished in Appendix IV.

Using statistical techniques, the coefficients are estimated. It is found that the time varying elasticity formulation fits the data better than the constant elasticity formulation.

Having obtained the elasticities, the optimal advertising budget sequences for the two models are found, by maximising the net contribution[8] over the entire period. The maximisation is done subject to the constraint that the total amount spent on advertising should not exceed the amount that was actually spent. In the case under study, the optimal net contribution, according to the time varying elasticity model is found to be almost double that obtained from the constant elasticity model. The optimal advertising budget sequences, arrived at from both the models reveal that advertising allocation should be much more in the earlier years than the appropriations that were actually made.

'Do advertising elasticities change, as the product passes through the various stages in its life cycle?' is the main question, the paper seeks to answer. The methodology developed has many limitations and therefore, the paper is not likely to be of much help to the practising world. In our judgement, there are three main limitations in the paper.

a) <u>Elasticities Related to Time Period and not to Stage in the Product Life Cycle</u>

The paper intended to study the changes in elasticities during the product life cycle. The elasticities shown in both the models, however, are related to time and not to the stage in the life cycle

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8 Net contribution is defined as Sales minus total costs

of the product. The paper does not even shows the demarcation of the 30 year time span into the various stages in the life cycle. From the data as presented, it is not possible to identify, which period will correspond to the introduction stage, which periods to growth and maturity and which to decline. Therefore, the main purpose of studying the changes of elasticities in the stages in the life cycle is defeated.

b) The Long Historical Data Required in the Model

Commenting on the data, the author observes that there are only 30 observations. Further, he points out that there are no observations covering the early years in the product life cycle. If even data ranging over 30 years is found inadequate, it is not clear as to what would be considered adequate. In practical applications, should the model require such a long past history, then its utility is limited, since the conditions in the market are changing very rapidly and the future is very likely to be substantially different from the past.

c) Not a Guide to media selection

This paper at best provides an answer to the question about the total allocation of funds for advertising. It does not answer the question of practical interest, namely "how should the advertising vehicles be selected?".

5.2. Rasmussen's Model for Determination of Advertising Expenditure

The purpose of the paper is to identify the conditions under which the optimum allocation of advertising will work out to a predetermined proportion of sales. The importance of the paper stems from the fact that, in practice, the advertising budget is very often decided upon as a predetermined proportion of sales, by many advertising practioners.

The paper shows that the percentage of sale method of determining advertising allocation is optimal only under certain conditions. These are :

i) Price and unit cost are constant. Stated alternatively, the condition stipulates that advertising should have no effect on the price that will be charged.

ii) Advertising elasticity of demand should be constant.

The mathematical formulation of the model is furnished in Appendix V.

The paper argues that the above assumptions are too restrictive. Advertising elasticity cannot be constant under all conditions and must fluctuate according to the size of the advertising expenditure, cyclical position, competitors' reactions and so on. Advertising elasticity must be shifting and not be constant. Thus the paper illustrates that the percentage of sale method is not the best way of determining advertising appropriation.

6.0. Selection Models that Attempt to Maximise average sales/contribution

In Section 5 we were concerned with the decision about the optimum allocation of funds for advertising. The approach adopted for resolution of this problem was that of setting up a relationship between advertising expenditure and sales and of using this relationship in choosing an expenditure level that will maximise sales. The main drawback of this approach is, as Kotler[9] points out, "The effect of advertising on sales is not simply a function of much is spent. Even more important may be, how it is spent-specifically, what is said, how it is said, where it is said and how often it is said.

This difficulty was partly overcome in the models discussed in Section 4. These models were designed to answer the question as to which media options should be selected and how many insertionsshould be taken in the in the options selected. In the Reach and Frequency approach, selection was done so as to achieve the desired level of reach and frequency at the lowest possible cost. The proponents of the reach and frequency approach, justify the use of this approach on the grounds that the purpose of advertising is communication and that it is impossible to quantify the relation between sales and advertising.

[9] Philip, Kotler, Marketing Management : Analysis, Planning, and Control, op.cit, p.356

This argument can be briefly summed up in the words of Wolfe, Brown and Thompson (43) as " Generally it is not considered reasonable to use sales results as a basis of measuring advertising effectiveness, except where advertising is the dominant sales force, where other factors affecting sales remain fairly constant, and where the results of the advertising are quickly reflected in shipments and billings. When these conditions do not exist, other yardsticks must be used".

This argument is not acceptable to many. It is their contention that the ultimate objective of advertising is to increase sales (or response) and as such the criterion for the choice in evaluating advertising alternatives should be whether or not the decisions result in increase in sales (or response). The controversy between these is highlighted by a letter written by R.L. Ackoff (4) We quote excerpts from the letter reproduced by Rao[10]. "It is argued that advertising causes a state of appreciation, which when certain other conditions are met, produces sales. But because of the chain of causes, it is said we cannot blame or credit advertising with the eventual purchase or failure to purchase. The issue is not whether advertising alone can be blamed or credited

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10 Ambar G Rao, <u>Quantitative Theories in Advertising</u> (New York : John Wiley & Sons, Inc.1970), pp. 9-10

with sales or lack of them - no one argues that it can be - but can it be blamed or credited with some effect on sales? Politz answers "No", with the following logic : "A (advertising) causes a state S (appreciation). If an individual is in state S and other conditions (C_1, C_2 ....) are met, a sale results. Hence A's effect on sales is indirect since it depends on conditions (C_1, C_2, ...) and hence cannot be measured."

However he is wrong. Suppose that advertising can increase the number f persons in state S by N and that of these, a fraction of meets the conditions (C_1, C_2 ...) If each individual who meets the condition buys a quantity q then the sales due to advertising will be fNq. Politz argues that even if f is zero we can credit advertising with producing N persons in state S. So what! If f is zero, we might as well save the advertising dollars. If f is not zero, we can measure the quantity fNq and decide whether the advertising expenditure is justified".

In this section, we describe some models in which the selection of media is done so as to maximise the response measured in terms of sales or net expected contribution by i) Little and Lodish and ii) Aaker. It might be recalled that Zangwill also used the approach of maximising response without explicitly saying so. Lee has also shown that maximising reach is equivalent to maximising response under certain assumptions.

6.1 MEDIAC Model by Little & Lodish

The model is based on the foundations stated below:

a. Sales potential and media habits are not uniform in the entire population. It is however possible to segment (stratify) the population into groups so that within a group, the sales potential and media habits are similar but differ substantially between groups.

b. The population is divided into different segments based on some criteria such as sex, education or life style.

c. An advertising schedule creates exposures. By exposure it is meant that a person has perceived the presence of the advertisement. The effect of exposure on a person in a segment differs, not only from one media to another but also from one vehicle to another in the same media class.

   For example, the advertiser may believe that the effect that an advertisement in the magazine 'Life' may have on a person would be different from the effect that the advertisement in the magazine 'look' may have on the same person. This belief, it is assumed can be quantified.

d. The response of individuals in a market segment increases with the exposure level, but at a diminishing rate at higher levels.

The mathematical reasoning used is similar to the one used by Zangwill. Suppose y_i[11] denotes the exposure level of a person in the i^{th} segment then the response from a a person in the i^{th} segment is denoted by $r(y_i)$. The total response therefore will be $N_i . r(y_i)$, summed over all the segments i, where N_i denotes the number of people in the i^{th} segment. Here again, y_i and hence $r(y_i)$ are uncertain quantities, depending on the types of vehicles that a person in the i^{th} segment is exposed to. Hence the average (expected) value of the total response is taken as a measure of the response that will be achieved from a media schedule. The media schedule is chosen so as to maximise the total expected response. For a detailed presentation the reader is referred to Appendix VI.

6.1.1. Critique of the MEDIAC Model

The major problem in the model is the formulation of the response function. Formulation of the sales response as a function of the exposure level of an individual is not convincing. This is so because the method of construction is not based upon the currently accepted reasoning that shows how advertising leads people to take the desired action by moving them through the series of states: awareness, comprehension, conviction and action.

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11 For ease of exposition, we have not introduced the time element. Strictly speaking, we should have used the symbol y_{it} to denote the exposure level of a person in the i^{th} segment, in the t^{th}/period. /time

The more appropriate way to construct the response function would have been to show how the exposure levels will change the transition probabilities of the persons in the different segments. To be more specific, let $p_{is_1s_2}$ denote the probability of a person belonging to the i^{th} segment moving from state s_1 to s_2. Here s_1 and s_2 refer to the states mentioned earlier, namely, awareness, comprehension, conviction and action. It would have been more appropriate to assess the transition probabilities and then work out the probability of a person being in a state of readiness to try. Such an approach was attempted by Jain and we believe that the manner of constructing such a response function would be more meaningful.

It can be argued that "the ideal approach to the problem of response function would be through the construction of transition probabilities. This ideal however, cannot be achieved in practice because of the difficulties involved in assessing the transition probabilities and because of the computations involved, even in measuring the response that a media schedule will generate in a single market segment. The computational difficulty in optimisation would be of a very high order. Therefore the approach adopted in MEDIAC is the next best approach to the problem of media selection, so as to maximise the expected response."

This argument is not convincing because the current practice of preparing a plan so as to achieve a minimum level of reach and frequency can also be considered a simplication of the main problem of preparing plans so as to maximise response. The minimum reach and frequency approach is based on the premise that a person needs a minimal level of exposures to enable him to comprehend message and to be convinced about it. For this reason, an advertiser= wants to know the entire probability distribution of exposures per person. The reason behind stipulating minimum reach is to ensure that for the desired response, at least a minimum number of persons are exposed to the message.

It therefore cannot be claimed that the approach used in MEDIAC for media selection is superior to the one currently followed. At least in the existing approach, the advertiser has before him, a complete probability distribution of exposures per person and has the flexibility to exercise his judgements regarding the response that will be generated from such an exposure pattern. This flexibility is denied to him in MEDIAC, in which he would have with him only the mean and the variance of the exposure level distribution.

Moreover, to estimate the expected sales response the MEDIAC model requires higher order overlaps among the media vehicles. The probabilities of exposures to every pair of media options in the schedule, are required. The authors of MEDIAC used some empirical studies to estimate them. To adopt this model in the Indian situation, similar studies have to be conducted. Such studies are very expensive.

Hence, looking at the limitations of the model, implementation of this model in the Indian context does not seem to be promising.

6.2 ADMOD MODEL

The ADMOD model, proposed by Aaker is designed to aid media allocation decisions. The author asserts that this model differs from most other media models in the three respects stated below:

i) It is based on a different operational conception of the advertising process,

ii) Its focus is not on aggregate vehicle audience, but upon sample populations selected from the various segments. In evaluating an insertion schedule, it examines the likely impact on every individual in the sample.

iii) It attempts to solve simultaneously, the media selection problem, the decision problem regarding the copy to be used, and the problem of deciding the budget allocation for advertising.

6.2.1 The Foundations of the Model

a) Different view of the Advertising Process

Aaker[12] contrasts, in the following words, the conception of advertising process used in the model, with that used in other models:

[12] David A. Aaker, "ADMOD, An Advertising Decision Model", Journal of Marketing Research, XII, 3 (February 1975), pp.37-45.

> "Most media models rely upon a model of advertising, which suggests that advertising creates an advertising exposure (or some similar construct), which, in turn, creates sales. The advertising exposure level will dissipiate over time, but can be maintained or built up by more advertising. The heart of the model, then, is an aggregate response curve, that relates exposure levels to sales, or perhaps, advertising expenditure levels to sales directly. ADMOD, in contrast, has a more disaggregative view of the advertising process. It focuses on specific consumer decisions, which have long term implications for the firm, that advertising is attempting to precipitate.... The concern of advertising is then upon some indicator of the probability of the consumer changing his cognitions in the desired way, or taking the desired action."

The model therefore builds a relationship between the probability of an individual taking the desired action, and the number of exposures the individual has received, the copy that is used, and the vehicles that are used in the media plan.

b) Focus on Samples from Different Segments

The author chooses a sample from each segment and assesses the impact of the schedule on each individual in the sample. The motivation behind using the method was to make the mathematical expression more tractable. The use of this model therefore requires a sample from each of the segments.

c) Interdependence of the Copy Decision, Media Selection and Budget Allocation

The author emphasises the interdependence between the budget decision, the media selection decision and the copy decision. The interdependence is formally incorporated in two steps. The first step uses the fact that the copy used will affect the probability of the individual being exposed to the insertion. The second step incorporates the fact that the probability of an individual taking the desired action will be determined by: i) the number of times he is exposed to the advertisement, ii) the vehicles through which he is exposed, and iii) the copy that is used.

In the first step, it is assumed that the probability that an individual will be exposed to the advertisement is a product of two probabilities. These are: i) the probability of an individual being in the audience of the vehicle carrying the advertisement, and ii) the probability that he notices the advertisement, given the fact that he is in the audience of the vehicle. The latter probability is governed by the copy approach. Symbolically, that is expressed as

$$p_{eij} = b_{ij} \cdot h_{cj}$$

P_{eij} = the probability that an individual i is exposed to the insertion of the copy approach c in vehicle j,

b_{ij} = the probability that an individual i is exposed to vehicle j, and

h_{cj} = the probability that anyone exposed to vehicle j will be exposed to the insertion of the copy approach c in vehicle j.

In the second step, the effect of each insertion in a media schedule is assigned a specific value, depending on the copy approach used and the vehicle in which the advertisement is inserted. This effect is called the vehicle source effect of the insertion. The number of insertion options in a schedule seen by an individual, however, cannot be known with certainty. As a result, the vehicle source effect of a schedule on an individual cannot also be known with certainty. In the model, therefore, the average vehicle source effect on an individual is computed. The probability of the individual taking the desired action is then expressed as an exponential function of the exposures received and weighted by the average vehicle source effect of the media schedule.

The net value of a media schedule is calculated in the following steps:

i) The target audience is divided into several segments of interest to the advertiser. The long term monetary value (w_s) of a member of the segment s taking the desired action is assessed for all the segments, using the judgement of the advertiser.

ii) A sample of suitable size is drawn from each segment. The behaviour of the individual in terms of taking the desired action cannot be predicted with certainty. The monetary value that will be received from the individual is therefore an uncertain quantity. The model, therefore uses the average monetary value (expected monetary value). This is computed by multiplying the long term monetary value resulting from the individual taking the desired action by the probability of the individual taking the desired action, as a result of being exposed to the advertisements.

iii) The average value of the schedule is summed over all the individuals in the sample and divided by the sample size. This is taken as the average value that will be received from a segment member by the media schedule.

iv) The average value from a segment member is scaled by the size of the segment and summed over all the segments in the schedule to obtain the total value of the media schedule.

v) The net value of the media schedule is obtained by subtract... the cost of the schedule from the total value.

The model chooses a media schedule so as to maximise the net value.

6.2.2 CRITIQUE OF THE MODEL

a) Different View of the Advertising Process

The author asserts that the model uses a different operational conception of the advertising process. This leads us to believe that the manner in which the probability of the individual taking the desired action is derived, will be based on the currently accepted view of how advertising works. More specifically, one expects that the probability of an individual taking the desired action, as a result of advertising will be obtained by assessing the probability of the transitions that the individual will undergo, from unawareness to awareness, comprehension, conviction and action. This unfortunately, is not the case. In fact, the approach of the model is not very different from that of MEDIAC. Just as the response function in MEDIAC relates exposure level to sales potential, in ADMOD, the probability of an individual taking the desired action is related to the number of exposures he has received.

b) Assumption of Independence Regarding Media Habits

A basic assumption of the model is regarding the chance that a person in the target population will be in the audience of a group of vehicles. It is assumed that the chance that a person is in the audience of k vehicles is a product of the chances of his being in the audience of the individual vehicles. Thus if b_{ij} (j = 1,2,3,---k) denotes the chance that the individual i is a reader of the jth vehicle then the chance that he is in the audience of all the k vehicles is $b_{i1}, b_{i2} --- b_{ik}$ This relation is assumed to be true for all values of k. This assumption is too restrictive and is certainly not valid in India.

c) Technical Details Involved in the Model

The treatment of the following technical details involved in the model is not satisfactory. i) The probability distribution of exposures is approximated by a binomal distribution. The efficacy of this approximation has not been demonstrated. This approximation does not seem to be necessary in the light of the restrictive assumption made about the media habits of target population. The exact formulae of the probability distribution will not be any more cumbersome to deal with than the binomial distribution. Further, some of the formulae in the paper can be expressed in a more compact form because of the restrictive assumption regarding media habits and the structure of the probability of the individual taking the desired action. As a matter of fact, this formulae can be shown to be a special case of the formula for response function developed by Lee. This aspect is dealt with in more detail in the Appendix. ii) The author does not clearly explain the reasoning behind the derivation of the average vehicle source effect of a media schedule. The result obtained by the author seems to be valid only under certain conditions. We have commented on this aspect also in the Appendix.

d) Samples from Each Segment

Choosing samples from the different segments involved and assessing the probability of the desired action for each individual will be too cumbersome when a large number of segments are involved. This approach does not seem to be necessary because of the strong assumption about media habits. The claim made that the approach will make the mathematics more tractable is difficult to understand.

In conclusion, the ADMOD model in essence, is not different from the models described earlier, namely MEDIAC and the model proposed by Zangwill. The treatment of some of the technical details involved in the model leaves many things to be desired. The model also requires the use of a large sample from each of the segments in the target audience. Hence, this model is not very promising.

7.0. Possible Approaches for Future Research

7.1. Introduction

In this section, we turn our attention to the problem of selecting vehicles so as to maximise the response that will be generated by the advertising campaign. Our objective in this section is two fold. The first one is to point out the approaches that can be adopted immediately for such a selection. The second is to suggest the directions for future research on this problem.

The first step in any selection process that is designed to maximise response will necessarily have to be that of setting up a yardstick by which response is to be measured. In our judgement, this step should not pose much difficulty. We have proposed, on the basis of the extensive review of literature, two possible measures that could be used. These are discussed in Section 7.2.

The second step in such a selection procedure is to design a computational scheme that will select the vehicles so as to maximise the response, measured in terms of the yardstick that has been decided upon. This step poses many problems. We have therefore tried to delineate what is possible to be done immediately from what might require further research.

In Sections 7.3, 7.4 and 7.5, we describe a method which can be adopted immediately. In section 7.6, we discuss the areas for future research.

7.2. Yardstick for Measurement of Response

There are two ways in which the response to an advertising campaign can be meaaured. These are listed below:

a. Probability of a Person in the Target Audience Taking the Desired Action.

Many experts in the field of advertising will agree with the proposition that the 'probability of a person in the target audience taking the desired action' is an appropriate yardstick for measuring the results of an advertising campaign. Persons who assert that the ultimate objective of advertising is to increase sales (response) will readily agree with the use of this proposed yardstick. It can be seen from Section 6 that such a yardstick has been explicitly used in the MEDIAC Model and in the ADMOD model developed by Aaker. Even persons who assert that it is not reasonable to use sales results as a basis for measuring advertising effectiveness, will have no objection to accepting this yardstick for measuring the results from advertising. This school of thought measures the effectiveness of an advertising campaign by the change that it brings about in the proportion of people in different states such as awareness, comprehension, conviction and action. The probability of a person taking the desired action,in this context, can be interpreted to mean the probability of a person being in the desired cognitive state.

b. Net Economic Gain Accruing from Advertising

The net economic gain accruing from an advertising campaign measures the results of the campaign in terms of the economic benefits it generates. There are two components in this measure. The first one is the economic gain that will accrue to the advertiser, as a result of the advertising campaign. The second one is the cost incurred for the advertising campaign. The net economic gain is the difference between the economic gain resulting from the campaign and the cost incurred for mounting the campaign.

The first element, namely, the economic gain accruing to the advertiser needs some explanation. As an illustration, consider the example of a manufacturer of infant foods. From past experience, it is known that if a person purchases this infant food for a child, then he will purchase approximately ten packs during the infancy of the child. The contribution (margin after variable cost) of this product is five rupees per pack. Thus the monetary benefit to the advertiser from a person in the target audience taking the desired action is Rs.50/-. It is not known with certainty whether a person in the target audience will take the desired action, as result of the campaign. The economic gain[13] resulting from a person taking such an action, therefore will be the product of the probability that the person will take the desired action and the monetary gain accruing from this person. Thus, if there are 200,000 potential buyers, then the economic gain for the advertiser will be defined as

Economic gain from the Advertising Campaign =

$$\left(\begin{array}{l}\text{Contribution/}\\ \text{person}\end{array}\right) \times \left(\begin{array}{l}\text{Number of}\\ \text{potential}\\ \text{buyers}\end{array}\right) \times \left(\begin{array}{l}\text{Probability that a}\\ \text{potential buyer will}\\ \text{take the desired}\\ \text{action.}\end{array}\right)$$

$$= 5 \times 200{,}000 \times p$$

where p is the probability that a person in the target audience will take the desired action.

The net economic gain to the advertiser, therefore, is the the economic gain less the cost incurred in advertising campaign.

Net Economic Gain =

$$\left(\begin{array}{l}\text{Contribution}\\ \text{per}\\ \text{person}\end{array}\right) \times \left(\begin{array}{l}\text{Number of}\\ \text{potential}\\ \text{buyers}\end{array}\right) \times \left(\begin{array}{l}\text{Probability that a}\\ \text{a potential buyer}\\ \text{will take the}\\ \text{desired action}\end{array}\right)$$

$$- \text{Total cost of the campaign.}$$

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[13] Strictly speaking we should have called this the average economic gain or the expected economic gain.

This yardstick may not find a very wide acceptance among the advertising experts. It would however find a greater degree of acceptance among marketing managers, who need the aid of such an economic indicator in their decision to commit expenditure for advertising.[14]

7.3. <u>The Approaches that can be adopted Immediately</u>

Any of the two yardsticks described above can be used in selecting vehicles within a medium. The main aim of this selection process would be to select the vehicles so as to optimise the results, measured either in terms of the probability of a person taking the desired action or the net economic gain.

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14 Stephen A Greyser of the Harvard Business School, in his foreward to the book "Cases in Advertising Communications Management in India", by Sen Gupta (38) remarks, "Of all the elements in the marketing mix, advertising combines the qualities of being most visible to the consumer and most baffling to other executives. Visible to the public because advertising is seen and/or heard by so many people every day. Baffling to other executives because advertising combines the mystique of successful creativity with/elusiveness of achieving precise measurement. Little wonder, then, that one of the most enduring concerns about advertising on the part of senior (non-advertising) management is that advertising funds are being wasted".

To design a selection procedure that will select the vehicles so as to optimise the results, we will need to set up a relationship between the probability of a person taking the desired action and the opportunities to see that were created in the advertising campaign. As a starting point, we propose a simple method of setting up this relationship. This method is based on the empirical work that has been done in United States and Britain, on the relationship between frequency and advertising effectiveness. The advantage of the proposed method is that it is based on empirical work and is simple to operationalise.

7.4. Relationship Between Frequency of Exposure and Advertising Effectiveness

The two major studies conducted abroad on the nature of the relationship between frequency of exposure (opportunities to see) and advertising effectiveness are :

a) An effective frequency pilot study by Colin McDonald (25) of the British Market Research Bureau Ltd., London.

b) Major Advertiser ADTEL study (35) in United States.

These two studies are described in some detail by Naples (27) The conclusions which emerge from these and similar studies form the basis of our proposed method for setting up the relationship between exposures to advertising and response. We therefore outline some of the major conclusions listed by Naples.

7.4.1. "One Exposure of an Advertisement to a target group Consumer within a purchase cycle has little or no effect in all but a minority of circumstances".

McDonald's study, as well as some of the other empirical studies conducted in United States agree that a single exposure provides only nominal advertising effect.

7.4.2. " Since one exposure is usually ineffective, the central goal of productive media planning should be to place emphasis on enhancing frequency rather than reach".

McDonald demonstrated effectively that scheduling for reach alone was inadequate. To quote McDonald, "One exposure has only a below average effect because it is not strong enough to overcome the competitive weight of the other brands on the housewife's list, for which she sees two or more exposures at the same time".

7.4.3. "The weight of evidence suggests strongly that an exposure frequency of two within a purchase cycle is an effective level".

In addition to the two studies mentioned above, based on some other studies conducted abroad, Naples draws the conclusion that, "It is often difficult to distinguish between advertising response at two or three exposure levels, suggesting that the lower number may be the more efficient target to aim for".

The strongest evidence in favour of two exposure effectiveness is offered by McDonald, since his study related directly to the purchase cycle. To quote McDonald "where a switch in brand occured on consecutive purchase occasions, the shopper was more likely to have been exposed to two advertisements than one for the brand switched to when people are making switch out of a brand, they are more likely, by 5 percentage points, to switch to the brand, (or less likely to switch from it) when, in the meantime, they have seen two or more advertisements for the brand".

The data in support of this assertion is furnished in Appendix VIII.

A word of caution in this respect is sounded by Naples:"It would be wrong to believe that such a finding would apply, without exception, to every category and every brand situation".

7,4.4. "<u>Beyond three exposures within a brand purchase cycle[15] or over a period of four or even eight weeks, increasing frequency continues to build advertising effectiveness at a decreasing rate, but with no evidence of a decline</u>".

McDonald's study did not show any declines upto four exposures within a purchase cycle. ADTEL study did not display any declines upto eleven or more exposures over four weeks.

The ADTEL study found that different probabilities of buying were associated with different household exposure levels. The study revealed that :

"User households show a sharp initial growth

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15 Here, purchase cycle refers to the period between two consecutive purchase occasions for the product.

and further steady increase in the probability of buying along with additional exposures.

Non user households show an initial growth, but very little growth in probability of buying with additional exposures".

7.4.5. "The frequency of exposure data from this review strongly suggests that wearout is not a function of too much frequency per se.

Frequency appears continually to enhance and/or maintain advertising effectiveness. However, too much frequency can be inefficient, considering that peak response can be achieved with as few as two or three exposures. It looks as though wearout and this may seem logical many - is strictly a copy or campaign content problem, and while excessive frequency can advance the decline of an effective campaign, frequency alone does not appear to cause declines".

7.4.6. "Large and Well known brands - and/or those with dominant market shares in their categories and dominant shares of category advertising weight - appear to differ markedly in response to frequency of exposure from smaller or more average brands".

In general, the smaller, less well known brands will virtually always benefit from frequency of exposure, while very large brands may, or may not, depending on how elose they are to advertising saturation levels. The supporting data for this conclusion is drawn from the ADTEL study.[15] Figures 7.1

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15 Naples (27), pp.49-50

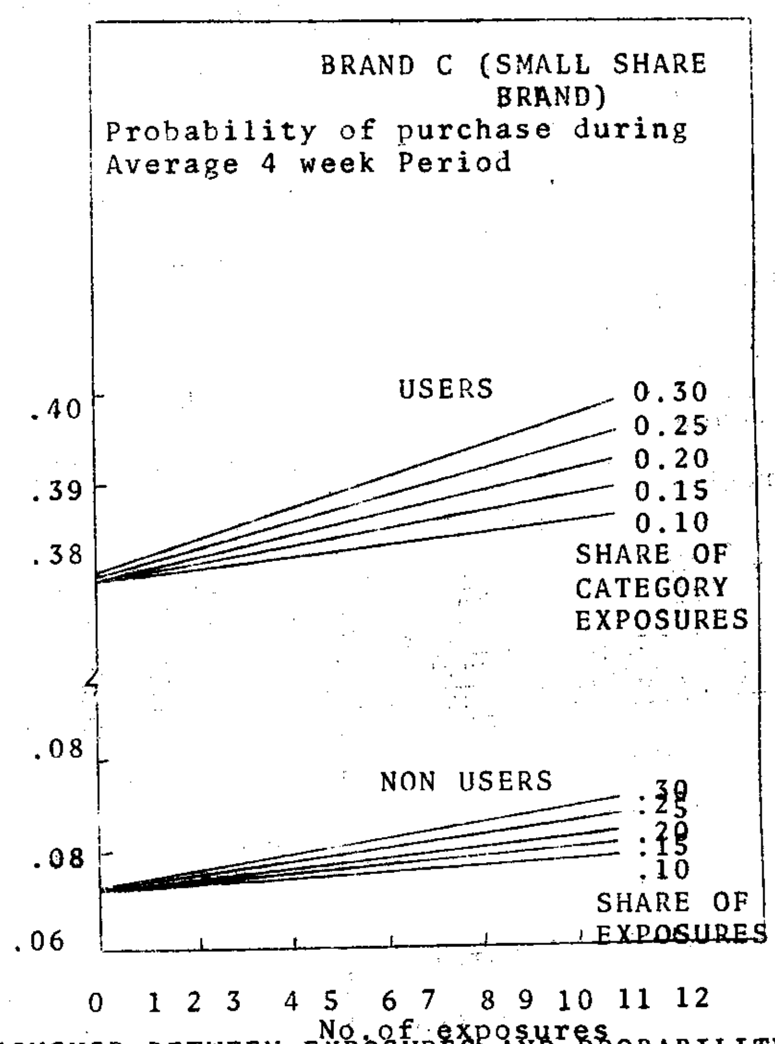

FIG 7.1 RELATIONSHIP BETWEEN EXPOSURES AND PROBABILITY OF PURCHASES FOR A SMALL SHARE BRAND

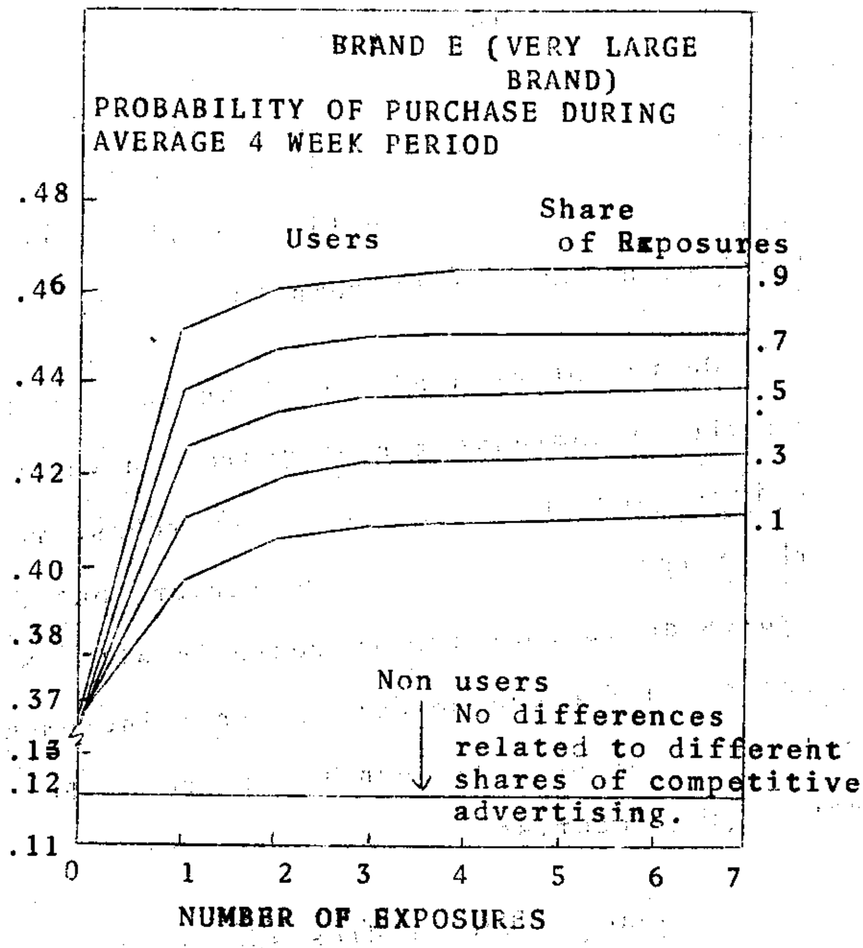

FIG. 7.2 RELATIONSHIP BETWEEN EXPOSURES AND PROBABILITY OF PURCHASE FOR A VERY LARGE BRAND

and 7.2 show that relationship between the probability of purchase and number of exposures for a small share brand C and a very large brand E.

In the case of a small share brand, as shown in fig 7.1 "one type of relationship is described by a mildly upward tilted straight line, where the increase in probability is not very large for each additional exposure. The rate of increase however remains nearly constant over a wide range of exposures. This type of response function, tends to be characteristic of a small brand such as C" remarks Naples.

On the other hand, as shown in figure 7.2, for a large brand with a dominant market share, in the words of Naples "The increases in probability of purchase resulting from each of the first few exposures are larger, but for incremental exposures above that the increase is nil. So, in this case whatever benefit can be derived from incremental advertising is derived quickly, as seen in figure 7.2. It represents a pattern characteristic of only very large share brands".

7.4.7. "None of the studies suggest that frequency response principles or generalisations vary by medium."

7.5. Stepping from Exposure to Response

7.5.1. Relationship Between Opportunities to See and Response

The conclusions (7.4.4) and (7.4.5) suggest that the response, measured in terms of the probability of a person taking the desired action, increases with the number of OTS (opportunities to see) received by a person, but at a decreasing rate. The nature of this relationship is shown in figure 7.3.

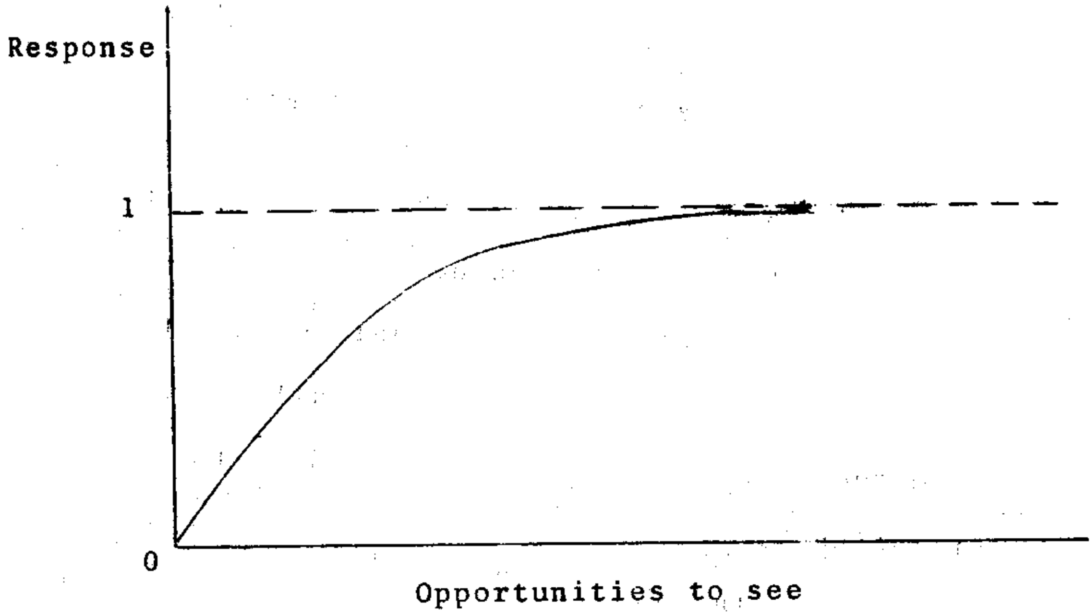

Figure 7.3.

RELATIONSHIP BETWEEN OPPORTUNITIES TO SEE AND RESPONSE

The relationship of the type shown in figure 7.3, can be expressed, in a simple way, as a mathematical relationship given by

$$p = 1 - A^{r}, \quad 0 < A < 1$$

where

r : Number of OTS received by a person,

P : Probability of a person who has received r OTS taking the desired action, and

A : a constant between zero and one, to be determined by the advertiser.

A relationship of the type shown in (7.5.1) is clearly revealed in the ADTEL studies, conducted for various brands, while the pioneering study done by McDonald does not explicitly show a relationship of this type. We shall now show that even the limited data available in McDonald's study, supports this relationship.

In 1966, McDonald carried out a diary study among housewives, from which, records of their purchasing sequences and opportunities to see (OTS) advertising for various brands were derived. The data for one respondent' s purchasing sequences for a brand of cereal, called Brand G and the OTS the person received, are shown in Table 7.1.

TABLE 7.1.

PURCHASE AND OTS FOR A BRAND G OF CEREALS

OTS in the Interval[16]	Number of Intervals	Number of interval s in which brand G was purchased
0	2	0
1	2	0
2	1	1
3	1	0

The data, expressed in the form of a Two by two table

16 An interval refers to a purchase cycle or the interval between two consecutive purchase occasions for the product.

(T... 7.2) brings out sharply McDonald's conclusion that an exp... re frequency of less tha two OTS, is ineffective.

TABLE 7.2.

CONDENSED OTS PURCHASE INFORMATION

	Lesst than two OTS	Two or more OTS	Total
Number of intervals after which purchased	0	1	1
Number of intervals after which not purchased	4	1	5
Total	4	2	6

Thus, from **Table** 7.2 , we see that in all the four intervals in which the person received less than two OTS, there were no purchases.

We now proceed to examine the validity of relationship (7.5.1) with the help of Table 7.1. According to the relationship (7.5.1) the probabilities that a person who has received zero OTS, one OTS, two OTS and three OTS, makes a purchase are given by 1-1, 1-A, $1-A^2$ and $1-A^3$ respectively. Let us denote by n_o the number of intervals with zero OTS.

n_1 the number of intervals with one OTS

n_2 the number of intervals with two OTS and

n_3 the number of intervals with three OTS in

Table 7.1. The average (expected) number of purchases will

therefore be given by

$$n_o(1-1) + n_1(1-A) + n_2(1-A^2) + n_3(1-A^3)$$

$$= 2(1-A) + (1-A^2) + (1-A^3)$$

We obtain the value of A, by equating the observed number of purchases with the expected nunber. It may be seen from Table 7.1 that the observed number of purchases is one. The equation that we obtain for estimating the value of A is given by

$$2(1-A) + (1-A^2) + (1-A^3) = 1,$$

$$\text{or } A^3 + A^2 + 2A-3 = 0 \qquad (7.5.2)$$

This equation has only one real root given by

$$A = 0.845$$

A therefore represents the probability of a housewife not making a purchase, when she receives exactly one OTS. There are two intervals during which the housewife received only one OTS. The probability of not buying in both the intervals, therefore, is given by the binomial expression

$$\binom{2}{2}(0.845)^2 = 0.72$$

The average (expected) number of intervals for which one OTS was received and no purchases were made is therefore

$$2(0.72) = 1.44$$

The average (expected) number is in close agreement with the observed number of intervals during which one OTS was received and no purchases were made. As has been shown in Table 7.1 the observed number of such intervals is two.

McDonald's conclusion that less than two OTS are ineffective and that advertising becomes more effective when the OTS received in a purchase cycle are two or more, is an important landmark in the field of media research. In our judgement, however, a media planner needs a measure to evaluate the effectiveness of less than two OTS versus that of OTS of two or more. In the absence of such a measure, it is difficult, if not impossible for a media planner, to decide how many OTS he should aim for. The relationship (7.5.1) provides us with this measure.

To see how this relationship provides us with such a measure, let us take six hypothetical purchase cycles such that in two of the intervals the housewife receives exactly one OTS, another two in which she receives two OTS and yet another two in which she receives three OTS. One of the measures that could be used to judge the effectiveness of one, two and three exposures is the probability of no purchase during the intervals in which one, two and three exposures were received. A related measure could be the probability of at least one purchase during the two intervals of each of these types. Taking the value of A as 0.845 with the same logic used earlier, we compute these probabilities, which are shown in Table 7.3.

TABLE 7.3.

PROBABILITIES OF NO PURCHASE AND AT LEAST ONE PURCHASE IN TWO INTERVALS WITH DIFFERENT OTS.

OTS received in each of the two intervals	Probability of no purchase after an interval of each type	Probability of At least one purchase after after either of the two intervals
1	0.72	0.28
2	0.52	0.48
3	0.36	0.64

As the table 7.3 shows , the probability of no purchases declines sharply as the number of OTS increase. Conversely, the probability of at least one purchase increases sharply with OTS. "How does this relationship (7.5.1) help us in deciding the advertising effort to be expended ?" is the next question to be answered. We shall discuss this question in Section 7.5.2.

7.5.2. MEDIA SELECTION SO AS TO MAXIMISE RESPONSE

Having set up a relationship between OTS and response, we now turn our attention to the problem of media selection. The best way to illustrate the practical utility of this approach will be to report here the results of some empirical work that we have done. We are reporting the results of using the net economic gain, as the criterion for selection.

As a part of our empirical work on media selection , Sarla Achuthan (3), was working on the problem of preparing an optimal cinema plan, for a manufacturer of infant foods. She prepared a cinema plan that costs the least, yet meets the desired advertising objectives and the scheduling constraints, given by the media planner. The advertising objectives were given in terms of reach, OTS per person and the OTS distribution. She prepared such a plan for the towns of interest to the advertiser in Gujarat region. We wanted to see whether the plan will change if the problem formulation was changed from that of finding a plan with the least cost to that of finding a plan which will maximise the net economic gain, as defined in Section 7.2.

We started with a low value of A = 0.1, which is indicative of very effective advertising. To see that A = 0.1 corresponds to a situation of very effective advertising, we note from relation (7.5.1) that the probability of a person who has received one OTS taking the desired action is given by,

$$1-0.1 = 0.9$$, a very high probability of purchase.

The purpose of starting with such a low value of A (assumption of very effective advertising_ was to see whether the size of the optimal plan prepared will be reduced because of this assumption. The highlights of the two plans, namely, the one that achieves the advertising objectives at the lowest cost and the other that maximises response, are shown in Tables 7.3 and 7.4. The plans were prepared for two selected towns in Gujarat, namely, Ahmedabad and Baroda.

Table 7.3.

CHARACTERISTICS OF THE TWO SELECTED TOWNS IN GUJARAT

CHARACTERISTICS	Towns AHMEDABAD	BARODA
1. Target audience	47,140	13,270
2. Potential candidate theatres.	51	18
3. Average cost per screening week (in rupees)	98	55

Table 7.4.

CINEMA PLANS FOR A LARGE MANUFACTURER OF INFANT FOODS USING THE TWO MODELS

(A = 0.1)

PARTICULARS	Minimum cost Model	Response Model
I. Town : AHMEDABAD		
a. Characteristics of the Plans		
1. Screening weeks	350	1224
2. Cost of the Plan (in lacs of rupees)	0.14	1.22
3. Average cost per screening week achieved in the plan (in rupees)	39	99

(Table 7.4 continued)

PARTICULARS	Minimum cost model	Response model
b. Achievements of the plans		
4. Reach	80%	94.4%
5. Frequency	4	11.9
6. Probability of response	0.78	0.94
7. Net Economic Gain (in lacs of rupees)	3.18	38.59
II. Town : BARODA		
a. Characteristics of the plans		
1. Screening weeks	140	432
2. Cost of the plan (in lacs of rupees)	0.05	0.24
3. Average cost per screening week achieved in the plan (in rupees)	36	56
b. Achievements of the plan		
4. Reach	83%	94.7%
5. Frequency	3.7	9.9
6. Probability of response	0.8	0.94
7. Net economic gain (in lacs of rupees)	0.91	11.01

The above tables show a very surprising result. Even with such effective advertising, the formal approach suggests that the advertising efforts should be increased very substantially. As can be seen from Table 7.4, the plan prepared so as to maximise response requires roughly three times more screening weeks, than those required in the minimum cost plan. The reasons for this are not far to seek. To illustrate, consider the case of the plan for the city of Ahmedabad. The size of the target audience in the city is 47,140 and the contribution from a potential buyer is Rs.90/-. Adding an extra screening week increases the probability of purchase, but on the other hand involves at the most an extra cost of Rs.98/-. Let us denote this increase in probability by f. So long as f is such that (47,140 x 90 x f) is greater that the average cost incurred because of an extra screening week, namely Rs.98/- it will be profitable to take that screening week. As can be seen, this condition implies that f should be greater that 0.00002. It so happens that the increase in the probability of purchase of a person, as a result of an extra screening week is much higher than 0.00002.

The formal approach recommends that the advertising effort therefore should be increased substantially. This analysis would suggest that the formal approach uses the well known principle of economics, that an action is optimal, when the marginal revenue from that action is equal to the marginal cost. In our approach, the advertising effort (screening weeks) is increased, until the marginal expected contribution is at least as high as the marginal cost of advertising (screening weeks). It seems reasonable therefore to increase the adverti-sing effort, if the contribution is high and the cost of advertising is low.

When these results were discussed with the user, he commented that the size of the target audience had perhaps been overestimated. He pointed out that the definition of the target audience perhaps needed some modification. The target audience was defined as women in the age group 15-45, income 500 plus and education SSC plus in the cities concerned. The target audience of interest however was women with children who would consume that infant food. In our judgement, the size of the target audience would be approximately ten per cent of the size that would be arrived at, by using the above mentioned definition of target audience. We therefore reworked the plan and the results are shown in Table 7.5.

TABLE 7.5.

CINEMA PLANS FOR A LARGE MANUFACTURER OF INFANT FOODS USING RESPONSE MODEL, WITH MODIFIED TARGET AUDIENCE SIZES

A = 0.1

Target Audience

Ahmedabad 4714
Baroda 1327

PARTICULARS	AHMEDABAD	BARODA
a. Characteristics of the Plan		
1. Screening weeks	808	284
2. Cost of the plan (in lacs of rupees)	0.36	0.10
3. Average cost per screening weeks achieved (in rupees)	45	36
b. Achievements of the plans		
4. Reach	91.5%	92%
5. Frequency	8	6.7
6. Probability of response	0.91	0.91
7. Net Economic Gain (in lacs of rupees)	3.48	0.99

It may be observed that even with the modified definition of the target audience, the Response Model still requires that advertising efforts should be doubled.

Even with a higher value of 0.3 for A, the results do not change substnatially. The cinema plans for Ahmedabad and Baroda with A = 0.3 are shown in Table 7.6.

TABLE 7.6

CINEMA PLANS FOR A LARGE MANUFACTURER OF INFANT FOODS

A = 0.3
Target audience:

Ahmedabad : 4714
Baroda : 1327

PARTICULARS	AHMEDABAD	BARODA
a. Characteristics of the Plan		
1. Screening weeks	812	288
2. Cost of the plan (in lacs of Rupees)	0.37	0.10
3. Average cost per screening week achieved by the plan (In Rs.)	45	36
b. Achievements of the plan		
Reach	91.5%	92.7%
Frequency	8	6.8
6. Probability of response	0.88	0.88
7. Net Economic Gain (in lacs of Rupees)	3.36	0.95

The use of this formal method, we believe, not only gives a practical aid to the media planner in deciding how much advertising effort should be expended, but also helps all those concerned with marketing the product, to think through in a systematic way about many complex factors that affect marketing the product.

FIGURE 7.4. FLEXIBILITY OF THE RESPONSE MODEL FOR DIFFERENT VALUES OF A

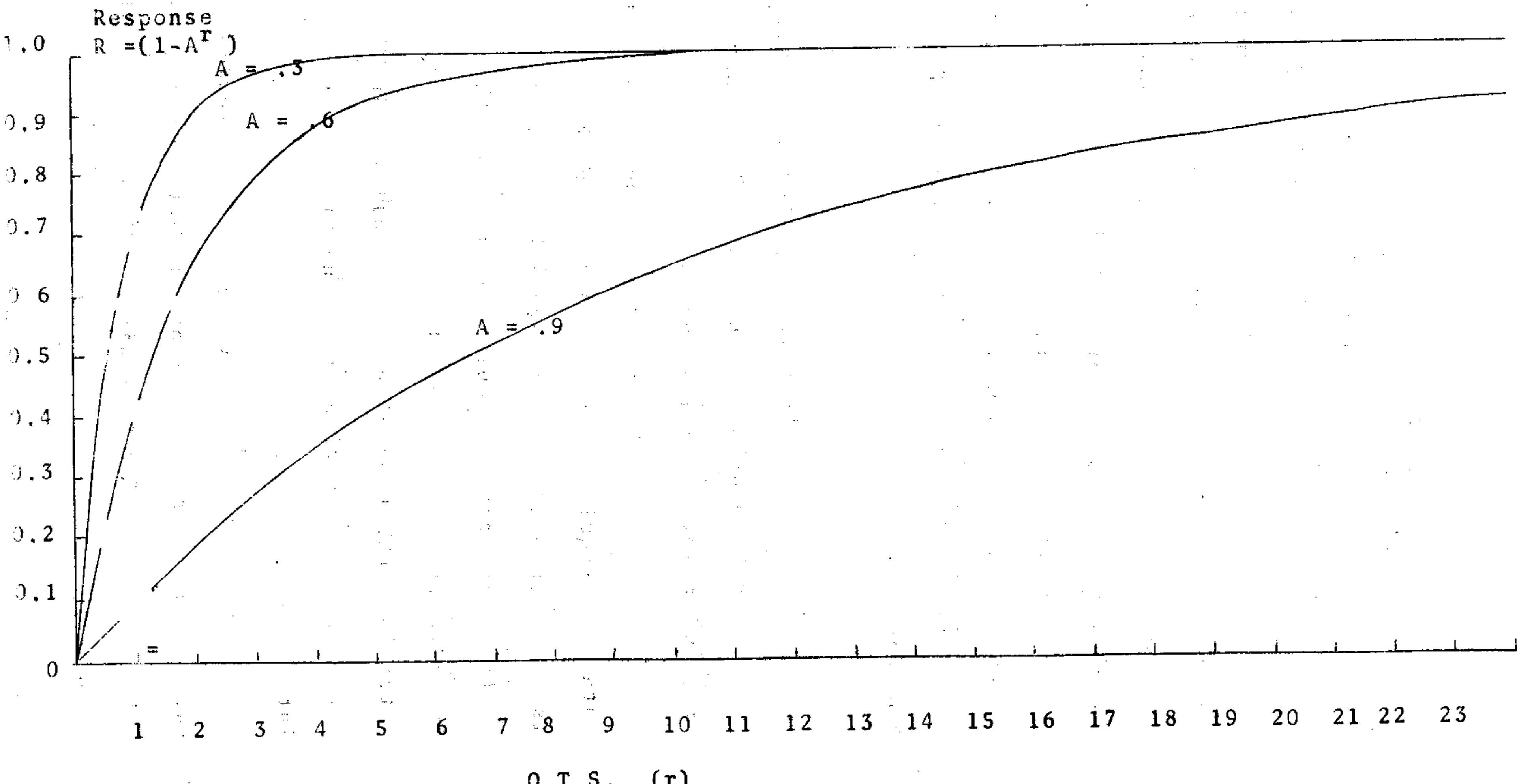

FIGURE 7.4. FLEXIBILITY OF THE RESPONSE MODEL FOR DIFFERENT

7.5.3. Flexibility of the Model

The relationship 7.5.1, which is the basis of the Response Model, offers us a great deal of flexibility in dealing with different brand positions and market conditions. We have shown in figure 7.4 three different types of response for different values of A. A low value of A generally indicates a brand which is the market leader and has the dominant market share, whereas a high value of A would correspond to a brand which is relatively new in the market and requires greater advertising efforts in order to create sufficient penetration and response.

It may be seen from figure 7.4 that when the value of A is as high as 0.9 even very high exposures do not lead to saturation levels of response.

A major advantage of the model is that once the data base required for our earlier Reach and Frequency models is set up,the additional data required is minimal. The only further information required is the subjective judgement of the advertiser regarding the value of A. Since the value of A determines the type of response, all the judgements which go into the determination of the nature of response, should go into determining A.

The ADTEL study gives us a few pointers regarding the marketing and communication factors which go into determining A. The following marketing factors would appear to be relevant.

i) Nature of the product (end use, cost etc.)
ii) Life cycle state of the product
iii) Brand dominance
iv) Brand loyalty
v) Purchase cycle
vi) Budget
vii) Competitive activity
viii) Target audience

The marketing manager's judgement on the above factors is required in the determination of A. However, marketing factors alone are not enough. Communication factors have to be considered in conjunction with marketing factors in order to fix the value of A. The basic communication factors which have to be taken in account are :

i) Message uniqueness
ii) Message complexity
iii) Message unit (half page ad, thirty second commercial etc.)
iv) Scheduling pattern
v) New versus continued campaign
vi) Wearout
vii) Media used

Advertising experts can provide their judgement on the above mentioned factors. This method therefore provides a more formal mechanism for linking advertising with the other elements of the marketing mix.

Mathematical procedures and inexpensive computer software have already been developed for developing a plan that maximises response in the medium of cinema. Similar procedures can be worked out for other media as well.

7.6. Directions for Future Research.

7.6.1 More complexity in the structure of the model

In section 7.5, we constructed the simplest form of response function, requiring minimum additional data. We can introduce more complexity into the structure, in order to model more effectively, the actual process involved in advertising communication. As a first step in this direction, we can introduce the following factors into the determination of the value of A.

a. Expected Net Effect of a Media Schedule

The effectiveness of an advertisement varies, depending on the copy approach used, and the vehicle in which the advertisement is inserted. This is because of the differences in editorial climate and vehicle audience for the different vehicles in the media schedule. When an individual is exposed to the advertisements in a media schedule a specific number of times, the net effect depends on the specific vehicles through which he was exposed to the advertisement. Aaker calls this net effect as the ' expected vehicle source effect' of the media schedule.

b. Probability of Actual exposure

Every person who has an opportunity to see an advertisement, does not necessarily get exposed to the advertisement as the person may or may not actually see the advertisement. For instance, a person who peruses the newspaper in a hurry in the morning may not actually see all the advertisements in the newspaper, though he gets an opportunity to see them. Thus, OTS is different from actual exposure.

The probability of actual exposure to the advertisement can be introduced into the model, instead of mere opportunities to see. It would seem more sensible to introduce this factor also in determining the value of A, instead of trying to derive actual exposures from OTS, as effective methods have already been developed to compute reach and frequency.

7.6.2. Transitional Probabilities and Opportunities to see

In section 7.5 we had set up a relationship between OTS and probability of the desired action. Such a relationship of course is very crude. Before a person takes the desired action, as currently accepted theory of advertising points out, he undergoes a transition from unawareness to awareness, comprehension, conviction and finally to the desired action. Conceptually, it will be more correct to set up a relationship between the probabilities of a person moving from one state to another and the OTS he receives. Though this ultimate goal is clear, its achievement is not very easy. The methodology for the assessment of these transitional probabilities will have to be developed in cooperation with communication experts. Having developed this methodology, the next task would be to develop the mathematical procedures and the computer software to develop a procedure so as to maximise response. This area offers considerable challenge to persons interested in inter-disciplinary work.

APPENDIX I

TOTAL ADVERTISING EXPENDITURE FOR 1976 - 77

ESTIMATED BY THE ADVERTISING AGENCIES ASSOCIATION OF INDIA

(Rupees in crores)

Sl.No.	Category of Media	1976	1977
1.	Press	60.00	67.50
2.	Cinema	7.00	7.50
3.	Radio	7.00	7.00
4.	Television	1.00	3.00
5.	Outdoor	11.00	10.00
6.	Expenditure on media selection (add items 1 - 5)	86.00	95.00
7.	Print	14.00	15.00
8.	Production	15.00	15.00
9.	Expenditure on creation of advertisements (add items 7 and 8)	29.00	30.00
10.	Total (additions 6 and 9	115.00	125.00

Source: <u>INFA Press & Advertisers Yearbook</u> (INFA Publications, New Delhi 1978-79)

Appendix II

TYPICAL FORMULATION OF THE LINEAR PROGRAMMING MODEL

a. MODEL BY KOTLER

Maximise :

$E = e_1X_1 + e_2X_2 + \ldots\ldots e_nX_n$ (objective function) (1)

subject to :

$a_1X_1 + a_2X_2 + \ldots. a_nX_n \leq B$ (budget constraint) (2)

$a_1Y_1 + a_2X_2 + \ldots. + a_nX_n \geq B_1$ (Media class usage constraint) and, (3)

$X_1 \geq K_{1L},$
$X_1 \leq K_{1U},$
$X_2 \geq K_{2L},$
$X_2 \leq K_{2U},$
.. ..
.. ..
$X_n \geq K_{nL},$ and
$X_n \leq K_{nU}$

(Individual medium usage constraints) (4)

where

E = Total exposure value (weighted number of exposures),

e_i = exposure value of medium i,

x_i = number of insertions taken in medium i,

a_i = cost per insertion in medium i,

B = total advertising budget,

B_1 = part of the advertising budget

K_{iL} = minimum number of insertions to purchase of medium i, and

K_{uL} = maximum number of insertions to purchase of medium i,

..contd..

The other variations of this approach are to take account of the size of the advertisement, the relative effectiveness of colour, the quantitative rating of the media etc., Such a formulation proposed by Brown and Warshaw is given below:

MODEL BY BROWN AND WARSHAW

Let N_i (i = 1, 2, ... n) be the number of times the ith advertising alternative is used per period.

Let the advertising budget be M dollars, and let the cost of one use of media i be c_i. Then the budget constraint is

$$C_1N_1 + C_2N_2 + C_3N_3 + \ldots\ldots + C_nN_n \leq M \qquad (1)$$

In addition, subjective constraints can be used which reflect management's conception of the limits to be placed on the media program. The objective is to maximise the number of effective exposures that can be attained, given the advertising budget :

Maximise :

$$Z = s_1c_1q_1e_1r_1N_1 + \ldots\ldots\ldots + s_nc_nq_ne_nr_nN_n \qquad (2)$$

with $(0 \leq s_i \leq 1)$, $(0 \leq c_i \leq 1)$, $(0 \leq q_i \leq 1)$ and $(0 \leq e_i \leq 1)$

where

s_i = the relative effectiveness of the size of the advertisement i, when compared with the largest sized advertisement under consideration, expressed as a decimal,

c_i = the relative effectiveness of the colour characteristics of the advertisement i, compared with the most effective colour advertisement available, expressed as a decimal.

..contd..

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q_i = qualitative characteristics rating coefficient of alternative i (e.g. appropriateness of editorial climate for the product or past ability to produce successful advertising readership)

e_i = effectiveness rating coefficient (importance weight times corresponding percent incidence in magazine readership), and

r_i = total readership for medium i.

Source:

a. Philip Kotler : Marketing Decision Making : A Model Building approach (Holt, Rinehart and Winston, Inc. New York, 1971), p.455.

b. Douglas B Brown and Martin R Warshaw, "Media Selection /by Linear Programming", Journal of Marketing Research, Vol.2, February 1965, pp.83-88.

APPENDIX III

MATHEMATICAL REASONING LEADING TO ZANGWILL'S MODEL

1.1. Introduction An explicit reasoning is not provided in Zangwill's paper for using the objective function that he has used. The purpose of this note is to provide a reasoning that will lead to the formulation of an objective function similar to the one used by Zangwill. The need for providing such an explicit reasoning was to highlight the assumptions under which the use of the objective function can be justified. These assumptions have not been clearly stated in the paper.

1.2. Notation

1. i : index for media selection (option) $i = 1, 2, 3, \ldots M$,
2. j : index for choice within a selection,
3. n_i : number of choices within the i^{th} selection,
4. n_{ij} : number of insertions in the j^{th} choice of the i^{th} selection,
5. x_{ij} : 1 if the j^{th} choice within the i^{th} selection is chosen, where $j = 0, 1, 2, 3, \ldots n_i$, 0 otherwise, and

   x_{io} : not choosing the i^{th} selection,
6. a : index for age group $a = 1,2,3,4$,

7. s : index for sex, s = 1, 2,

8. l : index for income group I = 1, 2, 3, 4, 5,

9. r : index for region, r = 1, 2, 3,

10. t : index for (a,s,I,r), t = 1, 2, 3, . . . 120 as there are 120 segments (a,s,I,r) and

11. F(t,i,j) rated effectiveness of the j^{th} choice within the i^{th} selection for a person belonging to the t^{th} segment.

1.3. Assumptions required for the Formulation of an Objective Function Similar to the One used by Zangwill

Assumption 1 (A_1) : A person in the audience of any selection is also in the audience of all the issues of that selection.

$$p(j/i) = 1, \qquad (1)$$

where p(j/i) is the probability (conditional) that a reader of the i^{th} publication is in the audience of the j^{th} choice. By virtue of this assumption, we have,

$$p(i,j) = p(j/i) \;\; p(i) = p(i),$$

where :

p(i,j) : probability that a reader is both in the audience of the i^{th} selection and the j^{th} choice within the i^{th} selection, and p(i) : probability that a reader is in the audience of the i^{th} selection.

Hence A_1 states that a reader of the i^{th} publication ($i = 1, 2, 3, ...M$) reads all the issues of that publication.

Assumption 2 (A_2) The rated effectiveness of a choice within a selection depends only on i) the segment in which it is read, and ii) n_{ij} .

$$F(t,i,j) = n_{ij} \quad v(t) \tag{2}$$

where v(t) is the sales potential weight (value) of a person belonging to the t^{th} segment.

Assumption 3 (A_3) : v(t) has a special structure.

$$v(t) = v(a,s,I,r) = \{\lambda_1 \; v(a,s) \; +\lambda_2 v(I)\} \quad v(r) \tag{3}$$

where :

v(a,s) : sales potential weight of a person belonging to the age group a and sex s,

v(I) : sales potential weight of a person belonging to the income group I,

v(r) : sales potential weight of a person belonging to the region r, and

λ_1 and λ_2 : constants to be determined by the advertiser, such that

$\lambda_1 + \lambda_2 = 1$, and

$\lambda_1 \geq 0$ & $\lambda_2 \geq 0$.

ASSUMPTION 4 (A_4) : Special structure for the conditional probability that a person belongs to the t^{th} segment given that he is in the audience of the i^{th} publication.

$$p(t/i) = p(a,s,I,r/i) = p(a,s/i)\, p(I/i)\, p(r/i) \quad . \qquad (4)$$

where :

$p(t/i)$: probability (conditional) that a person in the audience of the i^{th} selection belongs to the t^{th} segment,

$p(a.s/i)$: probability (conditional) that a person in the audience of the i^{th} selection belongs the age group a and sex s,

$p(I/i)$: probability (conditional) that a person in the audience of the i^{th} selection belongs to the income group I, and

$p(r/i)$: probability (conditional) that a person in the audience of the i^{th} selection belongs to the region r.

This assumption states that a,s,I and r are conditionally independent.

ASSUMPTION 5 (A_5) : The response generated by a media plan from a person in the t^{th} segment is a quadratic function of the rated effectiveness value generated by the plan in the t^{th} segment.

$$R(t,x_{ij}) = y(t,x_{ij}) - \{y(t,x_{ij}) - k(t,x_{ij})\}^2 \qquad (5)$$

where :

$R(t,x_{ij})$: response from a person in the t^{th} segment to a feasible media plan $\{x_{ij}\}$,

$y(t,x_{ij})$: rated effectiveness level generated by a feasible media plan $\{x_{ij}\}$ from a person in the t^{th} segment, and

$k(t,x_{ij})$: $E\{y(t,x_{ij})\}$, expectation of $y(t,x_{ij})$.

ASSUMPTION 6 (A_6) The media plan is to be chosen so as to maximise the expected value of the total response

Let

$$R(x_{ij}) = E[\sum_t \{y(t,x_{ij}) - [y(t,x_{ij}) - k(t,x_{ij})]^2\} N_r] \qquad (6)$$

where

$R(x_{ij})$ denotes the expected total response to the media plan $\{x_{ij}\}$, and

N_r denotes the number of people in the region r.

The objective is to choose the x_{ij}'s so as to maximise $R(x_{ij})$.

1.4. PROOF

We are required to prove that $R(x_{ij})$ is of the form

$$\sum_{ij} R_{ij}\, x_{ij} - \sum_{ijhk} x_{ij}\, r_{(ij)(hk)}\, x_{hk}$$

To prove this, we will first evaluate $y(t,x_{ij})$, and then $E\{y(t,x_{ij})\}$ and $E\{y(t,x_{ij}) - k(t,x_{ij})\}^2$. Having evaluated these terms, it is a simple matter to evaluate

$$R(x_{ij}) = \sum_t [E\{y(t,x_{ij})\} - E\{y(t,x_{ij}) - k(t,x_{ij})\}^2] N_r$$

1.4.1. Evaluation of $y(t,x_{ij})$ and $E\{y(t,x_{ij})\}$

Let

z_{ti} = 1, if a person belongs to the t^{th} segment and is in the audience of the i^{th} selection,

= 0, otherwise.

Then,

$$y(t,x_{ij}) = \sum_{ij}\sum F(t,i,j)\ z_{ti}\ x_{ij}$$

$$= \sum_{ij}\sum v(t)\ n_{ij}\ z_{ti}\ x_{ij}$$

by virtue of A_2.

Let

$$D_i = \sum_j n_{ij}\ x_{ij}$$

Then

$$y(t,x_{ij}) = \sum_i v(t)\ D_i\ z_{ti} \tag{7}$$

$$= \sum_i v(a,s,I,r)\ D_i\ z_{ti}$$

$$= \sum_i \{\lambda_1\ v(a,s) + \lambda_2\ v(I)\}\ v(r)\ D_i\ z_{ti}$$

by virtue of A_3.

Hence

$$E\{y(t,x_{ij})\} = \sum_i \{\lambda_1 v(a,s) + \lambda_2 v(I)\} v(r) D_i E(z_{ti}) \qquad (8)$$

Let p_i^t probability that a person belongs to the t^{th} segment and is in the audience of the i^{th} selection.

Then,

$$E(z_{ti}) = \{1.p_i^t\} + \{0 \cdot (1-p_i^t)\}$$

$$= p_i^t$$

$$= p(t/i)\, p(i),$$

by the definition of joint probability.

$$= p(a,s,I,r/i)\;\; p(i)$$

$$= p(a,s/i)\, p(I/i)\, p(r/i)\, p(i)$$

by virtue of A_4.

Hence from (8)

$$E\{y(t,x_{ij})\} = \sum_i \{\lambda_1 v(a,s) + \lambda_2 v(I)\} v(r) D_i \; p(a,s/i)\, p(I/i)\, p(r/i)\, p(i) \qquad (9)$$

1.4.2. Evaluation of $\sum_t E\{y(t,x_{ij})\} N_r$

From (9) we obtain

$$\sum_t E\{y(t,x_{ij})\} N_r$$

$$\sum_a \sum_s \sum_I \sum_r \sum_i \{\lambda_1 v(a,s) + \lambda_2 v(I)\} v(r) D_i\, p(a,s/i)\, p(I/i)\, p(r/i) p(i)\, N_r \qquad (10)$$

Since the summation is over a finite number of terms, the summation can be interchanged. Hence (10) can be written as

$$\sum_t E\{y(t,x_{ij})\}\ N_r$$

$$= \sum_{i}\sum_{r}\{D_i N_r p(i)p(r/i)v(r)\}\left[\sum_{a}\sum_{s}\sum_{I}\{\lambda_1 v(a,s) + \lambda_2 v(I)\}p(a,s/i)p(I/i)\right] \tag{11}$$

We now evaluate,

$$W_{1i} = \sum_{a}\sum_{s}\sum_{I}\{\lambda_1 v(a,s) + \lambda_2\ v(I)\}\ p(a,s/i)\ p(I/i) \tag{12}$$

This expression reduces to,

$$W_{1i} = \lambda_1 \sum_{a}\sum_{s} v(a,s)p(a,s/i) + \lambda_2 \sum_{I} v(I)\ p(I/i) \tag{13}$$

Since $\sum_I p(I/i) = \sum_a\sum_s p(a,s/i) = 1$

Hence from (11)

$$\sum_t E\{y(t,x_{ij})\}\ N_r = \sum_i\sum_r N_r\ p(i)\ p(r/i)\ v(r)\ \ W_{1i}\ D_i \tag{14}$$

Let $W_{2i} = \sum_r N_r\ p(i)\ p(r/i)\ v(r)$ (15)

Now $p(i)\ p(r/i) = p(i,r)$ and is the joint probability that a person belongs to the region r and is in the audience of the i^{th} selection.

Hence,

$$W_{2i} = \sum_r N_r\ p(r,i)\ v(r) \tag{16}$$

We note that $N_r\ p(r,i)$ represents the expected number of people in the region r, who are in the audience of the i^{th} selection.

Let,

$$C(r,i) = N_r\ p(r,i) \tag{17}$$

Hence from (16)

$$W_{2i} = \sum_r C(r,i)\ v(r) \tag{18}$$

Equation (14) can now be written as

$$\sum_t E\{y(t,x_{ij})\}\ N_r = \sum_i D_i\ W_{1i}\ W_{2i} \tag{19}$$

1.4.3. Evaluation of $\sum_t E\{y(t,x_{ij}) - K(t,x_{ij})\}^2$

Since,

$$K(t,x_{ij}) = E\{y(t,x_{ij})\},$$

it follows that

$$E\{y(t,x_{ij}) - k(t,x_{ij})\}^2 = V\{y(t,x_{ij})\}$$

where,

$V\{y(t,x_{ij})\}$ denotes the variance of the random variable $y(t,x_{ij})$.

To find $v\{y(t,x_{ij})\}$ we make note of the fact that $y(t,x_{ij})$ is a linear combination of the random variables z_{ti}.

We express (7) as

$$y(t,x_{ij}) = v(t)\ \underline{D}'(1 \times M)\underline{Z}_t(M \times 1) \tag{20}$$

where

$\underline{D}'\ (1 \times M) = \{D_1, D_2\ \ldots\ldots\ D_M\}$, and

$$\underline{Z}_t\ (M \times 1) = \begin{bmatrix} z_{t1} \\ z_{t2} \\ z_{t3} \\ \cdot\cdot \\ \cdot\cdot \\ z_{tM} \end{bmatrix}$$

Hence $v\{y(t,x_{ij})\} = \{v(t)\}^2 \; D'\Sigma D$

where Σ is the variance - covariance matrix of the random variable

$$\underline{Z_t} = \begin{bmatrix} z_{t1} \\ z_{t2} \\ \dots \\ \dots \\ z_{tM} \end{bmatrix}$$

It can be easily seen that

$$\Sigma = \begin{bmatrix} p_1^{\ t}(1-p^t_{\ 1}) & p_{12}^{\ t} - p^t_{\ 1}p_2^{\ t} & \dots\dots\dots\dots & p_{1M}^{\ t}-p^t_{\ 1}p_M^{\ t} \\ p_{21}^{\ t}-p_2^{\ t}p_1^{\ t} & p^t_{\ 2}(1-p_2^{\ t}) & \dots\dots\dots\dots & p_{2M}^{\ t}-p^t_{\ 2}p_M^{\ t} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ p^t_{\ M1}-p^t_{\ M}p_1^{\ t} & p_{M2}^{\ t}-p^t_{\ M}p_2^{\ t} & \dots\dots\dots\dots & p^t_{\ M}(1-p_M^{\ t}) \end{bmatrix} \quad (21)$$

where

$p_{ik}^{\ t}$: probability that a person belongs to the t^{th} segment and is in the common audience of the i^{th} and the k^{th} in publication.

Hence

$$v\{y(t,x_{ij})\} = \{v(t)\}^2 \{ \sum_{i=1}^{M} D_i^{\ 2} \; p_i^{\ t}(1-p_i^{\ t}) + 2 \sum_{i=1}^{M-1} \sum_{h=i+1}^{M} D_i D_h (p_{ih}^{\ t} - p_i^{\ t} p_h^{\ t})$$

Hence

$$\sum_t v\{y(t,x_{ij})\} N_r = \sum_{i=1}^{M} D_i^2 \sum_t \{v(t)\}^2 p^t_i (1-p^t_i) N_r$$

$$+ 2 \sum_{i=1}^{M-1} \sum_{h=i+1}^{M} \sum_t \{v(t)\}^2 D_i D_h \{p^t_{ih} - p^t_i p^h_i\} N_r \qquad (22)$$

Note that

$$D_i D_h = \sum_j x_{ij} n_{ijk} \sum x_{hk} n_{hk}$$

Hence

$$2 \sum_{i=1}^{M-1} \sum_{h=i+1}^{M} \sum_t \{v(t)\}^2 D_i D_h (p^t_{ih} - p^t_i p^t_h) N_r$$

$$= 2 \sum_{i=1}^{M-1} \sum_{h=i+1}^{M} \sum_t \{v(t)\}^2 (p^t_{ih} - p^t_i p_h^t) N_r \sum_{jk}\sum x_{ij} x_{hk} n_{ij} n_{hk}$$

$$= \sum_{i=1}^{M-1} \sum_{h=i+1}^{M} \sum_j \sum_k x_{ij} x_{hk} \sum_t 2\{v(t)\}^2 (p^t_{ih} - p_i^t p_h^t) n_{ij} n_{hk} N_r$$

$$= \sum_{i=1}^{M-1} \sum_{h=i+1}^{M} \sum_j \sum_k r_{(ij)(hk)} x_{ij} x_{hk} \qquad (23)$$

where

$$r_{(ij)(hk)} = 2\sum_t \{v(t)\}^2 (p^t_{ih} - p_i^t p^t_h) n_{ij} n_{hk} N_r$$

The expression for $R(x_{ij})$ can now be written using (6), (19), (22) and (23)

$$R(x_{ij}) = \sum_i D_i W_{1i} W_{2i} - \sum_{i=1}^{M} D_i^2 \sum_t \{v(t)\}^2 p_i^t (1-p^t_i) N_r$$

$$- \sum_{i=1}^{M-1} \sum_{h=+1}^{M} \sum_j \sum_k r_{(ij)(hk)} x_{ij} x_{hk} \qquad (24)$$

Note that $D_i = \sum_j n_{ij} x_{ij}$

Hence

$$\sum_i D_i W_{1i} W_{2i} = \sum_i \sum_j n_{ij} x_{ij} W_{1i} W_{2i}$$

Let

$$R_{ij} = n_{ij} W_{1i} W_{2i}$$

Then

$$\sum_i D_i W_{1i} W_{2i} = \sum_i \sum_j R_{ij} x_{ij} \quad (25)$$

Hence from (24) and (25) we get

$$R(x_{ij}) = \sum_{i=1}^{M} \sum_{j=0}^{n_i} R_{ij} x_{ij} - \sum_{i=1}^{M} D_i^2 \sum_t \{v(t)\}^2 p_i^t (1-p_i^t) N_r$$

$$- \sum_{i=1}^{M-1} \sum_{h=i+1}^{M} \sum_{j=0}^{n_i} \sum_{k=0}^{n_h} r_{(ij)(hk)} x_{ij} x_{hk} \quad (26)$$

It can now be seen that the above expression is very similar to the objective function that Zangwill has used. The expression for the objective function used by Zangwill is:

$$R = \sum_{ij} R_{ij} x_{ij} - \sum_{ijhk} r_{(ij)(hk)} x_{ij} x_{hk} \quad (27)$$

The difference is that the expression for $R(x_{ij})$ in (25) contains the term

$$- \sum_{i=1}^{M} D_i^2 [\sum_t \{v(t)\}^2 p_i^t (1-p_i^t) N_r]$$

which is not present in the objective function used by Zangwill.

All that remains to be shown is that $\frac{R_{ij}}{n_{ij}}$ is what Zangwill derives as the rated effectiveness of the i^{th} selection. In order to do this, let us examine the expressions for W_{1i} and W_{2i}.

From (13),

$$W_{1i} = \lambda_1 \sum_a \sum_s v(a,s)\ p(a,s/i) + \lambda_2 \sum_I v(I)\ p(I/i)$$

This expression for W_{1i} agrees with the computations done by Zangwill to obtain the combined weight of the age, sex and income factors.

From (17)

$$W_{2i} = \sum_r C(r,i)\ v(r)$$

This expression is similar to the regional weight used by Zangwill, except that $C(r,i)$ represents the expected number of people in the region r, who are in the audience of the i^{th} selection, whereas Zangwill uses the circulation figures of the i^{th} selection in the r^{th} region.

In our view, it is not correct to use the data on circulation, in the objective function, as Zangwill has used. Circulation only represents the copies that are sold. In the problem on hand our interest is in the expected number of persons who read the issue

The expression for $\frac{R_{ij}}{n_{ij}}$ agrees with the one used by Zangwill, except for the difference in the use of circulation. We illustrate the computations for W_{1i} from the example that Zangwill has provided.

ILLUSTRATION FOR THE CALCULATION OF RATED EFFECTIVENESS GIVEN BY ZANGWILL

The product advertised is a full line of quality sunglasses. The target audience is divided into four age groups, five income groups and three regions.

The sale potential values of the different segments of interest are furnished by the advertiser as follows :

TABLE 1

VALUES OF v(a,s), v(I), and v(r)

AGE GROUP	v(a,s) MALE	FEMALE	INCOME GROUP (I)	v (I)	REGION (r)	v(r)
10-17	1.05	1.20	Below $ 3000	.70	Los Angeles Latitude	1.30
18-34	1.05	1.20	$3000 - $ 5000	.85		
35-54	.95	1.05	$5000 - $ 7000	1.00	San Francisco Latitude	1.10
55 and above	.95	1.00	$ 7000 - $10000	1.10		
			Above $ 10000	1.20	Farther north	1.00

TABLE 2

LIFE MAGAZINE DISTRIBUTION p(a,s/i)

Age	Male	Female
10-17	10.5	9.7
18-34	11.8	14.8
35-54	18.4	18.9
55 and above	8.0	8.0
Percent of readers	48.7	51.3

Weighing the proportion of readers in the different age sex groups, obtained from table 2 by the relative v(a,s) values from table 1 and adding gives a combined age sex weight of 1.06.

Thus

$$\sum_{a,s} v(a,s)\, p(a,s/i) = 1.04,$$

A similar determination for income yields

$$\sum_{I} p(I/i)\, v(I) = 1.08$$

The relative importance of the income factor versus age sex factor is judged to be .475 to .525. Thus,

$$\lambda_1 = .475 \text{ and } \lambda_2 = .525$$

Hence from (12)

$$W_{1i} = \lambda_1 \sum_a \sum_s v(a,s)p(a,s/i) + \lambda_2 \sum_I v(I)p(I/i)$$

$$= (.475)(1.08) + (.525)(1.04)$$

$$= 1.06$$

Zangwill weights the Life circulation figure in each region by the v(r) values in table 1, to obtain an adjusted circulation figure 1,425,000. Thus,

$$W_{2i} = C(r,i)\ v(r) = 1,425,000.$$

C(r,i) in our equation (17) stands for the number of people in the region r, who are in the audience of the i^{th} selection, whereas Zangwill uses circulation figure of the i^{th} selection in region r for C(r,i). This may not be completely correct.

$$W_{1i} \cdot W_{2i} = (1.06)(1,425,000) = 1,510,500.$$

Thus the rated effectiveness of one issue of Life Magazine is 1,525,000/-.

For the choice four issues of Life, rhe rated effectiveness is given by

$$R_{ij} = n_{ij} \ W_{1i} \ W_{2i}$$
$$= 4 \times 1,510,500$$
$$= 6,042,000$$

APPENDIX IV

MODEL BY PARSONS

The elasticity of advertising is defined as :

$$\frac{dS}{dA} \times \frac{A}{S}$$

where

S = Sales, and

A = Advertising expenditure

Notations: S_t = Sales at time period t,

$S_{(t-1)}$ = Sales at time period (t-1)

A_t = Advertising expenditure at time period t,

P_t = General price index at time period t and

$\alpha_1, \alpha_2, \alpha_3$, β_1, β_2, and β_3 are constants to be estimated.

a. CONSTANT ELASTICITY MODEL

The co stant elasticity mo el fitted to the data is :

$$\ln S_t = \alpha_1 + \alpha_2 \ln (A_t/P_t) + \alpha_3 \ln S_{(t-1)} \quad (1)$$

Differentiating both the sides with respect to A_t/P_t

$$\frac{1}{S_t} \frac{dS_t}{d(A_t/P_t)} = \frac{\alpha 2}{(A_t/P_t)}$$

..contd..

$$\frac{dS_t}{d(A_t/P_t)} \quad \frac{(A_t/P_t)}{S_t} = \alpha_2$$

Thus elasticity of advertising at time t

$$= \frac{dS_t}{d(A_t/P_t)} \quad \frac{(A_t/P_t)}{S_t}$$

$= \alpha_2$, a constant.

Hence, in the constant elasticity model, the elasticity of advertising is assumed to be constant throughout the product life cycle.

b. TIME VARYING ELASTICITY MODEL

The time varying elasticity model fitted to the data is

$$\ln S_t = \beta_1 e^{-\beta_2 t} \ln (A_t/P_t) + \beta_3(1-e^{-\beta_2 t}) \ln S_{t-1} \quad (2)$$

Differentiating both the sides with respect to A_t/P_t

$$\frac{1}{S_t} \quad \frac{dS_t}{d(A_t/P_t)} = \frac{\beta_1 e^{-\beta_2 t}}{(A_t/P_t)}$$

$$\frac{dS_t}{d(A_t/P_t)} \quad \frac{A_t/P_t}{S_t} = \beta_1 e^{-\beta_2 t}$$

..contd..

Thus, the elasticity of advertising at time t

$$= \frac{dS_t}{d(A_t/P_t)} \cdot \frac{A_t/P_t}{S_t} = \beta_1 e^{-\beta_2 t}$$

Hence, in the time varying elasticity model the elasticity of advertising is expressed as a function of time and is assumed to change over a period of time.

APPENDIX V

RASMUSSEN'S MODEL FOR DETERMINATION OF ADVERTISING EXPENDITURE

Notations :

q = quantity demanded

p = price per unit,

v = variable cost per unit, excluding advertising expenditure

A = expenditure in advertising, and

e = elasticity of advertising,

By the assumption of constant elasticity, we have

$$\frac{dq}{dA} \; \frac{A}{q} = e \qquad (1)$$

where e is a constant.

Solving this differential equation, we obtain

$$q = mA^e \qquad (2)$$

where m is another constant. It should be noted that $m > 0$ since $q > 0$ and $A > 0$. Consider the profit function

$$\Pi(A) = (p-v)q-A \qquad (3)$$

We wish to choose A so as to maximise Π.

By the assumption that p and v are constants, not depending upon A, we have

$$\frac{d\Pi}{dA} = (p-v)\frac{dq}{dA} - 1 \qquad (4)$$

The first order condition for $\Pi(A)$ to be maximum is

$$(p-v)\frac{dq}{dA} - 1 = 0$$

contd..

$$(p-v)\frac{dq}{dA} = 1$$

From (2), we obtain

$$(p-v)\, emA^{e-1} = 1 \qquad (5)$$

Substituting from (2)

$$(p-v)e\frac{q}{A} = 1$$

$$q(p-v)e = A$$

$$qp(1-v/p)e = A$$

$A = kqp$ where $k=(1-v/p)e$

The second order condition for ¶(A) to be maximum requires that

$$(p-v)\frac{d^2q}{dA^2} < 0$$

$$(p-v)me(e-1)A^{e-2} < 0$$

which requires that

$p > v \quad m > 0 \quad$ and $\quad e < 1$

Thus under the assumptions of the model, the profit function ¶ (A) is maximum when

$A = kpq$

which means that advertising expenditure should be a constant proportion of sales.

APPENDIX VI

MATHEMATICAL REASONING DEVELOPED IN THE MEDIAC MODEL

Let

M : Number of media options under consideration,

T : Number of time periods in the planning horizon,

S : Number of market segments,

X_{jt} = 1 if an insertion is made in option j in time period t,

0, otherwise, and

Z_{ijt} = 1, if the person in segment i is exposed to an insertion in media option j, in time period t,

0, otherwise.

The probability distribution of Z_{ijt} is determined by media exposure probabilities and whether or not an advertisement is placed.

E_{ij} denotes the value of an exposure in media vehicle j to a person in market segment i.

$\sum_{j=1}^{M} E_{ij} Z_{ijt}$ is the increase in the exposure level of a particular individual in market segment i in time period t.

Y_{it} denotes the exposure level of a particular individual in market segment i in time period t.

...comtd..

Then

$$Y_{it} = \alpha Y_{i,t-1} + \sum_{j=1}^{M} E_{ij} Z_{ijt} \qquad \ldots\ldots\ldots \qquad (1)$$

where α is a memory constant

Let

N_i = Number of people in market segment i, and

W_{it} = Sales potential of a person in the market segment i in time period t.

The equation (1) will reduce to

$$Y_{it} = \sum_{s=-\infty}^{T} \sum_{j=1}^{M} \alpha^{t-s} E_{ij} Z_{ijs}$$

by taking infinite past period.

Having developed an expression for the exposure level of a particular individual, the authors develop the response function. Response from an individual belonging to a particular segment at a given point of time is measured in terms of his sales potential. Let $r(Y_{it})$ be the response from an individual belonging to the i^{th} segment in time period t, whose exposure level is Y_{it}.

Total response is the sum of individual responses over the entire time span under considera- tion. It is obvious that total response is a random variable, since Y_{it} is a random variable. $f_{it}(.)$ denotes the probability density of function of Y_{it}. Total expected market response R is given by

$$R = \sum_{i=1}^{S} \sum_{t=1}^{T} n_i W_{it} E\{r(Y_{it})\} \qquad (2)$$

The choice of the function r(y) is left to the users' judgement. In this paper, the authors have used an exponential function, which is of the form

$$r(y) = r_0 + a(1-e^{-by}) \qquad (3)$$

$$0 \leq y < \infty$$

where r_0, a and b are non negative constants specific to the product.

The reason for this choice is that the graph of this function increases at a decreasing rate.

The authors have then expanded the function r(y) in a Taylor series about the expected value of Y_{it}. Such an assumption is valid only if the variables Y_{it} are continuous. In this case for Y_{it} to be continuous E_{ij}'s must be continuous. Otherwise the response function falls in the category of step functions and hence not differentiable. In such a case, the use of Taylor series as done by the authors is not permissble. For ease of expression, dropping the subscripts for time and segment and using the Taylor's expansion, we get,

$$r(y) = r(\mu) + \sum_{k=1}^{n-1} \frac{1}{k!} r^k(\mu)(y-\mu)^k + \ldots \qquad (4)$$

$$+ \frac{1}{n!} r^n(y_1) \ (y-\mu)^n$$

..contd..

where $r^k(\mu)$ is the k^{th} derivative of $r(y)$ evaluated at $y = \mu$ and y_1 is some value between y and μ

By taking expectation,

$$E\{r(y)\} = r(\mu) + \sum_{k=2}^{n-1} \frac{1}{k!} r^k(\mu)\mu_k + \frac{1}{n!} E\{ r^n(y_1)(y-\mu)^n\} \quad (5)$$

It may be noted that,

$\mu = E(y) =$ mean of y

$\mu_n = E\{(y-\mu)^n\} = n^{th}$ moment of y about the mean.

The equation (5) can be written as

$$E\{r(y)\} = r(\mu) + \frac{1}{2!} r^{(2)}(\mu)\mu_2 + \sum_{k=3}^{n-1} \frac{r^k(\mu)}{k!}\mu_k + \frac{1}{n!}E\{ r^n(y_1)(y-\mu)^n\} \quad \ldots\ldots\ldots\ldots \quad (6)$$

The authors then express the **moments** of the exposure level distribution in terms of the media decisions X_{jt} and the probability of a person in the target audience being exposed to individual and pairs of options.

Now,

$$y = \sum_{j=1}^{M} e_j z_j \text{ , where}$$

$$Z_j = \begin{cases} x_j, & \text{if the individual is exposed to option } j \\ 0, & \text{otherwise.} \end{cases}$$

Let $p_j = P(Z_j = 1) =$ probability that a person in the target audience is exposed to option j.

Hence

$$E(zj) = x_j p_j \text{, and}$$

$$E(\cdot) = = \sum_j e_j x_j p_j \quad \ldots\ldots \qquad (7)$$

$$V(y) = = \sum e_j^2 V(z_j) + 2\sum e_j e_k \text{ Cov}(z_j, z_k)$$

Now,

$$V(z_j) = x_j^2 p_j - x_j^2 p_j^2$$

$$= x_j^2 p_j(1-p_j)$$

$$= x_j p_j(1-p_j) \text{ since } x_j = 0 \text{ or } 1.$$

Similarly

$$\text{Cov }(z_j, z_k) = x_j x_k(p_{jk} - p_j p_k)$$

where p_{jk} is the probability that a person is exposed to both option j and option k.

Hence

$$V(y) = \sum_j e_j^2 x_j p_j (1-pj) + 2\sum_{j,k} e_j e_k x_j x_k (p_{jk} - p_j p_k) \qquad (8)$$

Substituting these values in (6) we have,

$$E\{r\} = r(\Sigma e_j x_j p_j) + \frac{1}{2!} r^2(\Sigma e_j x_j p_j) \Big[\Sigma e_j^2 x_j p_j (1-pj) + 2\Sigma e_j e_k x_j x_k) \; (p_{jk} - p_j p_k) \Big]$$

$$+ \sum_{k=3}^{n-1} \frac{r^k(\mu)}{k!} \mu_k + \frac{1}{n!} E\{ r^n(y_1)\}(y-\mu)^n \qquad (9)$$

It can be seen that an expression for μ_3 will involve three way overlaps among the media. In general to compute μ_k we need the probabilities $p_{j_1 j_2 \ldots\ldots\ldots j_k}$, where $p_{j_1 j_2 \ldots\ldots jk}$ denotes the

the probability that a person belongs to the common audience of the vehicl s j_1, j_2 ....j_k. As higher ove-laps are expensive to collect, Little and Lodish have done some empirical work to relate the lower moments to the higher moments.

By substituting the exponential response function used by the authors in (6) we get,

$$E(r) = r_0 + a(1-e^{-b\mu}) + ae^{-b\mu}\{-\tfrac{1}{2}b^2\mu_2 + \tfrac{1}{6}b^3\mu_3\}$$

ignoring the fourth and higher order moments.

The MEDIAC model boils down to a mathematical programming problem, in which the expected response is maximised, subject to the budget constraint. The model is given by

Find X_{jt} (j = 1,2,3,.....M and t = 1,2,.....T) in order to maximise P.

$$R = \sum_i \sum_t n_i W_{it} \; E\{r(Y_{it})\}$$

subject to

$$\sum C_{jt} \, x_{jt} \leq B \qquad \text{..........(7)}$$

where B : the total budget available, and

C_{jt} : Cost of an insertion in media option j in time period t .

APPENDIX VII

MATHEMATICAL FORMULATION OF ADMOD

A_1 <u>Assumption 1</u> <u>The probability of exposure to an insertion option has a special structure</u>

Let

b_{ij} : probability that the individual i is exposed to vehicle j,

h_{cj} : probability that an individual exposed to vehicle j will be exposed to the insertion of copy approach c in vehicle j,

p_{cij} : probability that the individual i is exposed to the insertion of the copy approach c in vehicle j, and

$$Z_{cij} : \begin{cases} 1, & \text{if an individual in the target audience is exposed to the insertion of the copy approach c in vehicle j,} \\ 0, & \text{otherwise.} \end{cases}$$

It is clear that,

$$p_{cij} = P(Z_{cij} = 1)$$

p_{cij} is assumed to have the following structure.

$$p_{cij} = b_{ij}\, h_{cj} \tag{1}$$

A_2 <u>Assumption 2</u> The events A number of the target population is in the audience of a specific vehicle are independent for the different vehicles considered in the media plan.

Thus if b_{ij} ($j = 1,2, \ldots k$) is the probability that the individual i is in the audience of the j^{th} vehicle, then the probability that he is in the audience of all k vehicles is given by $b_{i1} \cdot b_{i2} \cdot \ldots \cdot b_{ik}$. This relation is assumed to be true for all values of k. Now, let us define the events,

A_{ij} : individual i is the audience of vehicle j, and

E_{cij}: individual i is exposed to the insertion of copy approach c in vehicle j.

It can be seen that $P(A_{ij}) = b_{ij}$.

The events A_{ij} are independent by virtue of this assumption. Hence probability that an individual i is in the audience of k vehicles ($j = 1, 2, \ldots k$).

$$= P\left(\bigcap_{j=1}^{k} A_{ij}\right)$$

$$= \prod_{j=1}^{k} P(A_{ij})$$

$$= \prod_{j=1}^{k} b_{ij}.$$

Now $A_{ij} \cap E_{cij}$ denotes the event 'individual i is exposed to the insertion of the copy approach c in vehicle j.'

Hence,

$$p_{cij} = P(Z_{cij} = 1) = P(A_{ij} \cap E_{cij})$$

Now,

$P(Z_{ci1} = 1,\ Z_{ci2} = 1,\ \ldots.\ Z_{cik} = 1)$

= probability that the individual i is exposed to the insertion of the copy approach c in k vehicles $(j = 1, 2, \ldots.\ k)$

$$= P\{\bigcap_{j=1}^{k} (A_{ij} \cap E_{cij})\}$$

$$= \prod_{j=1}^{k} P(A_{ij} \cap E_{cij})$$, the events $A_{ij} \cap E_{cij}$ $(j = 1, 2, \ldots.\ k)$ being independent, since the events A_{ij} $(j = 1, 2, \ldots.k)$ are independent.

$$= \prod_{j=1}^{k} P(A_{ij})\ \ P(E_{cij}/A_{ij})$$

$$= \prod_{j=1}^{k} b_{ij}\ h_{cj}$$

$$= \prod_{j=1}^{k} p_{cij}$$

$$= \prod_{j=1}^{k} P(Z_{cij} = 1)$$

Thus

$$P(Z_{ci1} = 1,\ Z_{ci2} = 1,\ \ldots\ldots,\ Z_{cik} = 1) = \prod_{j=1}^{k} P(Z_{cij} = 1)$$

Thus this assumption implies that the random variables Z_{cij}'s are independent.

A_3 Assumption 3 : The probability of an individual in the target audience taking the desired action depends on i) the number of exposures received, and ii) the effect of the media schedule, based on the copy approach and the vehicles used in the schedule.

Let

s = index for the segments into which the target population is divided ($s = 1, 2, 3, \ldots$)

ϑ_{cj} = vehicle source effect of the copy approach c when inserted in vehicle j, and

$$X_{cj} = \begin{cases} 1, & \text{if the copy approach c is inserted in vehicle j,} \\ 0, & \text{otherwise.} \end{cases}$$

$$M = \{j / x_{cj} = 1\}$$

If the probability that an individual i, who has received z exposures takes the desired action is demoted by $a_{ci}(z)$, then

$$a_{ci}(z) = [\, A_{cs} + B_{cs}(1 - \lambda^{z}_{cs}) \,] \left\{ \frac{\sum_{j \in M} \vartheta_{cj}\, p_{cij}}{\sum_{j \in M} p_{cij}} \right\}, \qquad (2)$$

where A_{cs}, B_{cs} and λ_{cs} are constants specific to the segment s.

Let us define,

$$a_{cs}(z) = A_{cs} + B_{cs}\,(1 - \lambda_{cs}^{z}) \qquad (3)$$

$$\mathcal{V}_{ci} = \frac{\sum_{j\varepsilon M} \mathcal{V}_{cj}\, p_{cij}}{\sum_{j\varepsilon M} p_{cij}} \qquad (4)$$

Aaker calls $\mathcal{V}_{ci}$ given in (4) as the expected vehicle source effect of the media schedule. He does not clearly explain the **reasoning** for this structure, beyond saying " The expected value of the vehicle source effect is simply a weighted average, with those vehicles having a higher **probability** of creating exposures having the higher weight. "

However we have developed the reasoning in the following manner :

The expected value of the vehicle source effect of a media schedule for an individual who has received at least one exposure is given by

$$E[\mathcal{V}_{ci}/\sum_{j\varepsilon M} Z_{cij} > 0] = \frac{E[\sum_{j\varepsilon M} \mathcal{V}_{cj}\, Z_{cij}]}{1-\pi_{j\varepsilon M}\,[1-E(Z_{cij})]}$$

$$= \frac{\sum_{j\varepsilon M} \mathcal{V}_{cj}\, p_{cij}}{1-\pi_{j\varepsilon M}\,(1-p_{cij})}$$

If we ignore the second and higher order terms in p_{cij} which represent the probabilities that the individual will see the advertisement in two vehicles or more, then we have,

$$E[V_{ci}/\sum_{j\varepsilon M} Z_{cij} > 0] = \frac{\sum_{j\varepsilon M} V_{cj}\, p_{cij}}{\sum_{j\varepsilon M} p_{cij}}$$

Special Structure for the probability distribution of Exposures

Let

$$Z = \sum_{j\varepsilon M} Z_{cij}, \text{ and}$$

$f_{ci}(z)$ = probability of an individual i receiving z exposures.

Aaker approximates $f_{ci}(z)$ by a binomial distribution. The method of approximation is as follows :

Since the random variables Z_{cij} is are independent, the mean μ and variance σ^2 of $f_{ci}(z)$ are given by :

$$\mu = \sum_{j\varepsilon M} p_{cij} \qquad (5)$$

and

$$\sigma^2 = \sum_{j\varepsilon M} p_{cij}(1-p_{cij}) \qquad (6)$$

Since $f_{ci}(z)$ is approximated by a binomial distribution,

$$\mu = n\pi \qquad (7)$$

$$\sigma^2 = n\pi(1-\pi) \qquad (8)$$

From (7) and (8) we get estimators of the parameters π and n in terms of μ and σ^2~

$$\left.\begin{aligned} \hat{n} &= \frac{\mu^2}{\mu - \sigma^2} \\ \hat{\pi} &= \frac{\mu}{\hat{n}} \end{aligned}\right\} \qquad (9)$$

The purpose of the author to make the computations less costly and cumbersome does not appear to be served by the binomial approximation. In view of the strong independence assumption regarding the Z_{ci}'s, by virtue of A_1, the generating function of Z is simply the product of the generating functions of the Z_{cij}'s. The generating function of Z_{cij} is given by

$$P_j(s) = p_{cij}' s + q_{cij} \quad (1$$

Since the random variables Z_{cij}'s are independent, the generating function of Z will then be

$$P(s) = \pi_{j \varepsilon M} (p_{cij}' s + q_{cij}) \quad (11)$$

The probability that an individual will be exposed to the advertisement Z times is given by the coefficient of s^Z in the above expression. This computation of the exact probabilities will not be any more cumbersome than the calculation of the binomial probabilities $\binom{\hat{n}}{z} \hat{P}^z (1-\hat{P})^{n-z}$ as the powers of $(1-\hat{P})$ will involve n, which is computed from the p_{cij} values, as can be seen from (5), (6), and (9).

THE MODEL

Let

W_s : Long term monetary value of a member of the segment s taking the desired action. (s = 1, 2, 3, ...S)

The value of the insertion schedule from an individual i belonging to the segment s, who has received Z exposures is given by

$$V_i' = W_s \, a_{ci} (z) \, f_{ci} (z)$$

Since the number of exposures received by an individual is an uncertain quantity, the expected value V_i of V_i' is taken.

$$V_i = E(V_i') = W_s \sum_{z=0}^{\infty} a_{ci}(z)\, f_{ci}(z)$$

Because of the special form of the function $a_{ci}(z)$, the above expression can be simplied further

$$V_i = W_s \sum_{z=0}^{\infty} a_{ci}(z)\, f_{ci}(z) \qquad (12)$$

$$= W_s \sum_{z=0}^{\infty} [A_{cs} + B_{cs}(1 - \lambda_{cs}^{z})]\, \gamma_{ci}\, f_{ci}(z)$$

$$\therefore V_i = W_s (A_{cs} + B_{cs})\, \gamma_{ci} - W_s B_{cs}\, \gamma_{ci} \sum_{z=0}^{\infty} \lambda_{cs}^{z} f_{ci}(z) \qquad (13)$$

since $\sum_{z=0}^{\infty} f_{ci}(z) = 1$.

The expression $\sum_{z=0}^{\infty} \lambda_{cs}^{z} f_{ci}(z)$ is the same as the generating function of Z with s replaced by λ_{cs}. Hence from (11) we have

$$\sum_{z=0}^{\infty} \lambda_{cs}^{z} f_{ci}(z) = P(\lambda_{cs}) = \prod_{j \varepsilon M} (p_{cij}\, \lambda_{cs} + q_{cij}) \qquad (14)$$

Now (13) reduces to

$$V_i = C_i - D_i \prod_{j \varepsilon M} (p_{cij}\, \lambda_{cs} + q_{cij}) \qquad (15)$$

where $C_i = W_s (A_{cs} + B_{cs})\, \gamma_{ci}$, and

$$D_i = W_s B_{cs}\, \gamma_{ci}.$$

If we were to use the binomial expression for $f_{ci}(z)$, then the expression in (15) will be

$$V_i = C_i - D_i \sum_{z=0}^{\infty} \lambda_{cs}^{z} \binom{\hat{n}}{z} \hat{P}^{z} (1-\hat{P})^{\hat{n}-z} \qquad (16)$$

which is not any less cumbersome computationally, than the expression in (15).

The exponential response function used by Aaker

$$a_{ci}(z) = A_{cs} + B_{cs}\ (1-\lambda_{cs}^{\ z})$$

is a special case of the response function used by Lee. It may be recalled that Lee uses the response function

$$R = (1-P^r)\ I_r,$$

where

- R : response to the advertising campaign
- r : Number of times an advertisement is seen by a member of the target audience.
- I_r : Probability that a member of the target audience sees the advertisement exactly r times.
- P : a constant to be determined by the advertiser.

With this structure for R, Lee simplifies the expected value of R to an expression very similar to that of the reach of a media schedule and therefore shows that maximisation of reach is equivalent to maximisation of response, with his assumptions and structures.

Let N_s denote the size of the segments. Let a sample of size n_s be drawn from each of the segments s.

The expected value V of the insertion schedule is obtained in the following manner :

The expected value V_i that will be received from an individual is summed over all the individuals in the sample from the segment s and scaled by $\frac{N_s}{n_s}$ to obtain the average value that will be received from the segment s. This value is summed over all the segments. The cost of the media schedule is subtracted to obtain the net value V. Thus,

$$V = \sum_{s} \frac{N_s}{n_s} \sum_{i=1}^{n_s} V_i - \sum_{j \varepsilon S} k_j x_{cj} \qquad (17)$$

where K_j is the cost of an insertion in vehicle j.
(j = 1, 2, ... J).

The model chooses a set of x_{cj} to maximise :

$$V = \sum_{s} \frac{N_s}{n_s} \sum_{i=1}^{n_s} [C_i - D_i \prod_{j \varepsilon S} (p_{cij} \lambda_{cs} + q_{cij})] - \sum_{j} K_j x_{cj} \qquad (18)$$

APPENDIX VIII

Relationship Between OTS during a Purchase Cycle and Switching of Brands

Number of OTS in the Purchase Interval	% O⟶X out of all Switches of Brands (i.e. O⟶X + X⟶O)	
	0 or 1	2 or more
	%	%
Washing Powders	49.6	52.4
Cereals	49.8	51.3
Tea	48.1	62.5
Soap	49.4	52.2
Margarine	49.9	51.0
Wrapped Bread	50.2	56.3
Tooth Paste	47.4	54.7
Shampoo	47.6	50.0
Milk drinks	53.7	55.9
Average	49.5	54.1

Note : X is defined as the brand being studied and O means any other brand except X.

Source: Colin Mc Donald, "What is the Short Term Effect of Advertising". Marketing Science Institute, London; Special Report No.71-142, 1971.

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