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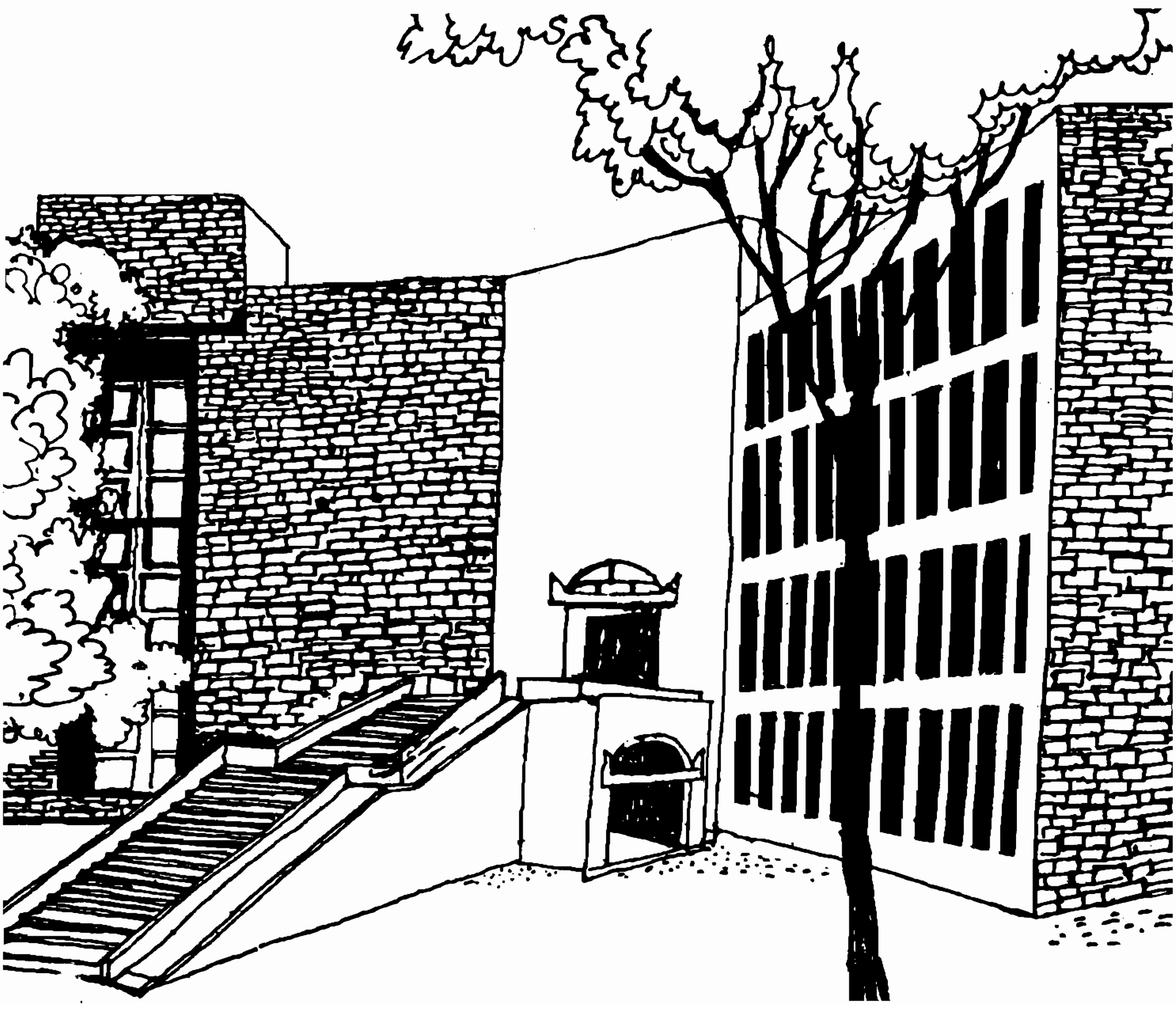

MARKET VALUATION MODEL UNDER DIFFERENTIAL TAXES, INFLATION, RECURRING INVESTMENTS AND FLOTATION COSTS

By

V. Raghunathan

&

Jayanth R. Varma

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INDIAN INSTITUTE OF MANAGEMENT
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Market Valuation Model Under Differential Taxes, Inflation, Recurring Investments and Flotation Costs

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V. Raghunathan & Jayanth R. Varma *

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* The authors are respectively Professor and Assistant Professor of Finance & Accounting at the Indian Institute of Management, Ahmedabad (India).

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Abstract

The extant literature on valuation identifies several important variables affecting the value of a firm. These include, corporate and personal taxes, reinvestments, leverage, dividend policy, and inflation. But, most of the papers have focussed only on a small number of these variables at a time and constructed comparatively simple valuation models designed to answer relatively limited questions in valuation theory. However, the fact remains that the above variables interact in quite complex ways, and it is necessary to have a comprehensive valuation model which captures most of the complexities and subtleties of real world corporate finance. This paper is an attempt at developing such a model. The model is capable of supporting both the Gordon and MM type assumptions about the investment policy of the firm. It allows for personal taxes with differential tax rates for dividends, interest and capital gains. The model also takes into account flotation costs on debt and equity. Further, unlike other models which define capital gains as the increase in the book value which in turn equals retained earnings, this model interprets capital gains as the increase in the market value of the share. Finally, the model is modified to take into account Lintner's concern about inflation eroding the real value of the firm's assets, particularly, net monetary working capital. The paper also numerically depicts the impact of manifold taxes on valuation and the complex interactions of different variables in influencing the firm value.

Market Valuation Model Under Differential Taxes, Inflation, Recurring Investments and Flotation Costs

Introduction

Ever since Modigliani and Miller [16] (hereafter MM) demonstrated the importance of corporate taxation on valuation in their classic paper, it has been recognized that any comprehensive valuation model must incorporate realistic assumptions about corporate and personal taxation. Several papers - Miller [13], Jaffe [10], DeAngelo and Masulis [3], Kim [11] and Modigliani [14,15] among others have addressed this question. Arditti and Pinkerton [2] studied the impact of growth and taxes. Amoako-Adu [1], Rashid and Amoako-Adu [18], Raghunathan and Srinivasan [17] and Howe [9] have dealt with the impact of differential tax rates on dividends vis-a-vis capital gains and equity income vis-a-vis interest income. Meanwhile, there has been sustained interest in the impact of growth and inflation on valuation (Lintner [12], Modigliani [14,15], Hochman and Palmon [8] etc.).

Some of these valuation models were constructed to answer some specific questions about the impact of dividend policy and capital structure on firm valuation, the valuation of growth prospects, or the extent to which common stock is a hedge against inflation. Many of these papers have, therefore, focussed on a small number of these variables at a time and constructed relatively simple valuation models. The fact is, however, that these variables interact in quite complex ways, and it is necessary to have a comprehensive valuation model which captures most of the complexities and subtleties of real world corporate finance.

We believe that this means that the following points need to be taken into account:

1. The model must be capable of supporting both the Gordon[1] and MM[2] type assumptions about the investment policy of the firm.

2. Personal taxes must be taken into account with differential tax rates for dividends, interest and capital gains.

3. Flotation costs on debt and equity must be explicitly recognized.

4. Capital gains must be treated with care. Many papers implicitly assume that capital gain means the increase in the book value of the share which in turn equals retained earnings (for example Amoako-Adu, 1983, Rashid and Amoako-Adu [18], Raghunathan and Srinivasan [17][3] and Howe [9]). In reality, capital gains is the increase in the market value of the share.

5. Inflation has the effect of eroding the real value of the firm's assets, particularly, net monetary working capital (Lintner [12]).

In our opinion, most of the existing models fail to capture all the above aspects in their totality. They make crucial simplifying assumptions which are inconsistent with the above realities of corporate finance. Our model attempts to relax these crucial assumptions, and develop a comprehensive model of corporate valuation.

However, since the number of variables introduced make the subsequent algebra rather complicated, in Appendix I we present a few basic propositions which are used to simplify the subsequent derivations in this paper.

To facilitate comparison with existing models, we derive our models under two assumptions. In section I we develop a general model in which we correctly assume that capital gains equal the change in market value of the firm rather than retained earnings. In section II we also extrapolate the model for the traditional (incorrect) treatment wherein the retained earnings are treated synonymous with capital gains. In section III we extend the model to account for the fact that in order to maintain the real value of assets under inflationary conditions, some additional investment is required. A fraction of the assets, which is tied up in monetary working capital of the firm, in fact erodes in real terms. Section IV captures the impact of this erosion on the actual growth rate of the firm and its value. Section V contains the summary conclusions.

For the sake of convenience, we delineate at the outset the symbols to be employed in developing our models.

I = Initial investment,

X_t = the expected Earnings before interest and taxes (EBIT), end of period t,

Y_t = the expected Earnings after taxes (EAT), end of period t,

V_t = value of the firm, beginning of period t,

F_t = fresh equity issue, end of period t,

μ = investment as fraction of EBIT,

δ = debt as fraction of total value,

b = dividend payout ratio,

f_b = flotation cost of bonds as fraction of amount issued,

f_e = flotation cost of equity as fraction of amount issued,

f_a = flotation cost advantage of bonds over equity as a fraction of amount issued

= $(f_e - f_b)(1 - f_e)$,

h = $1/(1 - f_e)$,

r = nominal pre-tax interest rate on bonds,

$\emptyset$ = productivity of capital (ROI) in real terms,

π = inflation rate,

g = growth rate of the firm,

t = tax rate: t_c, t_d, t_g, t_b respectively on corporate profits, dividends, capital gains and bond interest,

t_e = composite effective tax rate on equity,

k_E = nominal after-tax equity capitalization rate for a levered firm,

k_L = nominal after-tax overall capitalization rate for a levered firm, and

k_U = nominal after-tax capitalization rate for an unlevered firm.

I. When capital gains are correctly assumed to be determined at market values - Model 1

Let us consider a levered firm which maintains a constant debt to equity ratio (at market values) and yields a perpetual before tax expected earnings stream X_t and after tax stream Y_t. The firm is assumed to invest a fixed fraction μ of the EBIT stream X_t in every period, so that the investment at the end of period t is μX_t.[4] I is the initial investment of the firm, and the inflation rate is π. Under these conditions:

$X_1 = \emptyset I(1+\pi)$, and

$$X_t = X_{t-1}(1 + \pi) + \mu X_{t-1} (1 + \pi) \emptyset$$

$$= X_{t-1} ((1 + \pi + (1 + \pi) \mu \emptyset),$$

with the implied growth rate

$$g = \pi + (1 + \pi) \mu \emptyset,$$

and

$$Y_t = (X_t - r \delta V_t)(1 - t_c).$$

The firm pays out a fixed proportion b of its after tax earnings Y_t so that its retention in each period equals $(1 - b) Y_t$. Since the firm maintains a constant debt to equity ratio, it is assumed to issue new debt in each period equal to $\delta (V_{t+1} - V_t)$.

Now, since

Investment - (Retention + Fresh Debt + Fresh Equity - Flotation Cost of Debt - Flotation Cost of Equity) = 0, we must have

$$\mu X_t - [(1-b)Y_t + \delta(V_{t+1}-V_t) + F_t - f_b\delta(V_{t+1}-V_t) - f_eF_t] = 0$$

or $(1 - f_e)F_t = \mu X_t - (1-b) Y_t - (1-f_b) \delta (V_{t+1}-V_t)$

or $F_t = h [\mu X_t - (1-b) Y_t - (1-f_b) \delta (V_{t+1}-V_t)]$

[Note: For growing firms, F_t will be positive if $\mu > (1-b)(1-t_c)$. However, if $\mu < (1-b)(1-t_c)$, F_t may be negative, implying repurchase of stock. In such a case the flotation cost of equity, f_e, may be interpreted as the transaction cost of repurchase of stock, so that the flotation cost of equity expressed as a percentage of negative equity issue (i.e. equity bought back) will be negative.

Similarly if g is negative, the term $\delta (V_{t+1}-V_t)$ implies retirement of debt and f_b may be interpreted as the cost, if any, of retirement of debt. Again, the flotation cost of debt expressed as a percentage of negative debt issue (retired debt) will be negative.]

The initial wealth of the shareholders is $V_{t-1}(1-\delta)$, and the closing value of their wealth after one period is $V_t(1-\delta) - F_{t-1}$. Hence, the capital gains net of taxes at the end of period t are:

$[V_t (1-\delta) - F_{t-1} - V_{t-1} (1-\delta)](1-t_g)]$

and dividends net of taxes are:

$b Y_{t-1} (1-t_d)$.

Since the equity capitalization rate for a levered firm is k_E, we must have:

$bY_{t-1}(1-t_d) + [V_t\ (1-\delta)-F_{t-1}-V_{t-1}\ (1-\delta)](1-t_g)] = k_E\ V_{t-1}\ (1-\delta)$

Substituting for F_{t-1} and expanding:

$$[b\ (1-t_d) + (1-b)\ (1-t_g)\ h]\ Y_{t-1} - \mu\ h\ (1-t_g)\ X_{t-1} +$$
$$(1-t_g)\ \ ((1-\delta) + (1-f_b)\ \delta\ h)\ V_t -$$
$$[k_E\ (1-\delta) + (1-t_g)\ ((1-\delta) + (1-f_b)\ \delta\ h)]\ V_{t-1} = 0$$

Now if we define

$$w = [b\ (1-t_d) + (1-b)\ (1-t_g)\ h]\ (1-t_c),\ \text{and}$$
$$q = (1-t_g)\ [(1-\delta) + (1-f_b)\ \delta\ h]$$
$$= (1-t_g)h\ [(1-\delta)(1-f_e) + \delta(1-f_b)]$$

~~We have~~

$$(w/(1-t_c))Y_{t-1}-\mu h(1-t_g)X_{t-1} + qV_t - (k_E\ (1-\delta) + q)\ V_{t-1} = 0.$$

Substituting for Y_{t-1}, we get

$$wX_{t-1} - r\delta wV_{t-1} - \mu h(1-t_g)X_{t-1} + qV_t - (k_E(1-\delta)+q)V_{t-1} = 0$$

or

$$qV_t - [k_E(1-\delta) + r\delta w + q]\ V_{t-1} + [w - \mu h\ (1-t_g)]\ X_{t-1} = 0$$

Applying Proposition II with $a_0 = q$

$a_1 = k_E\ (1-\delta) + r\ \delta\ w + q$

$a_2 = w - \mu\ h\ (1-t_g)$,

we get

$$p = \frac{w - \mu\ h\ (1-t_g)}{(k_E\ (1-\delta) + r\ \delta\ w) - g\ q}\ ,\ \text{and}\ V_L = p\ X_1 = p\ \emptyset I(1+\pi)$$

so that

$$V_L = \frac{(1-t_e) - \mu\ h\ (1-t_g)}{(k_E\ (1-\delta) + r\ \delta\ (1-t_e)) - g\ q}\ X_1 \qquad (1)$$

where $t_e = (1-w) = 1 - (1-t_c)[b(1-t_d) + (1-b)(1-t_g)h] =$

$$q = (1-t_g)h[(1-\delta)(1-f_e) + \delta(1-f_b)]$$

and the condition for convergence is that the denominator be positive.

or

$$V_L = \frac{(1-t_e) - \mu h(1-t_g)}{(k_W) - g q} X_1$$

where $k_W = k_E(1-\delta) + r\delta(1-t_e)$, can be interpreted as a weighted average cost of capital.

Eq (1) may also be expressed in terms of the value of an unlevered firm and the leverage benefit. We apply eq (1) to the unlevered firm by setting δ equal to 0, to get:

$$V_U = \frac{(1-t_e) - \mu h(1-t_g)}{k_U - g(1-t_g)} X_1 \qquad (1a)$$

where V_U is the value of an unlevered firm and k_U is the cost of equity for the unlevered firm.

Appendix II derives the relationship between k_U and k_L and restates eq (1) to expreses V_L as the sum of V_U and the benefit of leverage:

$$k_L = k_U - \sigma(k_U - r(1 - t_b))$$

where

$$\sigma = (V_L - V_U)/V_L = \frac{r(t_e - t_b) + f_a(1-t_g)g}{r(1 - t_b) - g(1-t_g)} \delta \text{ , and}$$

where $f_a = (f_b-f_e)/(1-f_e)$

$$V_L = V_U + \frac{r\ (t_e - t_b) + f_a(1-t_g)g}{r\ (1-t_b) - g\ (1-t_g)} D, \quad (1b)$$

where $D = \pounds\ V_L$ is the debt component of the levered firm.

or

$$V_L = V_U + \frac{r\ [(1-t_b)-(1-t_c)(b\ (1-t_d)+(1-b)\ (1-t_g)\ h)] + f_a(1-t_g)g}{r\ (1-t_b) - g\ (1-t_g)} D$$

A. MM's [16] Result:

It can be seen that when flotation costs $f_e = f_b = 0$, tax rates $t_d = t_g = t_b = 0$, payout ratio $b = 1$, growth rate $g = 0$, and inflation rate $\pi = 0$, eq (1b) reduces to

$$V_L = \frac{X_1(1-t_c)}{k_U} + Dt_c$$

$$= V_U + D\ t_c$$

This is the standard valuation formula of Modigliani and Miller.

B. Miller's [13] Result:

When $b = 1$, $g = 0$, $\pi = 0$, $t_g = 0$, and $\mu = 0$, eq (1b) reduces to

$$V_L = V_U + \frac{1 - (1-t_c)(1-t_d) - t_b}{(1 - t_b)} D$$

where $V_U = \frac{X1\ (1-t_c)(1-t_d)}{k_U}$.

or

$$V_L = V_U + [1 - \frac{(1-t_c)\ (1-t_d)}{(1-t_b)}]\ D,$$

which is Miller's result.

II. When capital gains are wrongly assumed synonymous with retained earnings - Model 2

In this approach, the net cashflow stream to be discounted is:

Dividends +Capital Gains -Investment -Flotation Costs + Interest

$$= b\, Y_t(1-t_d) + (1-b)\, Y_t\, (1-t_g) - \mu\, X_t - f_b\, \Sigma(V_{t+1} - V_t)$$

$$- f_e\, F_t + r\, \Sigma\, V_t(1-t_b)$$

$$= (w^* - \mu\, h)\, X_t + \Sigma\, (r(t_e^* - t_b)\, V_t + f_a^*(V_{t+1} - V_t))$$

where $w^* = 1-t_e^* = [b(1-t_d) + (1-b)(1 - t_g + f_e\, h)](1-t_c)$.

and $f_a^* = (f_b - f_e - f_b f_e)/(1-f_e)$ is the flotation cost advantage of debt[5].

[Note that w^*, t_e^* and f_a^* correspond respectively to w, t_e and f_a of the Market Price Model (eq (1)).]

The first component of the above cashflow, $(w^* - \mu\, h)\, X_t$ is the net cashflow stream of the unlevered firm growing at a rate g (since X_t grows at the rate g) to be discounted at the rate k_U. We can obtain V_U^* by invoking Proposition 1 (Appendix I) with

$Z_t = (w^* - \mu h)\, X_t$, to get

$$V_U^* = \frac{w^* - \mu h}{k_U - g}\, X_1$$

$$= \frac{(1-t_e^*) - \mu h}{k_U - g}\, X_1 \qquad (2a)$$

The second component of the levered firm s cashflows, $\Sigma(r(t_e^* - t_b)V_t + f_a^*(V_{t+1} - V_t))$, is the tax and flotation cost advantage of debt to be discounted at $r(1-t_b)$. Hence, the value of the levered firm is given by:

$$V_L^*(s) = V_U^*(s) + \mathcal{E} \sum_{t=s}^{\infty} \frac{r\,(t_e^* - t_b)\,V_t + f_a^*(V_{t+1} - V_t)}{(1 + r\,(1-t_b))^{t+1-s}}$$

Apply Proposition III with $c_0 = r\,(t_e^* - t_b) - f_a^*$

$$c_1 = f_a^*$$

$$r = r\,(1-t_b)$$

to get

$$V_L^* = V_U^* + \frac{r\,(t_e^* - t_b) + f_a^* g}{r\,(1-t_b) - g}\, D \qquad (2b)$$

If flotation costs are absent and t_g is zero, then model 2 reduces to model 1. This means in particular that the MM model and Miller model can be derived from model 2 also as follows:

A. MM's [16] Result:

When flotation costs $f_e = f_b = 0$, tax rates $t_d = t_g = t_b = 0$,

we get

$$V_L = V_U + \frac{r\,t_c}{r - g}\, D,$$

which when $g = 0$, gives

$$V_L = V_U + D\,t, \quad \text{where } V_U = \frac{X\,(1)\,(1-t_c)}{k_U},$$

which is the standard MM model.

B. Miller's [13] Result:

When $b = 1$, $g = 0$, $\pi = 0$, and $\mu = 0$, eq (2) reduces to

$$V_L = V_U + [1 - \frac{(1-t_c)\ (1-t_d)}{(1-t_b)}]\ D,$$ which is Miller's result.

where $$V_U = \frac{X\ (1)\ (1-t_c)\ (1-t_d)}{k_U},$$

which is Miller's model.

C. Howe's [9] Result:

Howe's 1988 model is quite peculiar. His no-growth firm grows at the rate of inflation without any additional investment. To maintain a constant debt-equity ratio, the firm issues additional debt the proceeds of which are distributed to the shareholders. The shareholders pay tax on these receipts. One peculiarity of Howe's model is that it implies a payout ratio exceeding unity in violation of usual corporate laws. Howe's model has the unfortunate effect of making the payout ratio a function of the leverage. It is, therefore, difficult to disentangle the effects of leverage and dividend policy in Howe's model. The better approach is to use the proceeds of the debt issue to retire stock; the tax implications of this are quite different. Howe's growth model is even stranger. Under growth, the firm has to issue fresh equity to finance the new investment. The most natural thing to do with the proceeds of debt is to use it to reduce the fresh equity issue. What Howe does is to pay out the debt proceeds as dividends and then make a fresh equity issue to finance growth. This is surely unreasonable.

Howe's model (no-growth firm) can be derived from our model 2 by a simple trick. We set the payout ratio equal to 1, set t_d to t_s (Howe's notation), set t_g to 0, and set f_a^* equal to $-t_s$ (f_e = 0 and f_b = t_s). The reason for setting f_b equal to t_s is that whenever debt is issued, the proceeds are distributed to shareholders who pay tax at the rate of t_s on the amount. Setting f_b = ts, f_e = 0, g = π, μ = 0, and b = 1 gives

$$V_L^* = V_U^* + \frac{r\ (t_e^* - t_b) - ts\ \pi}{r\ (1-tp) - \pi} D$$

$$= V_U^* + \frac{r\ ((1-t_b) - (1-ts)\ (1-t_c)) - ts\ \pi}{r\ (1-t_b) - \pi} D$$

$$= V_U^* + \left[1 - \frac{r\ (1-ts)(1-t_c) - \pi\ (1-ts)}{r\ (1-t_b) - \pi} \right] D \qquad (3)$$

where

$$V_U^* = \frac{(1-t_c)\ (1-t_d)}{k_U - \pi} X_1,$$

which is Howe's model.

We believe that the better way to model Howe's situation is to assume that the debt proceeds are used to repurchase stock rather than pay dividends. Recognizing that capital gains are determined by market prices, we use model 1 with f_b = f_e = 0, b = 1, μ = 0, and t_d = t_g = t_s, to get:

$$V_L^* = V_U^* + \frac{r\ (t_e - t_b)}{r\ (1-t_b) - \pi(1-t_g)} D,$$

$$= V_U^* + [1 - \frac{r(1-t_s)(1-t_c) - \pi g}{r (1-t_b) - \pi g}]$$

where

$$V_U^* = \frac{(1-t_c)(1-t_s)}{k^U - \pi (1-t_g)} X_1 .$$

III. Comparison and Interpretation of Models 1 & 2

Both our valuation models state the value of the firm as sum of the value of the unlevered firm as sum of the value of the unlevered firm and the leverage benefit. We shall interpret these two terms separately.

To obtain an intuitive understanding of the formula for V_U, we start with the well known Gordon's Valuation Model[6]:

$$V_U = \frac{b(1-t_c)}{k_U - \emptyset(1-b)} X_1 \qquad (4)$$

in which $b(1-t_c)X_1$ is the dividend at the end of period 1 and $k_U - \emptyset(1-b)$ is the cost of capital less the growth rate.

A. Value of the Unlevered Firm

Model (1a) and (1)

It is easily verified that if we ignore personal taxes ($t_d = T_g = 0$) and flotation costs ($f_b = f_e = 0$, $h = 1$) and assume investment equals retention ($\mu = b(1-t_c)$), then both our eqs (1a) and (2a) reduce to the above valuation formula of Gordon.

To proceed to the more complex results, we shall first reinterpret the numerator, $b(1-t_c)X_1$, of Gordon's formula (eq 4), not as dividends but as after tax earnings $(1-t_c)X_1$ less the additional investment $(1-b)(1-t_c)X_1$.

$$V_U = \frac{(1-t_c)X_1-(1-b)(1-t_c)X_1}{k_U - \emptyset(1-b)} = \frac{\text{After tax earnings-Investments}}{\text{Cost of capital - growth rate}} \qquad (4a)$$

Ignoring flotation costs for the present, the numerator of eq (1a) also consists of two terms: $(1-t_e)X_1$ less $\mu(1-t_g)X_1$.

The first of these is again after tax earnings, but now the taxes include personal taxes as well:

$$X_1\ (1-t_E) = X_1((1-t_c)\ [b\ (1-t_d) + (1-b)(1-t_g)])$$

where the factor within the square brackets is the weighted average of the tax rates on dividends and capital gains.

The second term in the numerator of eq (1a) is the additional investment μX_1 times capital gains tax adjustment factor $(1-t_g)$. Unlike the Gordon model, we do not necessarily set investment equal to retentions. The only thing which needs explanation is the presence of the tax factor $(1-t_g)$. The reason for its presence is simple. Given that the wealth appreciation of the shareholders attracts capital gains tax at the rate of t_g, the investment μX_1 "costs" the shareholders only $X_1(1-t_g)$.

Coming to the denominator of eq (1a), we see that the growth term is multiplied by a factor $(1-t_g)$ which is not present in 4a. The presence of this term is explained by Proposition IV (Appendix 1) as an adjustment for the present value of the capital gains tax to be paid in future years. In other words, since the numerator is specified in terms of after capital gains tax cashflows, the growth rate in the denominator is also specified in after tax terms. We see that in the absence of flotation costs, the difference between eqs (1a) and (4a) are:

1. After tax earnings is redefined as being after corporate and personal taxes.

2. Investments are specified in after tax terms.

3. The growth rate is specified in after tax terms.

Thus, intuitively, all the terms make sense. When flotation costs are brought in, it is necessary to redefine the term "after tax" as "after tax and flotation costs".

In order to clarify this, let us rewrite the term $g\ (1-t_g)$ in eq (1a) as $g\ (1-t_g)\ h\ (1-\ f_e)$, to have

$$V_U = \frac{(1-t_e) - \mu\ h\ (1-t_g)}{k_U - g(1-t_g)\ h(1-f_e)}\ X_1$$

We now see that flotation costs enter the valuation formula in two ways:

1. The tax factor $(1-t_g)$, whenever it occurs, is multiplied by the factor $h = 1/(1-f_e) \geq 1$. It is as if the tax rate on capital gains has been reduced. The reason is that flotation costs make retention (internal financing) more attractive as it eliminates the flotation costs associated with external financing. This partly compensates for the capital gains tax induced by such retention.

2. The growth rate in the denominator is multiplied by a further factor $(1-f_e)$. Since, we earlier reduced the growth rate by the factor $(1-t_g)$ to reflect the fact that a fraction t_g is lost due to capital taxes, we now multiply it by $(1-f_e)$ to account for the fraction f_e which is lost in the form of flotation costs.

With the terms in eq (1a) having been explained, eq (1) becomes self-evident. We merely substitute the cost of equity and flotation cost of equity in the denominator of eq (1) by weighted average cost of capital and weighted average cost of flotation (of debt and equity).

<u>Model (2a)</u>

Now, let us consider eq (2a) in the absence of flotation costs. The numerator consists of the after tax earnings $(1-t_e)X_1$ (in the absence of flotation costs, t_e and t_e^* are identical) less the investment μX_1. The denominator is simply the cost of capital less the growth rate. Earlier we highlighted three adjustments made in going from eq 4a to (1a). Of these, eq (2a) makes only the first adjustment and ignores the other two.

When flotation costs are brought in, the effect is similar to what was observed in eq (1a). First, the taxation on capital gains, t_g, is reduced by $F_e h$ to reflect the increased attractiveness of retention. Secondly, the additional investment μX_1 is multiplied by the term h to reflect the fact that investment has become more "costly" due to the flotation costs that they entail.

To highlight the difference between eqs (2a) and (1a), we look at the simplest case of an unlevered firm assuming full payout (b = 1), zero flotation costs (h = 1) and no inflation (π = 0). With these assumptions, eq (1a) reduces to:

$$V_U = \frac{(1-t_c)(1-t_d) - \mu\ (1-t_g)}{k_U - \mu\emptyset(1-t_g)} X_1 \qquad (1c)$$

while, eq (2a) becomes:

$$V_U^* = \frac{(1-t_c)(1-t_d) - \mu}{k_U - \mu\emptyset} X_1 \qquad (2c)$$

Thus, the first difference between the two models is that in the market price approach, the growth rate g in the denominator gets multiplied by a factor $(1-t_g)$ reducing the value of the firm. The presence of this factor is explained by Proposition IV (Appendix I) as a result of a fraction t_g of the growth being taxed away. The point is that even though no earnings are retained, the market price of the equity shares does increase reflecting the present value of all future growth opportunities. The market price approach recognizes that this growth in value attracts tax at the rate t_g: the present value of all these future tax liabilities is taken into account in arriving at the market price as shown in Proposition IV. In the retained earnings approach, there is no capital gains tax as retention is zero and accordingly it overvalues the firm.

The second difference between the two models is that the entire fresh investment μX_1 is subtracted from the earnings in the numerator in the retained earnings approach, while in the market price approach, only $(1-t_g)X_1$ is subtracted. This has the effect of the value of the firm being understated in the retained earnings approach as compared to the market price approach.

While these two differences work in opposite directions, it is easy to see from eqs (1c) and (2c) that for high growth firms, whose capital productivity far exceeds the cost of capital $(\emptyset(1-t_c)(1-t_d) > k_U)$, the retained earnings approach significantly understates the tax burden on the investor, and, therefore, significantly overvalues the firm. The reverse is true if the capital productivity is low or moderate $(\emptyset(1-t_c)(1-t_d) < k_U)$. The proof of this is contained in Appendix III.

B. Value of the Levered Firm

Model (1b)

We now turn to eq (1b). In this equation, the value of the levered firm is obtained by adding the benefit of leverage to the value of the unlevered firm (eq (1a)). Eq (1b) values the leverage benefit at:

$$\frac{r\ (t_e - t_b) + f_a(1-t_g)g}{r\ (1-t_b) - g\ (1-t_g)}\ D$$

The classical MM expression for the leverage benefit is Dt_c, which is nothing but the tax advantage of debt rDt_c discounted at the riskfree rate r. The difference between this and eq (1b) are:

1. The tax advantage of debt also takes personal taxes into account so that t_c is replaced by $t_e - t_b$. The effective composite tax rate on equity is t_e while bond interest attracts only the personal taxes at t_b.

2. There is an added term representing the flotation cost advantage of debt over equity. The debt issued in every period is gD, the flotation cost saving is f_a times this, and this saving is taken net of capital gains tax to obtain $gDf_a(1-t_g)$.

3. Discounting is done at the after (personal) tax riskfree rate $r(1-t_b)$ instead of at r.

4. In the case of a growth firm, the tax shield continues to grow. Hence the growth rate must be subtracted from the riskfree rate in the denominator.

5. Since the growth in value attracts capital gains tax at the rate t_g, the growth rate must be multiplied by $(1-t_g)$ as explained in Appendix II.

Model (2b)

Eq (2b) differs from eq (1b) in that the growth rate is not multiplied by the factor $(1-t_g)$ and that the flotation cost advantage of debt is computed in a slightly different fashion. In other words, eq (2b) takes into account factors 1,3 and 4 listed above, but ignores factors 2 and 5. This is similar to the difference between eqs (1a) and (2a) for valuing unlevered firm.

Example:

The above discussion may be illustrated through a simple numerical example. Let I = 100, $t_c = 0.45$, $t_d = 0.35$, $t_g = 0.25$, $b = 0.40$, $k_u = 0.08$, $\pi = 0$, and $f_e = 0$ (or h = 1). The values obtained from models (1a) and (2a) are tabulated below in Exhibit 1 for different values of μ and $\emptyset$.

Exhibit 1: Valuation Under Models 1a (V_U) & 2a (V_U^*)

(About here)

C. Effect of Flotation Costs

It may be seen that the error introduced in valuation by ignoring flotation costs is larger under model 2 as compared to model 1. This is evident from comparing the values in Exhibit 2 with those in Exhibit 1. Exhibit 2 presents the values under models 1 and 2 for the same parameter values as those used in the above example, except that now the flotation cost of equity (f_e) is assumed to be 0.1 (or h = 1.11). Accordingly, models (1a) and (2a) stand modified to (1d) and (2d) respectively as under:

$$V_U = \frac{(1-t_e) - \mu h(1-t_g)}{k_U - \mu\emptyset(1-t_g)} X_1 \qquad (1d)$$

while, eq (2a) becomes:

$$V_U^* = \frac{(1-t_e^*) - \mu h}{k_U - \mu\emptyset} X_1 \qquad (2d)$$

Exhibit 2: Valuation Under Models 1a (V_U) & 2a (V_U^*)

when flotation Cost is Non-zero

(About here)

Exhibit 3 presents the values of unlevered and levered firms under model 1 (V_U and V_L from eqs (1a)and (1b) respectively) and model 2 ($V_U^*$and $V_L^*$from eqs (2a) and (2b) respectively) for different parameter values. Thus it highlights the difference in valuation between the two models. It also demonstrates the impact of manifold taxes on valuation and the complex interactions of different variables in influencing the firm value.

For the purpose of arriving at the values in the tables, r and k_U (which are specified nominally) are arrived at through tax adjusted Fisher effect Thus, if r' and k_U' are real riskfree rate and real capitalization rate for an unlevered firm respectively, r and k_U are expressed as:

$$1 + r\,(1-t_b) = (1 + r')\,(1 + \pi),$$

or

$$r = \frac{(1 + r')\,(1 + \pi) - 1}{(1-t_b)}$$

and

$$1 + k_U = (1 + k_U')\,(1 + \pi),$$

or

$$k_U = (1 + k_U')\,(1 + \pi) - 1.$$

[Note that r is a pre-tax rate, whereas k_U is an after-tax rate.]

Exhibit 3: Valuation Under Models 1 & 2

For Different Parameter Values

(About here)

IV. Inflation and Monetary Working Capital

According to Lintner (1975), even when a firm "maintains a fully synchronized steady-state rate of growth in real terms", there is some erosion to the real value of the firm on account of the fraction of investment tied up in monetary assets. This erosion in the value of a firm may be captured as under:

Let K_t be the capital stock at the beginning of period t, and

I_t be the investment at the end of the period t.

$K1 = I_0 = I$

In our analysis earlier it was assumed that all of K_t is in real assets which grow at the inflation rate. But usually, a fraction θ is in monetary assets (net working capital) which erodes in real terms.

Hence

$$K_t = K_{t-1}\ (1-\theta)(1+\pi) + \theta\ K_{t-1} + I_{t-1}$$

$$= K_{t-1}\ (1+\pi(1-\theta)) + I_{t-1}$$

$\pi\theta k_{t-1}$ is the erosion in the real value of the firm's assets during the year t-1. We can now make two alternative assumptions about I_t. The first is that $I_t = \mu X_t$ as before. The second assumption is that over and above μX_t, the firm invests an additional amount of $\pi\theta k_t$ to maintain the value of the firm's assets in real terms. We can subsume both these assumptions under a single model by letting α be a zero-one variable and specifying the investment to be:

$$I_t = \mu X_t + \alpha \pi \theta K_t, \text{ so that}$$

$$K_t = K_{t-1} [1 + \pi - \pi \theta (1-\alpha)] + \mu X_{t-1}$$

$$= X_{t-1} \left[\frac{1 + \pi - \pi \theta (1-\alpha)}{\emptyset (1+\pi)} + \mu \right]$$

We then have

$$X_t = (1+\pi) \emptyset \left[\frac{1 + \pi - \pi \theta (1-\alpha)}{\emptyset (1+\pi)} + \mu \right] X_{t-1}$$

$$= [1 + \pi - \pi \theta (1-\alpha) + \mu \emptyset (1+\pi)] X_{t-1}$$

so that the nominal growth rate = $\pi - \pi \theta (1-\alpha) + \mu \emptyset (1+\pi)$ and

the real growth rate = $\mu \emptyset - \dfrac{\pi \theta (1-\alpha)}{1 + \pi}$

Now, when $\alpha = 0$ (i.e. $I = \mu X_t$ as before), we have:

Nominal growth rate = $\pi (1-\theta) + \mu \emptyset (1+\pi)$

as against $\pi + \mu \emptyset (1+\pi)$ assumed in model 2, and

Real growth rate = $\mu \emptyset - \dfrac{\pi \theta}{1 + \pi}$.

When $\alpha = 1$ (i.e. $I = \mu X_t + \pi \theta K_t$, so that μX_t represents only new investment, not including maintenance of the real value of capital), we have

Nominal growth rate = $\pi + \mu \emptyset (1+\pi)$, and

Real growth rate = $\mu \emptyset$.

Thus, in this case, the real growth rate is independent of inflation rate.

Given that a fraction θ of the capital stock is tied up in working capital, a nominal growth rate of $\pi + \mu\emptyset(1+\pi)$ can be achieved only if the firm invests an amount $\pi\,\theta\,K_t$ over and above $\mu\,X_t$ in period t (i.e. $\alpha = 1$). Alternatively, if only μX_t is invested in period t (i.e. $\alpha = 0$), the nominal growth rate of the firm s capital is restricted to $\pi(1-\theta) + \mu\emptyset(1+\pi)$.

The only changes induced by the new assumptions of this part are that μ is replaced by μ^* and g by g^* in our earlier models,

where $g^* = \pi - \pi\,(1-\theta) + \mu\,\emptyset\,(1+\pi)$ and

$\mu^* = \mu + \dfrac{\alpha\,\pi\,\theta}{(1+\pi)\,\emptyset}$, α being 0 or 1 as the case may be.

Accordingly, model 1(b) becomes

$$V_U = \frac{(1-t_e) - \mu^*\,h\,(1-t_g)}{k_U - g^*\,e}\,X_1 \qquad (5a)$$

and

$$V_L' = V_U + \frac{r\,(t_e - t_b)}{r\,(1-t_b) - g^*q}\,D, \qquad (5b)$$

Model 2 becomes:

$$V_U^* = \frac{w^* - \mu^*\,(1 + f_e\,h)}{k_U - g^*}\,X_1 \qquad (6a)$$

and

$$V_L^* = V_U^* + \frac{r\ (t_e^* - t_b) + f_a^* g}{r\ (1-t_b) - g^*} D \qquad (6b)$$

V. Summary

In this paper, we presented a comprehensive valuation model taking into account various real life complexities, such as corporate and personal taxes, recurring investments, inflation, leverage, dividend policy and flotation costs. Different personal tax rates were assumed for dividends, interest and capital gains. The model was capable of supporting both the MM as well as Gordon type worlds. Further, unlike most of the prevailing models which treat increase in the book value of the shares as synonymous with capital gains, in our model capital gains were correctly treated as being the increase in market value of the shares. And finally, the model was modified to take into account Lintner's concern about inflation eroding the real value of the firm's assets, particularly, net monetary working capital. While most of the well known valuation models have been shown to be special cases of our general model, the paper especially highlighted the error in valuation which results when change in book value rather than market value of shares is treated as capital gains. The paper also numerically depicted the impact of manifold taxes on valuation and the complex interactions of different variables in influencing the firm value.

Appendix I

Some Simplifying Propositions

Proposition I

If the end of the period cashflows Z_t satisfy

$$Z_t = (1+g)\ Z_{t-1},$$

then the present value of the cashflow stream at the rate k, V, is

$$Z_1/(k-g).$$

Proof:

$$V = \sum_{1}^{\infty} Z_t/(1+k)^{(t)} = \sum_{1}^{\infty} Z_1\ (1+g)^{(t-1)}/\ (1+k)^{(t)}$$

$$= [Z_1/(1+k)] \sum_{1}^{\infty} ((1+g)/(1+k))^{(t-1)} = Z_1/(k-g)$$

The condition for the convergence being $(1+g)/(1+k) < 1$ or $g < k$.

QED.

Proposition II

If

$$a_0\ V_t - a_1\ V_{t-1} + a_2\ X_{t-1} = 0 \quad \text{and}$$

$$X_t = (1+g)\ X_{t-1}$$

then

$$V_t = p\ X_t, \text{ where } p = a_2/(a_1 - a_0\ (1+g))$$

Proof:

Substitute $V_t = p\ X_t$ to get

$$a_0\ p\ (1+g)\ X_{t-1} - a_1\ p\ X_{t-1} + a_2\ X_{t-1} = 0$$

or $p\ (a_0\ (1+g) - a_1) + a_2 = 0$

or $p = a_2/(a_1 - a_0\ (1+g))$.

QED.

Proposition III

If

$$V_L(s) = V_U(s) + \delta \sum_{t=s}^{\infty} [c_0 V_L(t) + c_1 V_L(t+1)]/[(1+r)\hat{}(t+1-s)]$$

and

$$V_U(s) = (1+g) V_U(s-1),$$

then

$$V_L = V_U + \frac{c_0 + c_1 (1+g)}{r - g} \delta V_L$$

where V_L and V_U are the current values of the levered and unlevered firms respectively, t and s are time variables and δV_L is the debt component of the levered firm.

<u>Proof:</u>

$$V_L(s+1) - (1+r) V_L(s) = (g - r) V_U(s) - \delta (c_0 V_L(s) + c_1 V_L(s+1)$$

or

$$V_L(s + 1) (1 + \delta c_1) - V_L(s) (1 + r - \delta c_0) + (r - g) V_U(s) = 0$$

Apply Proposition II with

$t = s + 1,$

$a_0 = 1 + c_1 \delta,$

$a_1 = 1 + r - c_0 \delta,$ and

$a_2 = r - g$

to obtain

$$V_L = V_U \frac{r - g}{1 + r - \delta c_0 - (1+c_1 \delta) (1 +g)}$$

$$= V_U \frac{r - g}{r - g - \delta(c_0 + c_1 (1 +g))}$$

or

$$V_L - V_U = \frac{c_0 + c_1 (1 + g)}{r - g} \; S \; V_L$$

or

$$V_L = V_U + \frac{c_0 + c_1 (1 + g)}{r - g} \; S \; V_L$$

QED.

Proposition IV

If the cash stream Z_t and Value V_t are growing at rate g, but a fraction c of the growth in value is taxed away, then

$V = Z/(r - g (1 - c))$

Proof:

$$V = \sum_{0}^{\infty} \frac{Z_t}{(1+r)^{t+1}} - \sum_{0}^{\infty} \frac{g \, c \, V_t}{(1+r)^{t+1}}$$

$= Z/(r - g) - g \, c \, V/(r - g)$ [by applying Proposition I to both summations]

or $V (r - g) = Z - g \, c \, V$

or $V = Z/(r - g (1 - c))$

QED.

Appendix II

Proposition

If V_L and V_U are given by eqs (1) and (1a) respectively, then

$$V_L = V_U + \frac{r\,(t_e - t_b) + f_a(1-t_g)g}{r\,(1 - t_b) - g(1-t_g)}\,D \qquad (a)$$

where $D = \delta V_L$ is the debt component of the levered firm,

and

$$k_L = k_U - \sigma(k_U - r(1 - t_b)) \qquad (b)$$

where

$$\sigma = (V_L - V_U)/V_L = \frac{r\,(t_e - t_b) + f_a(1-t_g)g}{r\,(1 - t_b) - g(1-t_g)}\,\delta.$$

Proof:

Let

$a_0 = (1-t_e) - \mu\, h\,(1-t_g)$,

$a_1 = g\, q$, and

$a_2 = g(1-t_g)$,

so that

$V_L = a_0/(k_E\,(1 - \delta) + r\,\delta\, w - a_1)$ and

$V_U = a_0/(k_U - a_2)$

where k_E and k_U are the capitalization rates appropriate to the levered and unlevered firms respectively, r is the interest rate on bonds, and $w = 1 - t_e$ (t_e being the composite effective tax rate on equity),

Also let $V_U/V_L = 1 - \sigma$ so that $V_L = V_U + \sigma V_L$, and S be the equity of th levered firm.

Since $V_U = (1 - \sigma) V_L$ and $S = (1 - \delta) V_L$, we have

$$S/V_U = (1 - \delta)/(1 - \sigma).$$

Therefore

$$\beta_L = \beta_U (1 - \sigma)/(1 - \delta)$$

By CAPM, using $r_f = r (1 - t_b)$, we have

$$k_E = r_f + \frac{1 - \sigma}{1 - \delta} \beta_U (r_m - r_f)$$

$$= \frac{1 - \sigma}{1 - \delta} k_U + \frac{\sigma - \delta}{1 - \delta} r_f$$

Since $k_L = k_E (1 - \delta) + \delta r_f$

$$= (1 - \sigma) k_U r_f + (\sigma - \delta) r_f + \delta r_f,$$

we have

$$k_L = (1 - \sigma) k_U + \sigma r_f = k_U - \sigma (k_U - r (1 - t_b))$$

QED.

$$\text{Now, } 1 - \sigma = V_U/V_L = \frac{k_E (1 - \delta) + r \delta w - a_1}{k_U - a_2}$$

Substituting for k_E, we have

$$(1 - \sigma) k_U + (\sigma - \delta) r_f + r \delta w - a_1 = (1 - \sigma)(k_U - a_2)$$

Since $a_1 - a_2 = f_a(1-t_g)g$, we have

$$\sigma = \frac{r (t_e - t_b) + f_a(1-t_g)g}{r_f - g(1-t_g)} = \frac{r (t_e - t_b) + f_a(1-t_g)g}{r (1 - t_b) - g(1-t_g)}$$

Using $V_L = V_U + \sigma V_L$, we have

$$V_L = V_U + \frac{r (t_e - t_b) + f_a(1-t_g)g}{r (1 - t_b) - g(1-t_g)} \delta V_L$$

or

$$V_L = V_U + \frac{r\,(t_e - t_b) + f_a(1-t_g)g}{r\,(1 - t_b) - g(1-t_g)}\,D$$

QED.

The above proof is equivalent to an MM type arbitrage argument that investing Re. 1 in the equity of the levered firm is equivalent to investing Re. $(1-\sigma)/(1-\delta)$ in the equity of the levered firm and selling bonds worth Re. $(\sigma-\delta)/(1-\delta)$.

Proposition

$$V_U \overset{<}{\underset{>}{=}} V_U^*$$

according as

$$\text{If } \emptyset \overset{>}{\underset{<}{=}} \frac{k_U}{(1-t_e)},$$

where

$$V_U = \frac{(1-t_e) - \mu\,(1-t_g)}{k_U - \mu\emptyset\,(1-t_g)}\,X_1, \text{ and } V_U^* = \frac{(1-t_e) - \mu}{k_U - \mu\emptyset}\,X_1 .$$

[V_U and V_U^* are obtained from eq (1a) and (2a) respectively, when $f_e = 0$ or $h = 1$, and $\pi = 0$.]

<u>Proof:</u>

Let $a = (1-t_e)$,

$m = \mu$,

$c = k_U$,

$d = g = \mu\emptyset$ and

$\tau = (1-t_g)$

The given Proposition may be restated as:

$$\frac{a - \tau m}{c - \tau d}\,X_1 \overset{<}{\underset{>}{=}} \frac{a - m}{c - d}\,X_1$$

according as

$$ad \overset{>}{\underset{<}{=}} mc.$$

Now

$$ad \gtreqless mc.$$

according as

$$ad\ (1-\tau) \gtreqless mc\ (1-\tau)$$

or, according as

$$ad - ad\tau \gtreqless mc - mc\tau$$

or, according as

$$ac - mc - ad\tau + md\tau \gtreqless ac - ad - mc\tau + md\tau$$

or, according as

$$\frac{a - \tau m}{c - \tau d} X_1 \lesseqgtr \frac{a - m}{c - d} X_1$$

QED.

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Exhibit 1: Valuation Under Models 1a (V_U) & 2a (V_U^*)

μ	$\emptyset$	V_U	V_U^*
0.20	0.20	96	95
0.20	0.25	141	159
0.20	0.30	206	286
0.10	0.20	98	97
0.10	0.25	129	132
0.10	0.30	165	174

[Observe that $\emptyset = k_U/(1-t_e) = 0.205$, so that model (2a) overvalues for $\emptyset > 0.205$ and undervalues for $\emptyset < 0.205$, as compared to model (1a). See Appendix III for proof.]

Exhibit 2: Valuation Under Models 1a (V_U) & 2a (V_U^*) when flotation Cost is Non-zero

μ	$\emptyset$	V_U	V_U^*
0.20	0.20	101	102
0.20	0.25	148	171
0.20	0.30	215	307
0.10	0.20	103	105
0.10	0.25	137	144
0.10	0.30	175	190

Exhibit 3: Valuation Under Models 1 & 2 for Different Parameter Values

f_b	f_e	b	π	μ	ϕ	k_u	k_e	τ	g	V_U	V_L	V_U^*	V_L^*
0.00	0.00	0.15	0.00	0.10	0.20	0.08	0.08	0.08	0.02	1.01	1.14	1.01	1.16
0.00	0.00	0.15	0.00	0.10	0.25	0.08	0.08	0.08	0.03	1.34	1.53	1.38	1.63
0.00	0.00	0.15	0.00	0.10	0.30	0.08	0.08	0.08	0.03	1.72	1.99	1.83	2.25
0.00	0.00	0.15	0.00	0.20	0.20	0.08	0.07	0.08	0.04	1.02	1.25	1.02	1.64
0.00	0.00	0.15	0.00	0.20	0.25	0.08	0.07	0.08	0.05	1.50	2.14	1.70	Undef
0.00	0.00	0.15	0.00	0.20	0.30	0.08	0.06	0.08	0.06	2.18	8.94	3.06	Undef
0.00	0.00	0.15	0.05	0.10	0.20	0.13	0.13	0.16	0.07	0.86	1.02	1.01	1.35
0.00	0.00	0.15	0.05	0.10	0.25	0.13	0.13	0.16	0.08	1.13	1.36	1.38	1.96
0.00	0.00	0.15	0.05	0.10	0.30	0.13	0.13	0.16	0.08	1.42	1.75	1.83	2.89
0.00	0.00	0.15	0.05	0.20	0.20	0.13	0.13	0.16	0.09	0.82	1.07	1.02	3.90
0.00	0.00	0.15	0.05	0.20	0.25	0.13	0.12	0.16	0.10	1.17	1.67	1.70	Undef
0.00	0.00	0.15	0.05	0.20	0.30	0.13	0.12	0.16	0.11	1.63	2.69	3.06	Undef
0.00	0.00	0.40	0.00	0.10	0.20	0.08	0.08	0.08	0.02	0.97	1.10	0.97	1.12
0.00	0.00	0.40	0.00	0.10	0.25	0.08	0.08	0.08	0.03	1.29	1.48	1.32	1.57
0.00	0.00	0.40	0.00	0.10	0.30	0.08	0.08	0.08	0.03	1.65	1.93	1.74	2.18
0.00	0.00	0.40	0.00	0.20	0.20	0.08	0.07	0.08	0.04	0.96	1.20	0.95	1.59
0.00	0.00	0.40	0.00	0.20	0.25	0.08	0.07	0.08	0.05	1.41	2.08	1.59	Undef
0.00	0.00	0.40	0.00	0.20	0.30	0.08	0.06	0.08	0.06	2.06	10.23	2.86	Undef
0.00	0.00	0.40	0.05	0.10	0.20	0.13	0.13	0.16	0.07	0.82	0.98	0.97	1.31
0.00	0.00	0.40	0.05	0.10	0.25	0.13	0.13	0.16	0.08	1.08	1.32	1.32	1.92
0.00	0.00	0.40	0.05	0.10	0.30	0.13	0.13	0.16	0.08	1.36	1.70	1.74	2.86
0.00	0.00	0.40	0.05	0.20	0.20	0.13	0.13	0.16	0.09	0.78	1.03	0.95	4.32
0.00	0.00	0.40	0.05	0.20	0.25	0.13	0.12	0.16	0.10	1.11	1.62	1.59	Undef
0.00	0.00	0.40	0.05	0.20	0.30	0.13	0.12	0.16	0.11	1.54	2.85	2.86	Undef
0.05	0.10	0.15	0.00	0.10	0.20	0.08	0.08	0.08	0.02	1.11	1.22	1.15	1.28
0.05	0.10	0.15	0.00	0.10	0.25	0.08	0.08	0.08	0.03	1.47	1.65	1.57	1.80
0.05	0.10	0.15	0.00	0.10	0.30	0.08	0.08	0.08	0.03	1.86	2.15	2.07	2.47
0.05	0.10	0.15	0.00	0.20	0.20	0.08	0.07	0.08	0.04	1.11	1.34	1.17	1.74
0.05	0.10	0.15	0.00	0.20	0.25	0.08	0.07	0.08	0.05	1.63	2.28	1.95	Undef
0.05	0.10	0.15	0.00	0.20	0.30	0.08	0.06	0.08	0.06	2.37	8.99	3.51	Undef
0.05	0.10	0.15	0.05	0.10	0.20	0.13	0.13	0.16	0.07	0.94	1.09	1.15	1.46
0.05	0.10	0.15	0.05	0.10	0.25	0.13	0.13	0.16	0.08	1.23	1.46	1.57	2.10
0.05	0.10	0.15	0.05	0.10	0.30	0.13	0.13	0.16	0.08	1.56	1.88	2.07	3.05
0.05	0.10	0.15	0.05	0.20	0.20	0.13	0.13	0.16	0.09	0.89	1.14	1.17	3.32
0.05	0.10	0.15	0.05	0.20	0.25	0.13	0.12	0.16	0.10	1.27	1.78	1.95	Undef
0.05	0.10	0.15	0.05	0.20	0.30	0.13	0.12	0.16	0.11	1.77	3.05	3.51	Undef
0.05	0.10	0.40	0.00	0.10	0.20	0.08	0.08	0.08	0.02	1.03	1.15	1.05	1.20
0.05	0.10	0.40	0.00	0.10	0.25	0.08	0.08	0.08	0.03	1.37	1.55	1.44	1.68
0.05	0.10	0.40	0.00	0.10	0.30	0.08	0.08	0.08	0.03	1.75	2.03	1.90	2.32
0.05	0.10	0.40	0.00	0.20	0.20	0.08	0.07	0.08	0.04	1.01	1.25	1.02	1.63
0.05	0.10	0.40	0.00	0.20	0.25	0.08	0.07	0.08	0.05	1.48	2.17	1.71	Undef
0.05	0.10	0.40	0.00	0.20	0.30	0.08	0.06	0.08	0.06	2.15	11.57	3.07	Undef
0.05	0.10	0.40	0.05	0.10	0.20	0.13	0.13	0.16	0.07	0.87	1.04	1.05	1.39
0.05	0.10	0.40	0.05	0.10	0.25	0.13	0.13	0.16	0.08	1.14	1.39	1.44	2.02
0.05	0.10	0.40	0.05	0.10	0.30	0.13	0.13	0.16	0.08	1.45	1.79	1.90	2.98
0.05	0.10	0.40	0.05	0.20	0.20	0.13	0.13	0.16	0.09	0.81	1.07	1.02	3.87
0.05	0.10	0.40	0.05	0.20	0.25	0.13	[illegible]	0.16	0.10	1.15	1.70	1.71	Undef
0.05	0.10	0.40	0.05	0.20	0.30	0.13	0.12	0.16	0.11	1.61	3.01	3.07	Undef

V_U and V_L are from model 1, V_U^* and V_L^* from model 2. Undef ind[illegible]s undefined.

In all the above, I=1, tc=0.45, td=tb=0.35, tg=0.25, δ=0.20

Real aftertax r and Ku are 5% and 8% respectively. Tax adjuste[illegible] Fisher effect is assumed.

Footnotes

1 In the "Gordon World" reinvestment equals retentions. Also see footnote 6.

2 In the MM world, investments for all time are specified at the outset.

3 An unpublished working paper which like many others regarded the retained earnings as capital gains. While the paper attempted to overcome the shortcomings of Rashid and Amoako-Adu (who ignored the impact of retentions and reinvestments in their model), the publication of the working paper was preempted by Howe.

4 The investment is specified as a fraction of EBIT rather than EAT to get around the problem of having different growth rates for the pure equity and the riskfree component of the firm.

5 $b\, Y_t(1-t_d) + (1-b)\, Y_t\, (1-t_g) - \mu\, X_t - f_b\, S(V_{t+1} - V_t)$

$- f_e\, F_t + r\, S\, V_t(1-t_b)$

$= (b(1-t_d) + (1-b)(1-t_g))\, Y_t - \mu\, X_t - f_b\, S\, (V_{t+1} - V_t)$

$- f_e\, h\, [\mu\, X_t - (1-b)\, Y_t - (1-f_b)\, S\, (V_{t+1} - V_t)]$

$+ r\, S\, V_t(1-t_b)$

$= \{b(1-t_d) + (1-b)(1 - t_g + f_e\, h)\}\, Y_t - \mu\, (1 + f_e h)\, X_t$

$+ RSV_t(1-t_b) - S\, (V_{t+1} - V_t)\, \{f_b - (1-f_b)\, f_e\, h\}$

$$= \frac{w^* Y_t}{(1-t_c)} - \mu h X_t + R \delta V_t (1-t_b) + \delta (V_{t+1} - V_t) f_a^*$$

$$= \frac{w^* Y_t}{(1-t_c)} - \mu h X_t + r \delta V_t (1-t_b) + \delta f_a^* (V_{t+1} - V_t)$$

$$= w^* X_t - w^* r \delta V_t - \mu h X_t + R \delta V_t (1-t_b) + \delta f_a^* (V_{t+1} - V_t)$$

$$= (w^* - \mu h) X_t + r \delta V_t (1 - t_b - w^*) + \delta f_a^* (V_{t+1} - V_t)$$

$$= (w^* - \mu h) X_t + \delta \{ r (t_e^* - t_b) V_t + f_a^* (V_{t+1} - V_t) \}$$

6 Gordon [6] himself presents a brief history of this model. While Durrand [4] considers this model to be a standard actuarial formula, according to Gordon the actuarial literature has no reference to the economic content involved in the derivation of the model. According to him, Williams [19] seems to have been the first in 1938 to have attempted a valuation formula on the lines suggested by the model. But the latter is said to have abandoned the equation before it could take its well known form, since he tried working with varying values of payout ratio and return on investment (payout ratio multiplied by the ROI being the growth rate), which could not yield him a closed form expression. According to Gordon, the model first appeared in its current form in Gordon and Shapiro [7]. We accept Gordon's claim and accordingly refer to the model under his name.