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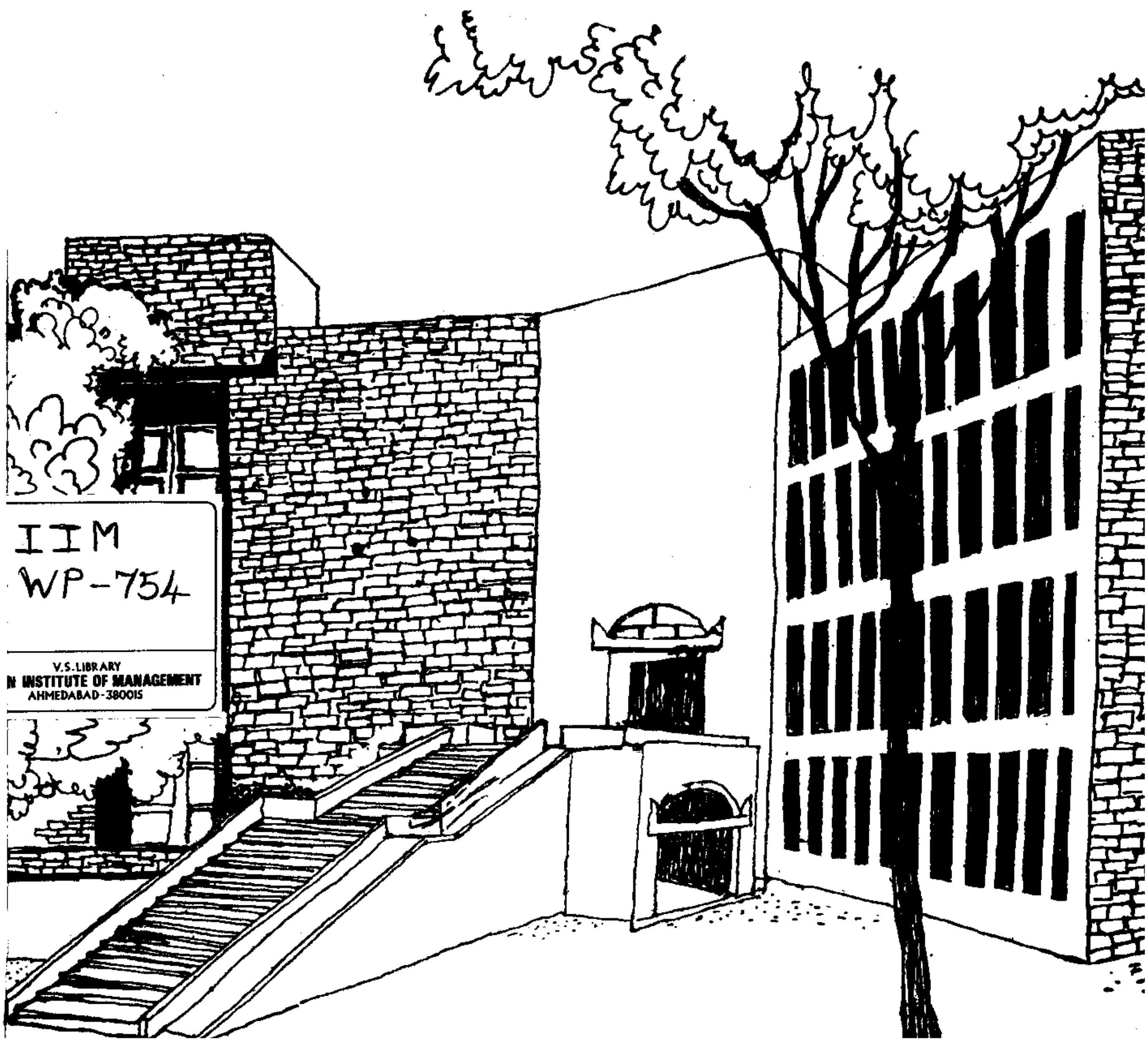

QUADRATIC PROGRAMMING APPLICATIONS: A REVIEW

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QUADRATIC PROGRAMMING APPLICATIONS: A REVIEW

This paper attempts to review applications of quadratic programming and quadratic integer programming. Major applications have been reported in the following areas:Finance, Agriculture, Economics, Production and Operations, Marketing, Public Policy, Water resource management and Transportation. Specific applications in each of these categories are briefly described.

Quadratic programming(QP) is concerned with the problem of optimizing a quadratic function subject to linear constraints. The study of QP problems has been of great theoretical interest in Operations Research. One possible reason could be that it is perhaps the simplest form of nonlinear programming. Moreover there are many application areas where QP formulations have been found very useful.

McCarl, Moskowitz and Furtan [40] have reviewed applications of quadratic programming in a paper published over a decade ago. Since no attempt has been made to study applications of quadratic programming eversince we felt that there was a need to review the OR literature to identify new application areas. In this paper, we attempt to incorporate major applications of quadratic programming and quadratic integer programming(QIP) as have been reported in the literature. We have attempted to classify the applications on the basis of area of application and have discovered that most applications fall in the following categories:

1. Finance
2. Agriculture
3. Economics
4. Production and Operations
5. Marketing
6. Public Policy
7. Water Resource Management
8. Transportation

We will now detail various applications in these categories.

1. FINANCE

Srinivasan and Narendran [48] have used quadratic programming to determine the maximum price that can be paid for a convertible debenture. The objective is to minimise the risk subject to a fixed return. They have developed a model for convertible debenture under the assumption that the redeemable portion is further convertible into equity shares. Since the terms and conditions of the subsequent conversions are not known with certainity, the problem becomes one of minimizing risk subject to a fixed return. Other applications of quadratic programming are in the areas of portfolio selection and capital budgeting.

1.1 Portfolio Selection

Selection of securities for investment purpose has been a problem of great interest since long time. This problem used to be formulated as a linear programming problem. LP formulation was based on maximization of expected net return subject to the fund constraint. Markowitz [39] rejected this formulation as it did not take the risk factor into account. He considered the rule that the investor does consider expected return a desirable thing and variance(as a measure of risk) of return as undesirable. But the tradeoff between return and risk depends on individual's utility function. This problem was formulated as a quadratic programming problem to identify those portfolios which represent the most efficient tradeoffs of risk return (called efficient frontier or E.V. frontier; E stands for expected net return and V stands for variance of return). What is meant by the most efficient tradeoff is identification of those portfolios with minimum V for given E or maximum E for given V.

Individual can select a point on E.V. frontier depending on his/her utility function. In this kind of formulation it is assumed that decision maker's utility function involves only mean and variance of return and the effects of higher moments of the outcome distribution other than the mean and variance are assumed to be negligible. The objective function in the mathematical formulation includes both expected return as well as its variance. Efficient frontier is generated by varying weight given to variance from zero to infinite. If weight given to variance is zero, then problem reduces to the standard linear programming problem. Sharpe [47] had discussed variations of this problem under different conditions using quadratic programming.

Gordon [19] has examined the problem of deriving the tangent (or market) portfolio from a given set of risky assets and a specified risk-free borrowing and lending rate. The problem of finding the portfolio line has been formulated as a QP problem.

Jacob [30] observed that the Markowitz portfolio models are inappropriate for the small investor. The portfolios they generate typically include a larger number of securities than the small investor has resources to manage. Because of the high commission rates he must pay on his odd lot purchases and sales, the small investor cannot afford to be as diversified as mutual funds. He formulated two mean-variance quadratic zero-one programming and one mean-variance zero-one integer programming portfolio selection models, each of which permits the investor to limit the number of securities in his portfolio. Since these models limit the number of securities which will actually be purchased, the amounts of commission to be paid will remain in control. She has formulated the problem of small investor as a mixed integer quadratic programming problem. The investor has to specify a desirable upper bound on number of securities he/she would like to handle. A particular attractive simplification of the model is obtained by requiring equal investment in all securities included in the portfolio enabling all decision variables to be binary valued which converts the mixed integer quadratic programming formulation into a zero-one quadratic programming formulation. Faaland [15] has suggested a model which is more general than model suggested by Jacob. In the formulation suggested by Falland, the investor's wealth is divided into specified number of equal parts, and the investor may invest any number of parts (rather than only one) in a particular security.

Cotner and Levary [9] have formulated the problem of determining short-term multiple currency portfolios using quadratic programming. Based on the Eurocurrency deposit interest rates and currency exchange rates, they have devolped a quadratic programming model for determining optimal short-term multiple currency denominated portfolios. This model can be applied in determinig efficient portfolios at various levels of expected returns. It is shown that investing in these multicurrency portfolios will provide higher risk-adjusted returns then comparable single currency holding. The major risks associated with the Eurocurrency deposits are the currency exchange risk and the inflation risk. If inflation rates among countries are not perfectly correlated across time then it follows from the basic Markowitz model that portfolios of forign currency are expected to be superior to single currency holdings. They also formulated a zero-one quadratic programming model to take care of the problem that most securities can be purchased only in some standard units. This includes a given minimum quantity and a given increment to the minimum.

1.2 Capital Budgeting

Traditional capital budgeting models assume that projects are independent. But as observed by Weingartner [53] project interrelationships such as mutual exclusion and interdependence do exist in reality. Mutual exclusion means that if one project is selected, the other project is rejected. Interdependent projects mean that selection of certain projects may depend on whether some other projects have been selected or not. He used zero-one variables to take care of such problems and formulated the capital budgetting problem as a mixed zero-one QP model.

2. AGRICULTURE

The linear programming models have been widely used for determining profit maximising combination of crop. The conventional deterministic models ignore uncertainty. Although income variability has long been recognised as an important component of farmer's decision making process, the first attempt to formulate this problem as a quadratic programming problem was made by Stovall [49]. He used quadratic programming formulation in a manner similar to portfolio selection. The expected income-variance (E.V.) criterion of QP assumed that farmer holds preferences among alternate farm plans solely on the basis of the expected income E and the associated income variance V. Given this assumption, a farmer rationally should restrict his choice among those farm plans for which the associated income variations are minimum for a given expected level of return. Hazell [24] has developed a set of feasible farm plans (E.V. plans) using quadratic programming formulation.

Manos and Kitsopanidis [37] have used quadratic programming models for a farm planning of a region in central Macedonia. Quadratic programming models were used because risk due to uncertainties of technical and economic coefficients, and the quantities and prices of resources. A quadratic programming model (the E-V model) was used to plan a Greek Farm region. It is reported that the resulting plan was preferred by the farmers to those resulting in linear and mixed integer programming models and to the previously used plans because it included crops expected to give the highest minimum total gross margin with the same fixed costs. The farmers preferred plans that achieved not only the highest but also the most stable economic results.

3. ECONOMICS

Quadratic programming formulations have been used in theory of firm models, utility theory and econometrics. Quadratic programming has also been used in spatial and temporal equilibrium analysis reported latter as applications in the category of public policy.

3.1 Theory of Firm Models

In theory of firm models, quadratic programming is of interest because of the flexibility and realism it provides over linear programming. In linear programming the unit cost (or profit) of each activity or process is assumed to be fixed, and so are the availability of the inputs, or the requirements of the outputs. In practice these elements are often not fixed. By postulating appropriate linear dependencies, for instance between demand and price, it is possible to expand linear programming into quadratic programming. Houthakker [29] used quadratic programming for the case of monopolist. Monopolist attempts to maximize his/her total revenue when faced with linear demand with output limited by scarce resources.

Chen [8] has applied quadratic programming to cost volume profit analysis under contribution margin uncertainty. He has presented three CVP probabilistic chance-constrained models based on various safety limit criteria for decisions under uncertainty.

The quadratic programming procedure offers both the technical relief from the computational difficulties posed by the probabilistic constraints and a desired flexibility in generating and presenting the relevant information for decisions under uncertainty.

3.2 Utility Analysis

Houthakker has also suggested using the quadratic programming formulation to describe and plan for a whole economy with linear technology where individual's preferences can be represented as quadratic objective function. Indifference curves (in consumption) can be also analysed by QP formulation of diminishing returns to scale.

3.3 Econometrics

The strict assumption of the classical linear regression model, which has justified the use of least-squares estimation, may be untenable in many economic situations.

Goldberger [18] describes some of the problems in economic situations where use of least square estimation cannot be justified. Economical disturbances may not be 'spherical'. For example, high-income families show much greater variability in their savings behaviour than low-income families. Therefore the assumption of a common disturbance variance would be inappropriate for a cross section saving income relationship. For another example, in the time series analysis a disturbance is likely to presist over several periods. Hence the assumption of independence may be inappropriate.

Range of the variation of the regression is inherently limited. Most of the time though the outside information is available, it cannot be exploited by the least square estimation. Thus in order to develop a less restrictive statistical framework quadratic programming formulation can be used. In quadratic programming formulation whatever restrictions are required can be put as constraints.

Zellner [54] has used quadratic programming to come out with simple forecasting decision rules which incorporates prior knowledge.

Buder and Buongiorno [20] have formulated econometric model of the North American newsprint industry forecasts, over longtime periods and by regions, the amount of newsprint produced, consumed, its price, and quantities transport between two regions. The core of the model is a recursive quadratic programming system that simulates the behaviour of competitive industry.

Carey, Hendrickson and Siddharthan [7] have used quadratic programming for direct estimation of origin destination trip matrices for the transportation studies. With linear demand functions, volume estimates have been obtained from a quadratic programming problem which minimizes the sum of squared errors from a demand function subject to constraints derived from observation of some travel volumes.

Nijkamp and Paelinck [41] have used quadratic programming for updating Input-Output coefficients.

4. PRODUCTION AND OPERATIONS

Quadratic Integer Programming has been used in location, layout, scheduling and assignment problems. Most of these problems have been classified as Quadratic Assignment Problems in literature.

4.1 Location and Layout

Burkard [6] has described so-called Backboard wiring problem and formulated the same as a QIP problem. In various technical applications the problem is to place certain modules on a board and to connect them by wires. The modules should be placed such that the total wire length becomes minimal. The problem formulation involves integer variables which take value one if a module takes the required position otherwise it takes value zero. He has also described the campus planning exercise where quadratic programming is used to decide facility location. There are n facilities to be located on n positions. The objective function is quadratic in nature as it involves minimisation of walking for facility users.

Koopmans and Beckmann [32] were perhaps the first to study the problem of locating indivisible economic activities to minimize total cost of movement. They formulated this problem as a binary QIP problem.

Elshafei [13] has also formulated the problem of hospital layout as a binary QIP problem where the objective is to minimise the longest way from a doctor to patient.

Bazaraa [4] has considered the problem of layout of warehouse or a factory floor or a floor of hospital. Different objects may be located in the area interacting with each other. The interaction may be information flow, material flow, employee movement, etc. The objective may be to minimize the total cost of interaction. In contrast to the standard problem, he has considered the problem of different sites and different area requirements of different objects.

Hodder and Jucker [27] have formulated the problem of uncapacited plant location under uncertainty in a mean-variance framework with prices in various markets correlated via their response to common random factor. This formulation results in a mixed QIP problem.

4.2 Scheduling

Hodgson, Kilpatrick and Longini [28] have considered the problem of scheduling multispeciality clinics in a university health centre using quadratic integer programming. The objective has been to maximize expected cross consultations. 30 clinics have to be scheduled for a five day cycle. Each clinic has its own medical staff but shares space, personnel and ancillary services with other clinics on rotating basis. If two specialities requiring cross-consultations do not meet at the same time, the consultation process is impaired. The clinics must be scheduled such that the capacities available act as constraints. The schedule must also consider the commitment of phlysicians to teaching, hospital rounds and operating room duties. This problem has been formulated as a binary QIP to maximise the expected cross-consultation volume subject to constraints on shared resources.

Geoffrion and Graves [16] have considered the problem of scheduling parallel production lines with changeover costs. Production orders for a number of products are to be scheduled on a number of similar production lines so as to minimize the sum of product dependent changeover costs, production costs and time constraint penalties. The problem has been formulated as a binary QIP problem.

Delporte and Thomas [12] have considered the problem of simultaneously making lot size and sqeuencing decisions for N products on one facility with deterministic demand. Mathematical programming formulation of entire problem is impracticable. They have given special formulation given that sequence is known and both (potentially unequal) lot sizes and idle time periods are treated as variables. The problem is formulated for infinite horizon with zero switch rule under the assumption of sequence independent changeover time and costs. The objective function is quadratic in nature and involves binary variables.

Biggs and Laughton [5] have discussed a problem of optimum scheduling of an electric power system. A realistic and moderately large problem is solved using quadratic programming.

Walas and Askin [52] have discussed the problem of minimizing part programmes for numerical control (NC) punch press. The part programme contains order in which the holes are punched into the part and the assignment of the tools to the indexing format. They have broken the problem into two classical problems: The travelling salesman problems and the quadratic assignment problem.

4.3 Assignment

Dutta, Koehler, and Whinston [11] have considered the problem of an optimal allocation in a distributed processing environment. One of the major problem in a distributed processing environment is the assignment of various sub-problems (modules) to various processors in an optimal fashion. Typically, the different modules comprising the problem interact with one another in varying degrees. For example, consider a distributed database system, processor at one site will need to refer to files at other site. They have formulated this as a QIP problem in which the total cost is minimized. Costs involved are cost of running modules and cost of communication between modules. Any module demand can be identified with exactly one source, constraints follows from the practical requirement that files not be split across several sites.

Heinhold [25] has developed a model to explain the allocation of clients to different locations of a certain class of service institutions. It can be used for all types of allocation problems which have the features: clients are travelling from their house locations to the service places, they can choose among several locations of the institution all of which offer the same services, they incur a constant travelling time or cost, and also incur different waiting times or costs at each locations which depend on the number of clients choosing that service station. The objective of the individual client is to minimise the total time or cost required. A QIP approach has been used to study the allocation of cars which must periodically be checked for traffic safety at official test stations.

4.4 Capacity Planning

Ammons and McGinnis [1] have used quadratic programming to estimate the annual production cost, which is a crucial component in long range planning for electric utilities. Traditional simulation based model are quite expensive in terms of computer resources (space and time). They have incorporated quadratic production cost for each unit and has used optimization sub-models to develop a system production cost versus output curve. The method is tested using data from a large South Eastern Utility.

4.5 Product Design

Ladany [34] has modelled the problem of optimal decision of key boards in which various characters have to be allocated to given keys. The problem is stated as zero one quadratic assignment problem. The solution depends on the joint probability distributions of the two string combinations of the character that describe the input data and on the key servicing time data.

Hiroaki, et. al. [26] have considered the problem of a generalised uniform processor design. The objective is to determine both the optimum speeds of processors and the optimum schedule in a preemptive multiprocessor environment. The jobs are independent and each processor can be assigned any speed. However, the cost associated with each processor is a function of the processor speed.

Burkard [6] has listed various applications of quadratic integer programming in product design. He describes use of quadratic integer programming to design control boards in order to minimize eye fatigue.

5. MARKETING

Brand switching model based on markov process is one of widely known model in marketing. Theil and Ray [51] have considered the problem of estimating transition probabilities in a markov process from a set of observations on one dimensional frequency distribution. There is no evidence on the number of persons who actually shift from one brand to another. Only available data is the market share of various brands in a given year. They have used quadratic programming to get the restricted regression, using constraints to make sure that probabilities are greater than or equal to zero and are less than or equal to one. They have also compared the results obtained by the least square method with the quadratic programming method.

6. PUBLIC POLICY

In the area of public policy, use of pricing policy to make certain changes in demand or supply have been studied using quadratic programming. Spatial and temporal equilibrium models have been developed using quadratic programming extensively.

6.1 Pricing Policy

Glassey [17] used quadratic programming to study the effect of price control policy adopted to mitigate the impact of quadrapling world crude prices in 1973-74. He has considered 21 sector input - output model of 1972 US economy and extended it to include consumer demands, imports and exports as endogenous variables. He has assumed that consumer demand as well as imports and exports vary linearly with prices. The equilibrium of the model economy is obtained by maximising net social payoff, which is quadratic in nature. Such a model can be used by policy makers to examine the impact of perturbations of the supply and prices of various fuel and energy sources. Effects of various policies can be studied and can help in determining the right policy.

Louwes, Boot and Wage [36] have studied the problem of optimal use of milk in Netherlands. Agricultural stabilization fund run by government was facing the problem of maintaining subsidies. Government assured a minimum price to producer. Production was rising but the export prices were lower than minimum price assured by the government. The objective of the study was to study the pricing policy and how the available quantity of milk should be split between milk for consumption, butter and cheese. The problem was formulated as a QP problem of maximising revenue. The model included a policy variable to study the effect of changes in price.

6.2 Economic Planning over Space and Time

Quadratic programming has been extensively used in models for economic planning over space and time. Enke [14] and Samuelson [44] were first to study the problem of spatially separated markets. The problem deals with two or more regions trading a homogenous commodity. Each region constitutes a single and distinct market. There are no physical restrictions on the movement of goods. For each region the function which relates local production and local use to local price are known. Given these trade functions and transportation costs, Enke - Samuelson were interested in finding out the following:

1. the net price in each region
2. the quantity of exports or imports for each region
3. volume and direction of trade between each possible pair of regions

Samuelson formulated this problem as minimum transport cost problem. It was suggested that problem can be solved by trial and error or by a systematic procedure of varying shipments in the direction of increasing social payoff.

In order to solve spatial problem most of the economists assumed that regional demands and supplies are fixed or predetermined. These assumptions were made mainly to get linear programming formulation. Takayama and Judge [50] have shown that within the framework of interconnected competitive markets by postulating appropriate linear dependencies between regional supply, demand, and price, it is possible to convert the Samuelson formulation into a quadratic programming problem. The constraints are identified to ensure that the difference in prices between any two regions can differ utmost by the cost of transportation. The social payoff function is maximized. The above model is also extended to multi-region and multi-product situation. It has also been shown that the same model can be used to obtain solutions over time dimension. The transportation costs between regions would be replaced by storage costs or carrying charges between time periods.

Sasaki [45] has extensively used this quadratic formulation to the spatial equilibrium analysis of the three important commodities in eastern Japan's agriculture. The commodities considered are milk, hogs, and poultry. The demand of these had been increasing sharply. The analyses were carried out for the purpose of inter-regional adjustment as to production, transportation and marketing for various form products. For the purpose of spatial equilibrium eastern Japan was divided into 15-20 regions depending on the product. Effect of various measures like adjusting regional production, improving transport facilities were studied on the price differentials. This kind of modelling has helped in evaluating various policies and their implications.

Lee and Seaver [35] have developed a positive model of spatial equilibrium of the broiler market in U.S.A. Farm prices of broilers were decreasing and feed prices were increasing in the north eastern states relative to that of southern states. In view of these changes in the broiler economy, the objective of the study was to analyse the effect of changes in feed price in each region on:

1. broiler production
2. price of broilers
3. shipment of broiler among regions
4. derived demand for feed.

For the purpose of the study, U.S.A. was divided into three regions. The model used quadratic programming formulation suggested by Takayama and Judge.

Schmitz and Bawden [46] have carried out spatial price analysis of world wheat economy based on Takayama and Judge quadratic formulation. They have extended the model to international trade. The world wheat economy which occupied more of the world's cultivated acrage than any other single cereal crop, was undergoing major changes. The objective of the study was to predict the future situation in each country under alternative economic and policy assumptions. They were also interested in studying the effect on world wheat economy of changes in U.S wheat programmes and U.S.A. trade policy. For their study, they divided world into 15 geographical areas.

Guise and Mensah [22] have tried to use the Takayama and Judge's quadratic programming formulation in examining possible ways of improving banana growers' returns by casting the problem into the framework of alternative spatial temporal price and allocation models. The banana industry in Australia regularly used to encounter periods of glut and shortages resulting in high and low prices, respectively. While in the past buffer stocks of such a perishable commodity for price stabilization purpose had been technologically infeasible, the recent technological advances made stocking of such bananas economically feasible. They have tried to study the effect of this technological development for the welfare of the banana producers. For the purpose of analysis markets were distinguished both spatially and temporally.

Guise and Flin [21] have used a spatial and temporal quadratic programming model to determine optimum pricing and allocation of water in a river basin for three different uses of water and over four seasons of year. The supplies of water in many river basins were limited, while users of water were increasing.They have divided users into three groups:

1. Power Generation requirement group
2. Urban water requirement group
3. Rural irrigation requirement group

They have tried to maximize economic efficiency of the use of water by properly allocating and pricing the water resources.

Pandey and Takayama [42] have used Takayama and Judge's quadratic formulation to carryout temporal equilibrium analysis of rice and wheat in India. In India the agricultural revolution brought about significant changes on the food front. This development called for reassessment of the situation with respect to consumption patterns of foodgrain over a finite time horizon in the future so as to make a proper reorientation of country's food policy. The objective was to conduct a temporal equilibrium analysis over the operation of Fourth Five Year Plan.

Kottke [33] has analyzed the problem of determining an optimal spatial and temporal price and allocation of dairy industry which was characterised as a combination of competitive and mixed components. To deal with multi-dimensions and diversity of competitions the quadratic programming model has been supplemented by set of recursive formulations.

Hall, Heady, Stoecker and Sposito [23] have used quadratic programming to derive spatial competitive equilibrium for the crop and livestock sectors of the U.S. agriculture economy. Agriculture commodities are clasified into three mutually exclusive classes: primary, intermediate and desired. Primary commodities represent available resources; intermediate commodities are produced only as a input for further production. acquired commodities are wanted either for consumption or for other uses outside the system (export). The 48 contiguous states and the district of Columbia are positioned into 10 markets, which are further partitioned into a total of 103 crop producing regions. In each market, demand for 10 desired commodities are presented by functions, linear in commodity prices. Production of crops and livestock, and inter market commodity shipments are represented by the linear activities. The objective function maximizes the aggregate producer profits. Since the demand function is linear, total revenue is quadratic hence the quadratic programming formulation.

Koo and Uhin [31] have used quadratic programming model to evaluate the spatial equilibrium conditions for U.S. wheat exports under alternative restrictions and transport costs.

6.3 Investment Decisions

Armstrong and Willis [3] have developed a water planning model which simultaneously considers investment and allocation decisions. They have questioned the existing formal decision framework in public policy used in large scale water investment and allocation planning. One of the major weakness of the existing framework is that it fails to consider simultaneously both water resource investment decisions for multiple supply source and allocation (pricing) of the resulting water supplies over region and uses. They have maximized the present value of the stream of net returns over time in terms of various investment options. Binary variables are used to make investment decisions and a formulatrion similar to Takamaya and Judge is used for allocation decisions. The model has been applied to a region in California.

7. WATER RESOURCE MANAGEMENT

Marino and Loaiciga [38] have developed a quadratic optimization model for reservoir management to obtain operation schedules. The model treats spillage and penstock release as decision variables and takes advantage of system dependent features to reduce the size of the decision space. The adequate fulfilment of other system functions such as flood control and water suply is guaranteed via constraints on storage and spillage.

Peralta and Killian [43] have modelled the problem of optimal regional potentiometric surface design so as to have least cost water supply. The model gives sustained groundwater withdrawal strategy. It maximizes the regional cost of attempting to satisfy the water needs of each finite difference cell from (a) groundwater and diverted surface water or (b) from groundwater and reduction of water needs achieved by reducing production acreages. Groundwater elevations, withdrawals, and recharges form constraints to satisfy legal and hydrological requirements. The technique is applicable for assuring a regional sustained yield of groundwater.

Arikol and Basak [2] have modelled the problem of stream water quality management. A new criterion of equity is introduced in the model and a quadratic programming problem with two objectives, namely equity among dischargers and minimization of total cost, is formulated.

8. TRANSPORTATION

Daskin, Schofer, and Talley [10] have devised an optimization model using quadratic programming for designing and evaluating distance-based and zone fares for urban transit. Public transit operation is facing the problem of balancing service requirements against financial resources. They have devised a model that finds the fixed charge, milage charge, and transfer charge to maximize gross revenue subject to constraints on ridership and the form of the fare equation. They have assumed that transit demand is inelasitic to prices which results in linear demand curve. This results in a quadratic term in the objective function. The model is also used for exploring a variety of fare policies.

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