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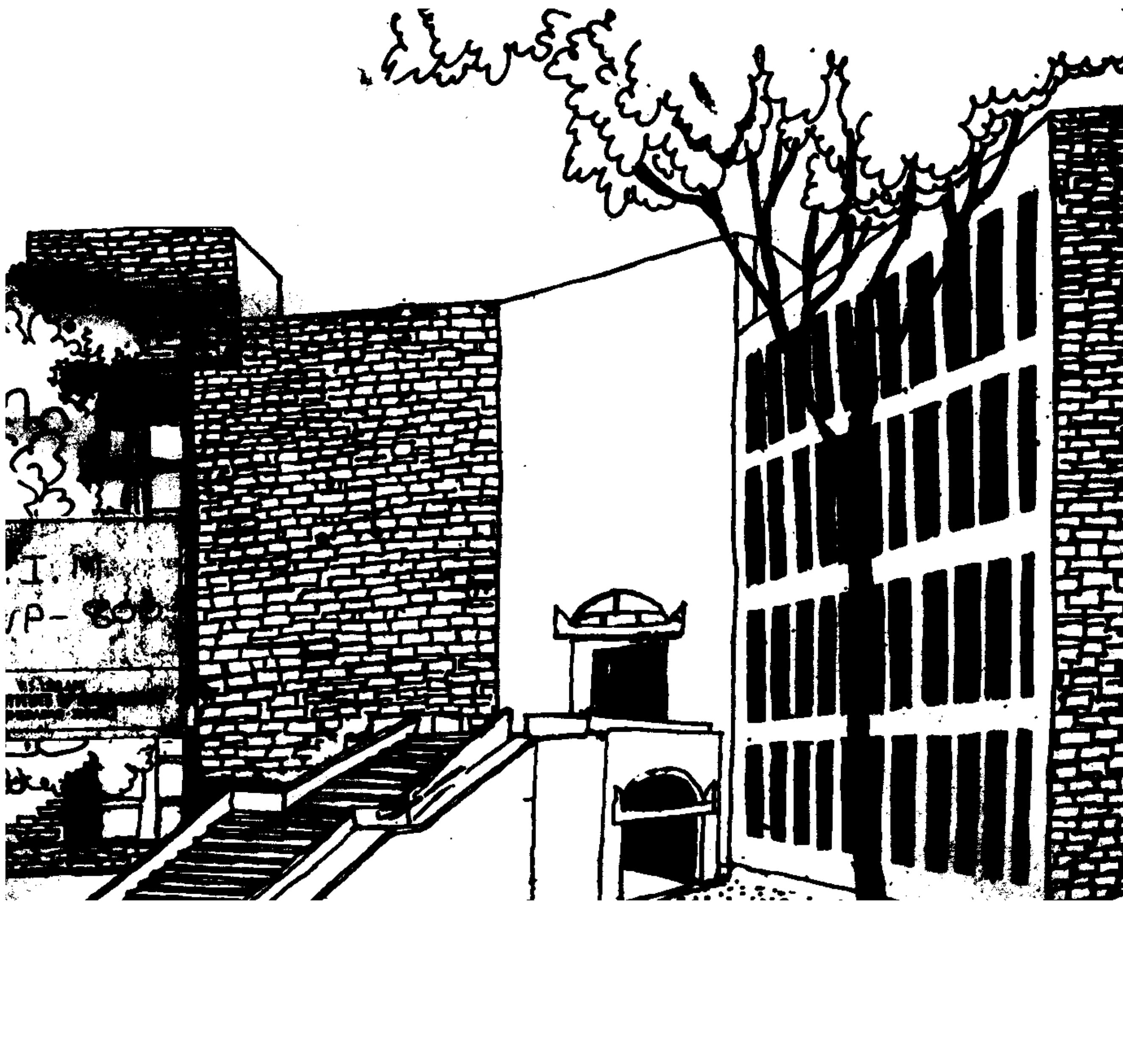

A DYNAMIC PROGRAMMING APPROACH
TO DETERMINE OPTIMAL MANPOWER
RECRUITMENT POLICIES

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ABSTRACT

The manpower planning models available in literature have dealt with how changes takes place in a manpower planning system, under various operating and policy constraints. However none of these models have identified the manpower system costs. In this paper we have identified various manpower system costs. Further, we have developed a manpower planning model with the objective of minimizing the manpower system costs. The model has been found to be analogous to Wagner-Within model in Production/Inventory management. A numerical illustration has been given to validate the model.

Key Words: Manpower Planning, Dynamic Programming

INTRODUCTION

The manpower planning models available in literature have extensively dealt with the description of how changes takes place in the system. Models have been developed for various constraints and operating policies under which the system operates. Bartholomew and Forbes[1] have described the state of art in various facets of manpower planning. Edwards[2] has reviewed the various models which have been developed with emphasis on their assumptions and applications and concluded that good presentation of results and ease of use are more important than theoretical sophistication. Price and Piskor[3] have formulated a goal programming model of the manpower planning system for officers of the Canandian forces with the objective of minimizing the weighted sum of deviations of the constraints viz., financial, manning promotion and manpower accounting. Penalities have been assigned in the decreasing order to financial, promotion, manning and manpower accounting. However the problem formulated by Price and Piskor[3] needs to assign 1244 weights which are subjective in nature. As a result the final solution is sensitive to the weights assigned to the deviational variables.

Zanakis and Maret[4] have formulated a markovian goal programming model with preemptive priorities and provided a more flexible and realistic tool for manpower planning problem. Mehlmann[5] has developed optimal recruitment and transition strategies for manpower systems using dynamic programming and has shown that these strategies are linear function of present state and of present and future goals. He formulated a dynamic programming recursion with the objective of minimizing a quadratic penalty function which reflects the importance of correct manning of each grade under preferred recruitment and transition patterns.

While the models developed in manpower planning literature have considered financial, labor costs and various recruitment policy objectives as indicated above, surprisingly no work has been reported with the objective of minimizing the manpower system costs. Manpower system costs depends upon the various factors outlined in the next section. Of late, due to increased competition cumbersome recruitment procedures have been evolved as a result of which recruitment costs have increased enoromously, which influence the manpower system costs.

In this paper a mathematical model has been developed with the objective of minimizing the manpower system costs. The model resulted in the form of a dynamic programming recursion which is analogous to the well known Wagner-Within model in Production/Inventory management to determine the economic lot size. The model takes the manpower requirements for future periods (which is determined by the changes that take place in the system) as input and basing on the cost data generates optimal recruitment schedules for the future periods.

MANPOWER SYSTEM COSTS

In this section an attempt has been made to identify the various costs associated with the operation of a manpower system. The relevant costs in a manpower system consists of the following:

1. Recruitment Cost
2. Overstaffing Cost
3. Understaffing Cost
4. Firing/Retirement Cost

<u>1. Recruitment Cost:</u> This is the cost incurred in the process of recruitment. The recruitment costs can be broadly classified into two categories viz., fixed costs and variable costs of recruitment which is proportional to the number of people recruited. The following is the different components of recruitment cost.

a) Cost of Advertising
b) Cost of administrative authority which makes recruitment policy
c) Cost of manpower working on the processing of applications
d) Cost of information processing
e) Cost of conducting written test
f) Costs incurred in the form of payment to the interview committee members(if hired from outside) or the wages of the people in the interview committee(if the members are internal)

g) Travelling expenses paid to the candidates

h) Cost of medical examination done by the organisation

i) Cost of training people

j) Miscellaneous expenditure which includes postage, telephone calls etc.

While the above components of cost are only indicative, the actual components of the recruitment cost depends upon the recruitment procedure followed by the organization. Often the organizations charge the applicants in the form of application processing fee. But it has been found that in many organizations the revenue generated from this fee is disproportional to the actual recruitment costs. Of the various components of cost identified above, the cost of advertisement, the cost of administrative authority form a fixed component, which is independant of the number of people recruited. Some costs like cost of interviewing the people, costs of travelling expenses paid to the candidates depends upon various factors like the turnout of the candidates, suitability of the candidates etc. Moreover these costs also depends upon the policy of the organization to determine the number of candidates to be interviewed etc. If the management's policy is to call people in a certain predetermined ratio these costs will be proportional to the number of candidates called and hence will be a constant. The costs of conducting written test, cost of manpower working on the processing of applications, information processing costs, the cost of medical examination and the cost of training people will have a certain fixed component and a variable component per recruitee. In a typical military recruitment process where the selection process is in groups the fixed costs are going to be higher.

2. Overstaffing Costs: Overstaffing costs are the costs resulted due to the unutilized workforce. These costs are analogous to the inventory costs in a Production/Inventory situation.

3. Understaffing Costs: Understaffing costs are the costs resulting due to decreased productivity and loss of goodwill(in a profit motive organization) as a result of the non-availability of the workforce.

4. Firing/Retirement Costs: These costs result in due to the retrenchment or retirement of the employee.

MATHEMATICAL MODEL

We make the following assumptions while formulating the manpower planning problem to determine optimal recruitment policies.

a) The recruitment size is known and fixed.

b) Recruitment at a particular grade is considered.

c) Recruitment and overstaffing costs are known and fixed.

d) Understaffing is not allowed.

Notations:

R_t Requirement in any period t

S_t Fixed recruitment cost in period t.

i_t Cost of overstaffing per recuitee per period.

x_t Number of people recruited in period t.

I_t Number of people recruited in an earlier period for requirements of period t.

We need to satisfy all requirements on time, so that understaffing is prohibited. The cost structure consists of the following components in period t.

a) Recruitment cost in period t is given by the concave function: $S_t \delta(x_t) + vx_t$.

where v is the unit variable cost of recruitment and is assumed to be constant for all periods.

$$\delta(x_t) = 0 \text{ if } x_t = 0$$
$$= 1 \text{ if } x_t > 0$$

b) The overstaffing cost $i_t I_t$

The total cost of recruitment for the T-period planning interval is

$$\sum_{t=1}^{T} [S_t \delta(x_t) + vx_t + i_t I_t] \qquad \ldots (1)$$

We take $I_0 = I_T = 0$ without loss of generality. The problem is to minimize this sum, subject to the constraint that all requirements must be met on time and since the variable cost of recruitment is constant we have $v\sum_{t=1}^{T} R_t$ is a constant in equation(1).

The problem may therefore be stated as follows:

$$\text{Minimize} \quad \sum_{t=1}^{T} [S_t(\delta_t) + i_t I_t]$$

$$\text{subject to} \quad \sum_{K=1}^{t} x_K \geq \sum_{K=1}^{t} R_K \ , \ t = 1,2,\ldots,T.$$

DYNAMIC PROGRAMMING FORMULATION

In this section we characterize the analogy of the well known Wagner-Within model to determine economic lot size with the model which we have developed in the previous section. It is easy to see that the fixed recruitment cost which we have described earlier is analogous to the set-up (or ordering) cost and overstaffing cost is analogous to cost of carrying inventory in an inventory system.

We give below the propositions of the Wagner-Within model which facilitate formulation of a dynamic programming recursion.

<u>Theorem 1:</u> There exists an optimal program such that

$I_t x_t = \emptyset$ for all 't'.

<u>Theorem 2:</u> The minimum cost policy has the property that x_t takes the following values: $\emptyset$, R_t, $R_t + R_{t+1}$, ...,$R_t + R_t + \ldots + R_T$

<u>Theorem 3:</u> There exists an optimal program such that if R_{t^*} is satisfied by some $x_{t^{**}}$. $t^{**} < t^*$, then R_t, $t = t^{**} + 1, \ldots, t^* - 1$ is also satisfied by $x_{t^{**}}$.

<u>Theorem 4:</u> Given that $I_t = \emptyset$ for period t, it is optimal to consider periods 1 through t - 1 by themselves.

For the detailed proofs of the above theorems the reader is adviced to refer Wagner-Within[6].

Let F(t) denote the minimal cost program for periods 1 through t. Then

$$F(t) = \text{Min.} \left[\min_{1 \leq j < t} \left[S_j + \sum_{h=j}^{t-1} \sum_{K=h+1}^{t} i_h R_K + F(j-1) \right], \; S_t + F(t-1) \right] \qquad \ldots (2)$$

The above recursion stated in words means, the minimum cost for the first 't' periods comprises a fixed recruitment cost in period j, plus the charges for satisfying requirements R_K, k = j+1,.....,t by recruiting manpower in period j, which results in overstaffing cost, plus the cost of adopting an optimal policy in periods 1 through j-1 taken by themselves.

We give below the manpower planning horizon theorem analogous to Wagner-Within planning horizon theorem which further simplifies determination of optimal policies.

<u>The Manpower Planning Horizon Theorem:</u> If at any period t^* the minimum in (2) occurs for $j = t^{**} < t^*$, then in periods $t > t^*$

it is sufficient to consider only $t^{**} \leq j \leq t$. If $t^{*} = t^{**}$, then it is sufficient to consider programs such that $x_{t^{*}} > 0$.

The proof for this theorem follows on the same lines of Wagner-Within planning horizon theorem.

ALGORITHM

In this section we give below the algorithm prosposed by Wagner-Within which can be made use of to determine Optimal recruitment policies.

The algorithm at period t^{*}, $t^{*} = 1,2,\ldots,N$ may generally be stated as

1. Consider the policies of recruiting at period t^{**}, $t^{**} = 1,2,\ldots,t^{*}$ by this order.
2. Determine the total cost of these t^{*} different policies by adding the fixed recruitment costs and overstaffing costs associated with the recruitment at period t^{**} and the cost of acting optimally for periods 1 through $t^{**} - 1$ considered by themselves. The latter cost has been determined previously in the computations for periods $t = 1,2,\ldots,t^{*} - 1$.

3. From these t^{*} alternatives, select the minimum cost policy for periods 1 through t^{*} considered independantly.

4. Proceed to period $t^{*}+1$ (or stop if $t^{*} = N$).

NUMERICAL ILLUSTRATION

Table 1 gives hypothetical data for a 10 year planning period of a manpower system. Table 2 summarizes the calculations of the manpower planningproblem presented in Table 1.

Table-1 to be inserted here

To illustrate the optimal plan for period 1 alone is to recruit in period 1 (incurring a fixed recruitment cost of 728). Two possibilities must be evaluated in period 2: recruit in period 2 and use the optimal policy of period 1, which costs 1433(728+705); or recruit in period 1 for both periods, which incurs a overstaffing cost of 510 in period 1 apart from the fixed recruitment cost of 728. The total cost of this policy is 1238(728+510). Thus in the two period problem the optimal policy is to recruit in period 1 for both periods. In period 3 three alternatives are to be evaluated: recruit in period 3 for period 3 and follow the optimal policy of period 2 (at a cost of 1936), recruit in period 2 for periods 2 and 3 and follow the optimal policy of period 1 (at a cost of 2057) or recruit in period 1 for periods 1,2, and 3 (at a cost of 2642). Thus in period 3 the optimal policy is to recruit for period 3 alone. It may be noted in period 4 it is sufficient to consider only two alternatives (This follows from Manpower planning horizon theorem): recruit in period 4 for period 4 while following the optimal policy of period 3 or recruit in periods 3 and 4 (at a cost of 2650) and follow the optimal policy of period 2 (at a cost of 2912).

Table-2 to be inserted here

Thus the optimal policy is

1. Recruit in period 9, x_9 = 48+34 = 82 and use the optimal policy for periods 1 through 8, implying
2. Recruit in period 7, x_7 = 56+29 = 85 and use the optimal policy for periods 1 through 6, implying
3. Recruit in period 6, x_6 = 89 and use the optimal policy for periods 1 through 5, implying
4. Recruit in period 4, x_4 = 61+25 = 86 and use the optimal policy for periods 1 through 3, implying
5. Recruit in period 3, x_3 = 52 and use the optimal policy for periods 1 through 2, implying
6. Recruit in period 1, x_1 = 79+34 = 113.

The total cost of this policy is 5966.

CONCLUSIONS

In this paper an attempt has been made to identify the various costs associated with a manpower planning system: Recruitment cost, Overstaffing cost, Understaffing cost and Firing/Retirement cost. Two categories of recruitment costs have been identified viz., fixed and variable costs. Further we have formulated a manpower planning problem with the objective of minimizing the manpower system costs. The model has been found to be analogous to Wagner-Within model in Production/Inventory situation. The major limitation of the model is the fact it is considered in isolation of the various constraints and operating polices under which a manpower system operates. An interesting extension is to develop an integrated model which minimizes the manpower system costs in presence of the system constraints and operating policies. A simple extension of the model is to consider a k-graded manpower system. Yet, another extension could be to allow the possibility of understaffing, which would result in a situation similar to backordering in Production/inventory models for which the algorithms are readily available.

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Table 1

Year	R	S (in 'ØØØs)	i (in 'ØØØs)
1	79	728	15
2	34	7Ø5	12
3	52	698	16
4	61	714	14
5	25	7Ø8	16
6	89	739	12
7	56	695	15
8	29	7Ø4	17
9	48	689	12
1Ø	34	712	13

Table 2

Year t	1	2	3	4	5	6	7	8	9	10
S	728	705	698	714	708	739	695	704	689	712
i	15	12	16	14	16	12	15	17	12	13
R	79	34	52	61	25	89	56	29	48	34
	728	1433	1936	2650	3358	3739	4434	5115	5558	6270
		1238	2057	2912	3000	4782	4411	4869	5931	5966
			2642			5670		5194	6405	
Minimum Cost	728	1238	1936	2650	3000	3739	4411	4869	5558	5966
Optimum Policy	1	1,2	3	4	4.5	6	6,7	7,8	9	9.10