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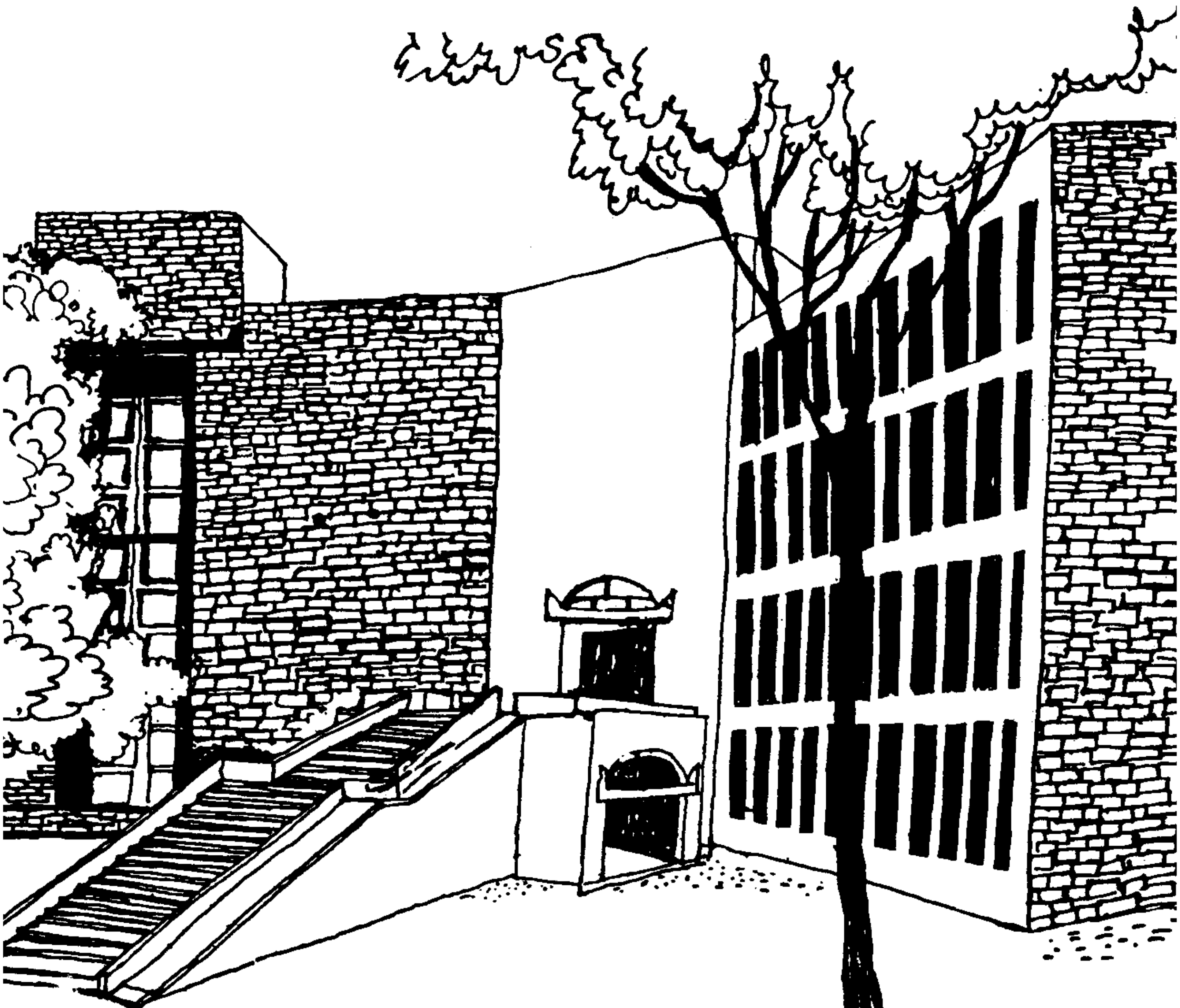

A NEGOTIATION PROCEDURE CONVERGING TO THE NASH BARGAINING SOLUTION

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W P No. 980
November 1991

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Abstract

In this paper we propose a negotiation procedure for the bilateral monopoly problem, solutions of which converge to the Nash bargaining solution.

1. Introduction :- The simple analytical framework of a bilateral monopoly problem as formulated in Chaterjee (1988) for instance, reduces to a problem involving division of money between two agents as discussed in Roth (1979). The problem is as follows :

Consider a situation with two agents 1 and 2 with utility functions $u_i : \mathbb{R}_+ \to \mathbb{R}$, $i=1,2$ for money respectively. Let $w_i \geq 0$ be the initial wealth of agent i and suppose they bargain over the division of Q units of money. We will assume that an individual's utility function for money is increasing (i.e. more money is preferred to less), concave (i.e. risk-averse) and has both first and second derivatives. Thus $u_i' > 0$, $u_i'' \leq 0$ for $i=1,2$. A <u>feasible proposal</u> is a proposed split (c_1, c_2), such that $c_1 + c_2 \leq Q$ and $c_1 \geq 0$, $c_2 \geq 0$, which leaves player i with total wealth $w_i + c_i$.

Following Roth (1979), we define player i's <u>boldness</u> with respect to the proposal (c_1, c_2) as

$$b_i(w_i, c_i) = u_i'(w_i + c_i)/[u_i(w_i + c_i) - u_i(w_i)], \quad i=1,2,$$

and the inverse of this quantity is called his fear of ruin. Roth (1979), discusses the intuitive basis and implications of this definition.

The unique Nash bargaining solution to the above problem is a <u>feasible proposal</u> (c_1^*, c_2^*) such that (c_1^*, c_2^*) solves the following programming problem

$$[u_1(w_1+c_1)-u_1(w_1)][u_2(w_2+c_2)-u_2(w_2)] \to \max$$

$$\text{s.t. } c_1+c_2 \leq Q$$

$$c_1 \geq 0, \; c_2 \geq 0.$$

Our assumptions about u_i ensure that $c_1^*>0, c_2^*>0$ and $c_1^*+c_2^*=Q$.

The purpose of this paper is to suggest a negotiation (or adjustment) procedure which converges to (c_1^*, c_2^*).

2. Main Results :- Roth (1979) establishes the following theorem

Theorem 1 :- (c_1^*, c_2^*) is the Nash bargaining solution if and only if $b_1(w_1, c_1^*)=b_2(w_2, c_2^*)$.

Proof :- Proof of Theorem 7 in Roth (1979).

The following property is easily verified :

$$\frac{\partial b_i}{\partial c_i}(w_i, c_i) < 0, \quad i=1,2. \qquad (1)$$

Our negotiation procedure assumes that the bolder of the two players is capable of extracting a concession from his opponent. Thus we suggest the following adjustment mechanism :

$$\left.\begin{aligned} \frac{dc_1}{dt} &= b_1(w_1, c_1)-b_2(w_2, Q-c_1) \\ c_2(t) &= Q-c_1(t), \ \forall t \geq 0 \end{aligned}\right\}$$

It is clear from Theorem 1, that the only critical point (or equilibrium) of (2) is (c_1^*, c_2^*) i.e. the Nash bargaining solution. The main result of this paper asserts that all solutions of (2) converge to (c_1^*, c_2^*) i.e. negotiation procedures consistent with (2) converge to the Nash bargaining solution.

Theorem 2 :- Let $(c_1(t), c_2(t))$ solve (2). Then

$$\lim_{t \to \infty} (c_1(t), c_2(t)) = (c_1^*, c_2^*)$$

Proof :- The proof proceeds by constructing the function

$V(c_1)=[b_1(w_1, c_1)-b_2(w_2, Q-c_1)]^2, \ 0 \leq c_1 \leq Q.$

$V(c_1^*)=0$ and $V(c_1)>0$ for $c_1 \neq c_1^*$

$$\frac{dV(c_1(t))}{dt} = 2[b_1(w_1, c_1(t)) - b_2(w_2, Q-c_1(t))]\left[\frac{\partial b_1(w_1, c_1(t))}{\partial c_1} + \frac{\partial b_2(w_2, Q-c_1(t))}{\partial c_2}\right]$$

$$\frac{dV(c_1(t))}{dt} < 0 \text{ for } c_1(t) \neq c_1^* \text{ by (1).}$$

Thus V is a Lyapunov function. Further c_1 moves in a compact set $[0, Q]$ with equilibrium c_1^*. Thus by the theorem on global asymptotic stability of equilibrium in Varian (1981),

$$\lim_{t \to \infty} c_1(t) = c_1^* \text{ and } \lim_{t \to \infty} c_2(t) = Q - c_1^* = c_2^*.$$

Q.E.D.

3. Conclusion :- In this paper we have established a convergent negotiation procedure for the bilateral monopoly problem. A negotiation procedure considerably different from ours has been suggested by Maschler, Owen and Peleg (1988) which converges to the Nash-set for a bargaining problem with a larger number of agents. Their procedure is independent of the underlying set of physical alternatives and thus lacks the economic insights that our adjustment procedure affords.

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