Fan Charts as Useful 'Maps' for an Inflation-Targeting Central Bank

An Illustration of the Sveriges Riksbank's Method for Presenting Density Forecasts of Inflation

In this study I illustrate the usefulness of Fan Charts for a central bank and show how they can be used to present its viewpoint on likely paths of future inflation. Exploiting a bivariate unobserved components model, I use the methodology followed by Blix and Sellin (1998) to demonstrate how subjective judgements can be systematically incorporated into model-based forecasts and effectively presented in a graphic manner.

I. Introduction

Starting February 1996, the Bank of England (BoE) has been publishing the probability distribution of (constant interest rate) inflation forecasts in its *Inflation Report* in the form what has now come to be known as the Fan Chart. Sveriges Riksbank (BoS), following suit, started publishing similar density forecasts of inflation in its *Inflation Report,* starting December 1997[1].

Since forecasts are inherently uncertain, the Fan Chart serves the two fold purpose of presenting the bank's subjective view of future inflation, and, as Britton, Fisher and Whitley (1998) at the BoE put it, also allows it to convey that same information *"without suggesting a degree of precision that would be spurious.[2]"*

Though the methodology of the BoE and the BoS is quite similar in the way they 'construct' the Fan Chart, while the BoE's is more of a 'top-down' approach[3], at the BoS, as Blix and Sellin (1998; hereafter referred to as the BoS method) share, "*the initial assessments* [of the inflation forecast distribution] *and the aggregation is done at the Economics department, and then filtered upwards.[4]"*

To the extent there is subjectivity involved in judgement of skewness in the distribution of future inflation, the approach of the BoS appears more attractive. In this study I illustrate how the BoS explicitly incorporates subjective judgements into the model-based forecasts and arrives at its Fan Chart. As the Reserve Bank of India (RBI) increasingly moves towards an Inflation Targeting regime, understanding of the key issues related to presenting probability distribution of inflation forecasts would become indispensable.

In **Section II** I briefly describe how the BoS goes about assessing uncertainty about future inflation. In **Section III** I outline an unobserved components model for generating inflation forecasts. In **Section IV** I demonstrate the 'process' of constructing a fan chart and present a few examples. **Section V** concludes with a recapitulation of importance of density forecasts in clearly bringing out the risks in the estimation of future macro variables.

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[1] Blix, M. and P. Sellin, "Inflation Forecasts with Uncertainty Intervals," *Sveriges Bank Quarterly Review,* 2, 1999

[2] Britton, E., P. Fisher and J. Whitley, "The *Inflation Report* projections: Understanding the Fan Chart," *Bank of England Quarterly Bulletin,* 1998

[3] *ibid*

[4] Blix, M. and P. Sellin, "A Bivariate Distribution for Inflation and Output Forecasts," *Sveriges Bank WP* 09-09, 1998

II. The Preliminaries

The most crucial assumption in the study is that the central bank has structural or reduced form models for the macroeconomic aggregates that it believes affect inflation. The idea is to separate subjectivities from the scientific exercise, so that discussion regarding future developments in inflation can focus on assumptions underlying the models, the sources of uncertainty and their quantitative importance, rather than on 'point' forecasts and subjective judgments.

It is also assumed that the central bank has fairly reliable model/s for forecast of inflation for various horizons. In the discussion that follows these are taken as given.

The density forecasts presented both by the BoE and the BoS are 'centered' on the *mode* rather than the *mean*. The advantage of using *mode* rather than the *mean* is that while *mean* as a measure of central tendency is affected by extreme observations, *mode* by definition is the 'most likely' value of the random variable[5]. On the downside, however, *mode* does not use all the information contained in the sample.

Uncertainty Assessment

This section is a summary of the description given in BoS as to how it assesses uncertainty in future inflation.

Denoting by $X_j s$ the macro variables (including inflation) that the central bank believes to affect the future level of inflation, both the BoE and the BoS assume all X_j to belong to a two-piece normal (*2PN*) distribution given by:

$$f(x;\mu,\sigma_1,\sigma_2) = \begin{cases} C\exp\left(\dfrac{-1}{2\sigma_1^2}(x-\mu)^2\right) & x \le \mu \\ C\exp\left(\dfrac{-1}{2\sigma_2^2}(x-\mu)^2\right) & x > \mu \end{cases} \qquad [1]$$

$$;\ C = k/(\sigma_1+\sigma_2)\ ;\ \ k = \sqrt{2/\pi}\ ;\ \ \text{and } \mu \text{ is the mode}$$

The most attractive feature of this two-piece normal distribution is that with addition of just one parameter to the standard Gaussian distribution, both skewness and heteroskedasticity can be incorporated and wide varieties of shapes result. Also, if there is a reason to believe that the forecast distribution is multi-modal, *2PN* can handle those special cases.

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[5] A special case is the Gaussian distribution *mean*, *median* and *mode* all coincide

Borrowing from BoS, for the *2PN* distribution under use, it can be shown that:

$$Pr[a \leq x \leq b] = \frac{2\sigma}{\sigma_1 + \sigma_2}\left[\Phi\left(\frac{b-\mu}{\tilde{\sigma}}\right) - \Phi\left(\frac{a-\mu}{\tilde{\sigma}}\right)\right]$$

where $\Phi(.)$ *represents the standard normal cumulative distribution function*

$$\begin{cases} \tilde{\sigma} = \sigma_1 & \text{if } a \leq b \leq \mu \\ \tilde{\sigma} = \sigma_2 & \text{if } \mu \leq a \leq b \end{cases}$$

Also,

$$E[(x-\mu)^2] = (1-k^2)(\sigma_2 - \sigma_1)^2 + \sigma_1\sigma_2$$

$$E[(x-\mu)^3] = k(\sigma_2 - \sigma_1)\left[(2k^2-1)(\sigma_2 - \sigma_1)^2 + \sigma_1\sigma_2\right]$$

$$\gamma \equiv \tilde{\mu} - \mu = k(\sigma_2 - \sigma_1)$$

where $\tilde{\mu}$ *is the mean of the distribution*

After specifying the above distribution of the macro-variables affecting inflation, the BoS proceed by asking the following two questions:

- What is the probability $P_j = Pr[X_j \leq \mu_j]$
- Is there a reason for the uncertainty of the forecast in the variable X_j to be different from the historical; if yes, what is the factor h_j by which to adjust the historical standard deviation of the variable X_j

Though above are fairly subjective questions, a central bank, with its expertise and the wealth of data that it possesses can be expected to be in a position to be put concrete numbers to these judgements. For example, if there is oil crisis expected, than there would be a probability greater than 0.5 that oil prices would be up in the future and that with more uncertainty than historical, i.e. an $h > 1$.

Then given $P_j = Pr[X_j \le \mu_j]$ and h_j, we have[6]:

$$\sigma^2_{1,j} = \omega_j \left[(1-k^2)\left(\frac{1-2P_j}{P_j}\right) + \left(\frac{1-P_j}{P_j}\right)\right]^{-1}$$

$$\sigma^2_{2,j} = \omega_j \left[(1-k^2)\left(\frac{1-2P_j}{1-P_j}\right) + \left(\frac{P_j}{1-P_j}\right)\right]^{-1} \qquad [2]$$

$$; \omega_j = \left(h_j \sigma_j\right)^2$$

As can be expected, in the above equations, a high P_j and a high h_j imply a high σ_1, i.e. a negatively skewed distribution.

Note that though the above discussion refers to the macroeconomic aggregates *including* inflation, to generate the Fan Chart for inflation, BoS make an additional assumption that the skewness in inflation is linearly related to the macro variables *affecting* inflation, i.e.

$$\gamma_\pi = \sum_{j=1}^{n} \beta_j \gamma_j \qquad [3]$$

$$where\ \gamma_j\ is\ skewness\ in\ distribution\ of\ the\ variable\ X_j$$

The weights β_j in the above equation can be interpreted as the elasticity of the macroeconomic variable X_j w.r.t inflation and can be obtained from a suitable macroeconometric model, which the central bank is assumed to possess.

Thus, having obtained the skewness in the distribution of inflation as above, as BoS show, it is rather straightforward to get the 'left' and 'right' variances. The two unknowns are σ_1 and σ_2, and the two equations:

$$\sigma^2_\pi = (1-k^2)(\sigma_2 - \sigma_1)^2 + \sigma_1\sigma_2$$

$$[4]$$

$$\gamma_\pi \equiv \tilde{\mu} - \mu = k(\sigma_2 - \sigma_1)$$

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[6] Note that here h_j and P_j both refer to forecast for a particular horizon; For brevity dependence on t has been suppressed

III. A Bivariate Unobserved Components Model for Inflation and Output

- *Output:* Following Watson (1986), output is separated into a trend and a cycle. The trend component is assumed to follow a random walk with drift and the cyclical component is assumed to follow an *AR(2)* process (much popular with the real business cyclical theorists; see Romer, 1996, Ch. 4). Thus, (natural logarithm of) output is specified as:

$$y_t = y^*_{t-1} + z_t$$

$$y^*_t = \alpha + y^*_{t-1} + \varepsilon_t$$

$$z_t = \varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \eta_t \qquad [5]$$

$$\varepsilon_t \sim N(0, \sigma^2_\varepsilon)$$

$$\eta_t \sim N(0, \sigma^2_\eta)$$

- *Inflation:* As found by Kuttner (1994) for the U.S., a parsimonious backward looking Phillips curve specification with *MA(2)* errors fits well for inflation in India too[7]:

$$\pi_t = \pi^*_t + \beta z_{t-1} + \nu_t + \delta_1 \nu_{t-1} + \delta_2 \nu_{t-2}$$

$$\pi^*_t = \pi^*_{t-1} + \varsigma_t \qquad [6]$$

$$\nu_t \sim N(0, \sigma^2_\nu)$$

$$\varsigma_t \sim N(0, \sigma^2_\varsigma)$$

where, following Domenech and Gomez (2003), core inflation (π^*_t) is modeled as a random walk without drift.

Note how 'restriction' on the coefficient of core inflation as above allows for its interpretation as that level of inflation when the output gap, z_{t-1} is zero. If in first equation in *[6]*, z_{t-1} is 0, with $E(\nu_t) = 0$, it follows that $E(\pi_t) = \pi^*_t$.

[7] Other specification for inflation were also checked; *MA(2)* was selected using the *general to specific* criterion

Above equations can be conveniently cast as a State Space Model (SSM), facilitating estimation of the latent variables by Maximum Likelihood (ML) using Kalman Filter. Details can be found in Harvey (1993). For above specification, the SSM is:

$$\alpha_t = c + A\alpha_{t\text{-}1} + \xi_t$$

$$y_t = H\alpha_t \qquad [7]$$

where,

$$\alpha_t = \begin{bmatrix} y_t^* \\ z_t \\ z_{t-1} \\ z_{t-2} \\ \pi_t^* \\ v_t \\ v_{t-1} \\ v_{t-2} \end{bmatrix} \qquad c = \begin{bmatrix} a \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \varphi_1 & \varphi_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad \xi_t = \begin{bmatrix} \varepsilon_t \\ \eta_t \\ 0 \\ 0 \\ \varsigma_t \\ v_t \\ 0 \\ 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 1 & 1 & \delta_1 & \delta_2 \end{bmatrix}$$

Estimation and Results

After running the Kalman Filter recursions as given in Harvey (1993), the state vector along with their associated Mean Squared Errors *(MSEs)* and the hyperparameters can be estimated using ML. The likelihood function is proportional to:

$$L(\theta) = \text{-}ln\,|\,F\,| - \sum_{t=1}^{n} e_t^{'} F_t^{-1} e_t$$

where θ is the vector of the hyperparameters, and *F* is the *MSE* associated with error *e.*

To estimate the vector of hyperparameters, we minimize the negative of the likelihood function $L(\theta)$ using the Nelder-Mead simplex search method available in MATLAB[8]. Although Nelder-Mead is one of the slower search routines, it is more reliable provided the initial values are not too off-mark, which is not a concern for the problem at hand.

[8] Using the function *fminsearch* available in the Optimization Toolbox of MATLAB 6.5

Data

For output, quarterly estimates of GDP at factor cost (1993-94 = 100) constructed by Virmani and Kapoor (2003) have been used after seasonally adjustment by the TRAMO/SEATS[9] method[10]. Inflation is alternatively taken to be based on seasonally adjusted WPI-All Commodities (1993-94 = 100) Index.

Initialization of the Hyperparameters

Running the Hodrick and Prescott (1980; hereafter *HP*) filter on the output and the inflation series, and estimating *OLS* gives initial estimates of the hyperparameters. Results are reported in *Table 1* below:

Table 1
Initialization of the Hyperparameters

Hyperparameter	φ_1	φ_2	a	β	δ_1	δ_2	σ_ε^2	σ_η^2	σ_ς^2	σ_ν^2
Initial Value	0.54	0.18	-0.0094	-0.2	-0.7	-0.2	0.0000012	0.00011	0.0000018	0.002

Initialization of the State Vector

Since both potential output and core inflation have been modeled as nonstationary, unlike for a stationary state space model, initial conditions for the Kalman Filter are not well defined. However, since we have first estimates for potential output from running the *HP* filter, and that of output gap from the *OLS* estimates, we can treat the initial condition as 'known' for our purpose. Taking first three values from the *HP* filtered output series, cyclical output is initialized as the residual, $y_t - y_t^*$. For the *MA* terms corresponding to inflation their expectation (zero) is used to for initialization. *MSE* of the initial state vector (taken to be diagonal) are taken from OLS estimates from *[1]* and *[2]*. Since inflation and cyclical output have been modeled as *MA(2)* and *AR(2)* process respectively, essentially filtering starts from the fourth observation. Initial values are reported in *Table 2* below:

Table 2
Initialization of the State Vector

State Variable	y_t^*	z_t	z_{t-1}	z_{t-2}	π_t^*	v_t	v_{t-1}	v_{t-2}
Initial State Value (α_0)	11.67	0.0056	0.0032	0.0093	0.061	0	0	0
Initial State MSE (P_0)	0.096	0.0002	0.0002	0.0002	0.0003	0.0002	0.0002	0.0002

[9] Time Series Regressions with *ARIMA* Noise/Signal Extraction in *ARIMA* Time Series
[10] Using the software DEMETRA made available by the European Statistical Institute (EUROSTAT)

Maximum Likelihood Estimates of the hyperparameters are given in *Table 3* below:

Table 3
Maximum Likelihood Estimates of the Hyperparameters

Hyperparameter	φ_1	φ_2	a	β	δ_1	δ_2	σ_ε^2	σ_η^2	σ_ς^2	σ_ν^2
Estimate	0.57	0.34	0.014	-0.39	0.19	-0.24	0.0000022	0.00013	0.000002	0.0022

IV. Constructing Fan Charts: Step by Step

In this section I exemplify step-by-step how fan charts are constructed.

- Generate the skewness in forecasts of the macro-aggregates (using equations *[1]* and *[2]*) that are believed to affect inflation
- From suitable macroeconometric models, get elasticities of inflation to the above macro-aggregates
- Use equation *[3]* to get implied skewness in the distribution of inflation for each horizon
- With the mean and skewness, the 'implied' mode results from equations *[4]*
- Generate the forecasts for inflation and output using suitable structural/reduced form econometric models
- Use equations *[4]* to get σ_1 and σ_2
- Plot the Fan Chart given μ, σ_1 and σ_2

For the *MSE* of inflation forecasts for the next eight quarters results from the above SSM are used. Results are presented in *Table 4* below

Table 4
Forecasts and the Associated MSEs

Forecast Horizon/Prediction and the MSE	*Inflation Forecast and the Associated MSE*	
t = 1 (Quarter)	0.0673	0.0023
t = 2	0.0588	0.0024
t = 3	0.0590	0.0025
t = 4	0.0592	0.0025
t = 5	0.0593	0.0025
t = 6	0.0594	0.0025
t = 7	0.0596	0.0025
t = 8	0.0597	0.0025

Only a central bank can be expected to have detailed data and information to get the implied values of skewness in the distribution of inflation. In *Figure 1* below I present examples of Fan Charts for inflation with γ taking the values as given in *Table 5* in various scenarios.

Table 5
Scenarios for the 'Path followed by Skewness' γ

Scenario 1	0.07	0.06	- 0.10	- 0.02	0.03	- 0.14	0.11	0.02
Scenario 2	- 0.07	0.09	0.03	- 0.06	-0.04	0.06	- 0.04	0.07
Scenario 3	0.08	0.08	0.03	0.07	0.02	- 0.02	- 0.02	- 0.16
Scenario 4	- 0.06	0.03	- 0.06	0.04	- 0.08	0.05	0.02	- 0.09

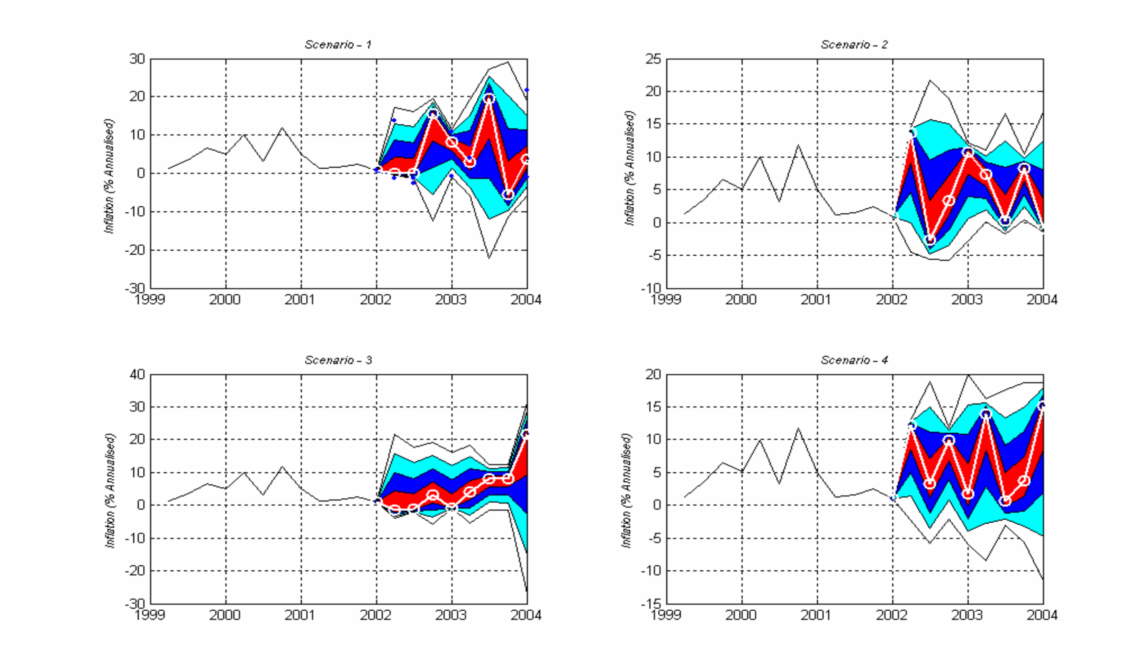

Figure 1

V. Conclusion

Forecasts of macro-variables like inflation are inherently uncertain and even with the most sophisticated models all the risks can't be quantitatively captured. The method used by the BoS is attractive because it allows subjectivities to be included in a systematic manner. The above analysis can be easily extended to present similar Fan Charts for future paths of output too. Furthering their study, Blix and Sellin (2000) present bivariate forecasts of inflation and output in the form of Contour Plots. Presenting information in the form of charts, after incorporating subjectivities into the historical standard deviation of macro-aggregates, would be useful in not only facilitating discussion at the central bank, but also informing the market participants and the general public about the paths of likely future of inflation and output.

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