Independent Project

On

*Estimating Term Structure models for India*

Submitted to

Prof. S. K. Barua

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By

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PGP II
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Introduction

Models of the term structure play a vital role in the pricing of risks associated with fixed income securities. Instruments like European options, European swaptions can be valued assuming that the probability distribution of interest rates or bond prices is lognormal. However, unless the stochastic behaviour of these variables is described it is not possible to value instruments like American options, callable bonds and others. Term structure modelling aims to solve this problem by describing the evolution of the yield curve over time.

Among the various models available we use the Cox, Ingersoll and Ross (CIR) framework. This is a one-factor equilibrium model.

An equilibrium model starts with assumptions about economic variables and then derives a process for the interest rate. Moreover, the process is studied in a risk-neutral world in which investors expect to earn no more than the risk-free rate.

A one-factor equilibrium model allows only one source of uncertainty to influence the process of the rate. This only implies that all rates move in the same direction over a very short period of time, but not necessarily by the same amount.

Cox, Ingersoll and Ross model

The model

The CIR model is based on the following stochastic process for the interest rate:

\[ dr = \alpha (\mu - r) dt + \sigma \sqrt{r} dz \]  

(1)
Here ‘r’ is the interest rate, ‘μ’ the long-term mean of the interest rate, ‘σ’ the volatility of the interest rate and ‘α’ the parameter that pulls the rate back to the long term mean.

The first term describes the trend in the rate process, while the latter term describes the fluctuations by including the Weiner process.

The model features mean reversion and the square root term on the volatility ensures that the interest rate will not fall below zero. Interest rates are distributed as a chi-square. This model, like that of Vasicek, does not fit the current term structure but rather provides the current term structure. As a result, its prices, though internally free from arbitrage, would not be consistent with arbitrage free prices in the market. Consequently, users of this model could suffer arbitrage losses.¹

An Application

Most applications of the CIR model assume the local expectation hypothesis. This is in fact the same as working with martingales or risk neutral valuation. Thus, for example, the value of a USD 1 face value pure discount bond is given as:

\[
P(t,T) = A(T-t)e^{-B(T-t)r}
\]

where \( A(T-t) = \left[ \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + \alpha)(e^{\gamma(T-t)} - 1) + 2\gamma} \right]^{2\alpha^2/\sigma^2} \)

and \( B(T-t) = \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + \alpha)(e^{\gamma(T-t)} - 1) + 2\gamma} \)

\[ \gamma = \sqrt{\alpha^2 + 2\sigma^2}. \]

(2)

¹ [www.cob.vt.edu/finance/faculty/dmc/Courses/TCHnotes/Tn97-04.pdf](http://www.cob.vt.edu/finance/faculty/dmc/Courses/TCHnotes/Tn97-04.pdf)
Working within the framework of this model it is possible to get upward sloping, downward sloping and even slightly humped curves. Thus, its assumptions are not too restrictive.

**Indian Debt Market**

The debt markets in advanced countries are significantly larger and deeper than equity markets. But in India, the trend is just the opposite. The development of debt market in India has not been as remarkable as in the equity market. However, the debt markets in India have undergone considerable change in the last few years. Characterised by regulated interest rates, limited players and lack of trading earlier, the markets have become more integrated and less regulated.

The debt market in India can be divided into two categories, viz. Government securities market consisting of Central Government and State Government securities; and Bond market consisting of FI bonds, PSU bonds and Corporate bonds/debentures. The Government Securities segment is the most dominant category in the debt market.

*Application of the CIR model*

Given the above background the primary aim of this project was to try and fit a one-factor model to the Indian interest rates and estimate the parameters of the model. Given the parameters, the model can then be used for application like pricing of interest rate contingent claims.

To estimate these claims, parameters estimated through use of the CIR model are used to create recombining trees of interest rates. With the associated probabilities available, any claim dependant upon expected interest rates can be priced. It is worth reiterating here that the CIR model provides the term structure and does not fit the
current term structure. The procedure for construction of these trees is explained below:

A transformed variable, \( x = (2\sqrt{t})/\sigma \) is used to construct a tree. The variable \( x \) will follow a stochastic process given by

\[
dx = \mu(x)dt + dz. \tag{3}
\]

The binomial tree will then expand according to

\[
x^+ = x + (\Delta t)^{1/2} \quad \text{and} \quad x^- = x - (\Delta t)^{1/2}, \tag{4}
\]

where \( \Delta t \) is the time step increment for the tree under construction.

Transforming this variable back using the equation

\[
r = \sigma^2 x^2 / 4 \tag{5}
\]

gives the expected interest rate applicable for a single period. To complete the analysis, probability of an up-move at any node can be calculated using the equation:

\[
p(r) = \frac{a(u - r)\Delta t + (r - r^-)}{(r^* - r^-)} \tag{6}
\]

**Research Questions**

The research questions addressed were as follows:

- What is the Behaviour of interest rates in India?
- Can the Model be used to price Interest rate contingent claims?

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\(^2\) This explanation is based upon material obtained from www.cob.vt.edu/finance/faculty/dmc/Courses/TCHnotes/Tn97-04.pdf
Methodology
Data used for the analysis, was provided by Dr. Gangadhar Darbha. Yield curve data for the Indian market from January 1 1997 to August 31 2001 was provided. The data consisted of rates with maturities of 0.25, 1, 3, 5, 7, 10, 15 and 20 years. As liquidity in the Indian markets, especially at the longer end, is not very good, the data was adjusted by Dr. Darbha to eliminate the liquidity premia associated with the rates observed.

Data was analysed using the software package ‘SAS’. The CIR equation was used for each maturity and parameters estimated for each.

Findings
Results obtained from the analysis conducted are presented in table below:

<table>
<thead>
<tr>
<th>Maturity (in Years)</th>
<th>A</th>
<th>B</th>
<th>Sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.05972</td>
<td>8.72438</td>
<td>0.187804</td>
</tr>
<tr>
<td>1</td>
<td>0.01907</td>
<td>8.966125</td>
<td>0.109013</td>
</tr>
<tr>
<td>3</td>
<td>0.00364</td>
<td>8.74478</td>
<td>0.050688</td>
</tr>
<tr>
<td>5</td>
<td>0.00201</td>
<td>8.381095</td>
<td>0.038918</td>
</tr>
<tr>
<td>7</td>
<td>0.00147</td>
<td>8.319048</td>
<td>0.035535</td>
</tr>
<tr>
<td>10</td>
<td>0.0022</td>
<td>8.836818</td>
<td>0.04042</td>
</tr>
<tr>
<td>15</td>
<td>0.00787</td>
<td>11.05705</td>
<td>0.060654</td>
</tr>
<tr>
<td>20</td>
<td>0.01398</td>
<td>11.32403</td>
<td>0.074181</td>
</tr>
</tbody>
</table>

The parameter ‘a’ represents the rate at which the interest rates move towards (regress to) the long-term mean, which is given by the parameter ‘b’. A plot of the long-term interest rate means is given in the chart below:
The plot shown above can be construed as a ‘yield curve’ of long-term interest rates. This depicts rather unique characteristics. For interest rates up to a maturity of 7 years, the curve is flat and slightly downward sloping. From then onwards, interest rates increase with increasing maturity, reaching a high of 11.32% for a maturity of 25 years. The shape of the curve obtained can be attributed to the following factors:

- The data used for the analysis was processed for illiquidity in the markets. This might not have been sufficient to remove the liquidity premium attached to rates of greater than 7 years maturity. This argument would also imply that sufficient liquidity exists in Indian markets for maturity periods of up to 7 years.

- The data used for the analysis was for the period January 1 1997 to August 31 2001. The downward sloping nature of the long-term rates could arise if the daily yield curves in the market were downward sloping for a large proportion of the period under consideration. Given the macroeconomic
conditions of slowing growth and a decrease in expected inflation during the period, this might be the case.

Using the data obtained on the behaviour of the 3 month interest rate, a recombining tree was constructed using equations 3 through 6 as an illustration. The starting or 'zero-period' rate was assumed to be 6%. The results obtained are given below:

<table>
<thead>
<tr>
<th>Maturity (in months)</th>
<th>A</th>
<th>B</th>
<th>Sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.05972</td>
<td>8.72438</td>
<td>0.187804</td>
</tr>
</tbody>
</table>

Current rate assumed 6% Time Period 0.25 Years

Modified Variable 26.0856

<table>
<thead>
<tr>
<th>Period</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>28.0856</td>
<td>27.5856</td>
<td>27.0856</td>
<td>26.5856</td>
<td>26.0856</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
### Probability of Up move

<table>
<thead>
<tr>
<th>Period</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.557294</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.565885</td>
<td></td>
<td></td>
<td>0.57466</td>
<td></td>
</tr>
<tr>
<td>0.583627</td>
<td></td>
<td>0.583627</td>
<td></td>
<td>0.5926</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5928</td>
<td></td>
<td>0.602189</td>
<td></td>
<td>0.611808</td>
</tr>
</tbody>
</table>

### State Probabilities

| Period | Rates Predicted | 0     | 1     | 2     | 3     | 4     | 6     | 0.040675 | 6.080743 | 6.120212 | 6.159092 | 0.105759 | 0.189791 | 0.335387 | 0.43453 | 0.333729 | 0.367543 | 0.308231 | 0.166777 | 0.067448 | 0.026183 |
|--------|-----------------|-------|-------|-------|-------|-------|-------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0      |                 |       |       |       |       |       | 0.583627 | 0.495066 | 0.416373 | 0.416373 | 0.069547 | 0.069547 | 0.067448 | 0.067448 | 0.056183 | 0.056183 | 0.056183 | 0.056183 |

The movement of spot rates can also be used to predict interest rates of more than 3 month maturity. Assuming forward rate equals expected spot rate, rates with maturity 3, 6, 9 and 12 months can be calculated.
Limitations of the Study

Estimation of the behaviour of the interest rates was carried out for each maturity using the CIR model. This assumes that each of these rates behaves stochastically in a manner independent of the behaviour of other rates. This assumption might hold true for rates with short maturity, but may not hold for longer maturity rates. The CIR model also provides the term structure rather than fit the observed term structure. Use of rates predicted by the model to price interest rate contingent claims of longer maturities may not be representative of the actual price.

The analysis was conducted on data set for a period during which there was a clear trend in macroeconomic indicators. This causes a trend in the movement of interest rates over the period in question. To get an unbiased estimate, the analysis should be conducted over a larger data set.

Conclusion

The CIR model can be applied to model interest rate behaviour in India, and the results be used to price interest rate contingent claims. It is advisable to model the shorter rates and use these to forecast the term structure.
Acknowledgements

We would like to thank Dr. Gangadhar Darbha for the data and the guidance provided. We would also like to express our gratitude to Prof. S. Baner, under whose supervision this project was carried out.

Student Background

Arko Sen holds a Bsc degree in Economics from St. Xaviers College, Calcutta. As part of his summer internship, he worked with Lehman Brothers, London. There he worked at the credit research and CDO desks.

Sanjay K. Mookim is an Instrumentation engineer from Jadavpur University, Calcutta. He worked with the supply chain vertical of Cognizant Technology Solutions as part of his summer internship.
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