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**W.P. No. 2017-10-01
October 2017**

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Mathematical Modelling for Time-of-Use Pricing of Electricity in Monopoly and Oligopoly

Nidhi Kaicker^a, Goutam Dutta^b,
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Abstract

This study establishes the feasibility condition for efficiency gains to arise from time-of-use pricing in the electricity market in a monopolistic and oligopolistic set up using constrained optimization. In an oligopolistic set-up, the strategic interaction between producers depends on the level of demand. In case of high demand, the producers compete on the basis of output they will produce, resulting in a Cournot-type competition. On the other hand, in case of low demand, an oligopolistic structure may break with only the most efficient firm operating, or results in the emergence of leader firms and follower firms, i.e. the Stackleberg model.

Keywords: Time-of-use pricing, Constrained Optimization, Monopoly, Oligopoly, Cournot, Stackleberg

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1. INTRODUCTION

Revenue management and dynamic pricing is one of the emerging research areas in the service industries. This paper is an attempt to study the efficiency gains for time-of-use (TOU) pricing over flat-rate pricing in the electricity sector. The United States of America has an installed capacity of about 1063000 MW¹. Similarly, an emerging economy like India has an installed capacity of 303083² MW of power. By application of TOU pricing, we can shift the demand from the peak period to the non-peak period and thus, may be able to avoid large capital investments. Even a reduction of 5 per cent in the peak demand can result in savings of \$36 billion in investment cost. Cost savings for India will be to the tune of \$9 billion³. This paper is motivated by the application of TOU pricing in the electricity sector in different types of market structures – monopoly and oligopoly.

Electricity pricing can be based on average cost, with no variation in prices during the day, or on marginal cost, where the price adjusts to the actual demand-supply balance during the day. The former is also known as flat-rate pricing, and the latter, dynamic pricing. Between these two extremes, lies the TOU pricing, where a day is divided into two or more periods, and the price that will be charged in each period is pre-determined. In the case of flat-rate pricing, the entire price risk is borne by the producer, and in real time dynamic pricing, this risk is passed on to the consumer. Since dynamic pricing reflects the actual demand supply balance, it can give the right price signals which enable customers to decide whether the price is high enough to curtail their usage during peak hours.

This paper makes fundamental contributions in the following areas in TOU pricing:

1. Using different prices for peak, shoulder and off-peak periods, reflecting actual demand, we determine the efficiency gains that can be captured over using a flat price throughout the day.
2. We observe that in the electricity market, there may be a monopoly in some cases, where a single firm continues to enjoy market power, or an oligopoly, where two or more firms

¹ CIA World Factbook, 2015

² https://en.wikipedia.org/wiki/Electricity_sector_in_India#Installed_capacity, retrieved on August 7, 2016

³ Authors' calculations based on US Department of Energy Data

compete against one another through strategic interaction. We capture the efficiency gains in both a monopolistic set-up and an oligopolistic set-up using constrained optimisation techniques.

3. In an oligopolistic set-up, the strategic interaction between producers depends on the level of demand. In case of high demand, the producers compete on the basis of the output they will produce. On the other hand, in case of low demand, an oligopolistic structure may break with only the most efficient firm operating, or result in the emergence of leader firms and follower firms. All these situations are captured in this paper.

We organise the paper as follows. Section 2 discusses related academic contributions and the techniques used in the past by researchers to model pricing in the electricity market. Section 3 describes the economics and assumptions behind our model. In Section 4, a model is formulated to explain the concept of TOU pricing in a monopolistic set up. In Section 5, the model is discussed for an oligopolistic set up, where we first take the case of a duopoly and then generalise it to n firms. Section 6 deals with the results, conclusions and future scope for research.

2. LITERATURE REVIEW

Dynamic pricing is a relatively new field which has attracted attention from researchers. When firms use dynamic pricing in order to match demand with the inventory or capacity, the creation of price differences between segments may cause consumers to switch (from higher priced segments to lower priced segments). Thus, there is demand leakage from the market segment with a higher priced product to the market segment with a lower priced product due to price driven substitution (Kim and Bell, 2011). However, the success of this practice of revenue management depends on the ability of the firm to fence customers into different market segments, making it difficult (or costly) for the consumer to switch from one market segment to the other (Zhang and Bell, 2007). In case of electricity, this may be feasible, when the market is segmented on the basis of use – commercial versus non commercial, or household versus industrial use. However, if we treat all consumers alike, but charge different prices for the same

product at different points in time, is there a leakage possible? In other words, can consumers shift demand from the peak period to the off peak period? Since, it is not possible (or difficult) to store electricity, that is, the consumer is charged at the time of use, this demand substitution may be not be possible.

So what benefits does dynamic pricing bring? Faruqui and George (2005) find that dynamic pricing of electricity incentivises customers to lower loads during peak periods, as a result of which the market-clearing price is reduced along with a reduction in market power. Specifically, with an increase in prices, the reduction in peak-period energy use increases but at a decreasing rate, the percentage reduction being greater in hotter climatic zones than in colder zones. They note that the benefits of dynamic pricing depend on factors which are uncertain, such as the impact on customer load and on avoidable supply-side costs. However, the installation of digital interval meters is a costly affair and thus dynamic pricing is feasible only if its benefits exceed the costs (Faruqui et. al., 2009a). Faruqui et al. (2009b) opine that dynamic pricing can lead to avoidance of purchase of expensive combustion turbines and installation costs, thereby prohibiting an increase in costs for all consumers. If dynamic rates are offered to customers on a default basis, they find that a much higher fraction of customers would remain on the rates than if these rates were offered on an optional basis. Faruqui et. al. (2007) find that a reduction in peak period demand is associated with a reduction in the peak period generation capacity, installation needs, and transmission and distribution capacity. An associated short term benefit is the reduction in wholesale market prices which comes along with the demand reduction during critical times.

Borenstein (2007) investigates the wealth transfer that would occur in the event of a shift from simple retailing to that of time-of-use pricing. He considers consumption patterns of 1142 large customers in northern California and concludes that such a shift would cause large wealth transfers. He also looks into how much of the potential loss of cross subsidy can be overcome by being more price responsive and shows that Real Time Pricing would eliminate cross subsidies across time and also across classes of consumers. The rate structure considered by Borenstein in his study raises a total revenue that is exactly equal to the wholesale cost of the power they

consume. The two TOU rates used are – one, that permits no subsidy across TOU periods and in the second case, the ratio of rates between the periods is almost equal to the ratio of the rates in the utility taken into account. In both cases, winners tend to be large consumers in terms of electricity consumption.

While there are several benefits to the time of use pricing structure, as highlighted in recent studies, there are also barriers to its proper implementation. The most common concern being that such a pricing scheme if made mandatory would create problems for those unwilling to join in. Borenstein (2012) suggests increasing the acceptance of this type of pricing, using Critical Peak pricing which combines a static price structure along with dynamic structures when the demand is high. The unresolved issues associated with dynamic pricing which make its implementation difficult have been highlighted by Joskow and Wolfram (2012) in their study. For residential and small commercial customers, metering is thought to be extremely expensive, thus, billing costs are expected to increase as the rates become more complex. There would also be larger redistributions of income as consumption patterns vary with changing rate designs. These unresolved issues in dynamic pricing are responsible for its slow implementation.

Cellibi and Fuller (2007) build a model to estimate ex-ante TOU prices while adhering to the marginal cost principle. They illustrate it by taking four time periods, four types of generation facilities and three demand blocks into account to calculate the prices. The hourly demand derived from Ontario's Independent Electricity System operator is grouped into 9 hours of off-peak, 8 hours of shoulder (or mid peak) and 7 hours of on peak demand. The results show that TOU prices for some months are equal to the operating cost of the generator that serves the last unit of energy and for some months it is the weighted average of the hourly prices within a certain demand block. The authors observe a 24% increase in off-peak, an 18% increase in mid-peak and a 11% decrease in the on peak demand under TOU as compared to the historical data.

Cellibi and Fuller (2012) develop a model consisting of three parts – the independent system operator's (ISO) problem, the supply and the demand side. For each demand block and each period, the ISO chooses its decision variable 'energy flowing from generator to the demand

point' to minimise the operating cost of all facilities subject to a set of constraints. The supply side maximises the firm's total profit in different demand blocks. With the help of a computational analysis, the authors show that perfect competition provides the lowest TOU prices for different blocks while monopoly gives the highest prices. The results of the Nash-Cournot model are between these two extremes. When single prices are computed for different months, they are found to be the lowest for perfect competition and highest for monopoly.

While a considerable amount of work has been done on dynamic pricing in electricity, our contribution lies in capturing the efficiency gains arising out of a special case in dynamic pricing, that is, time-of-use pricing when the market is imperfectly competitive.

3. THE ECONOMICS OF THE MODEL

Our model for time-of-use pricing is based on the following assumptions:

- a) Each day is divided into three periods, namely, peak period (having the highest demand), off peak period (lowest demand) and shoulder period (in between the highest and the lowest), and prices for each period are set in advance. There is no block pricing of electricity.
- b) The consumers of electricity include households, firms, as well as the government, with each category having different demands for electricity at different points of the day and varying price sensitivity. We do not take this classification of consumers into account, and assume all consumers of electricity to be a single entity.
- c) We assume linear demand and cost structures.
- d) For producers of electricity, we first assume the case of monopoly. In the next case, due to de-regulation there would be more than one producer of electricity, leading to an oligopolistic market. The firms strategically interact with each other. The competitive model they follow depends on the market demand.
- e) In the case of oligopoly, we initially take up the case of two firms, that is, the case of duopoly. Both the firms are assumed to have different cost structures depending on the technology used in the process of production and distribution of electricity. The firm using better technology faces a cost curve, which has low marginal cost, and is hence more

efficient as compared to the other firm. Each firm is constrained by some capacity, beyond which it is economically not profitable to produce. We consider the capacity of the more efficient firm to be more than the less efficient one.

- f) In calculating the efficiency gains, we look at the contribution margins in all cases, and deduct the fixed cost from the total margin. Another approach could have been to allocate all fixed costs to the peak period only; however, we do not make that distinction.
- g) We also assume that there is government regulation, which prevents the firms in an oligopolistic setup to collude with each other and form cartels.
- h) It is technologically possible for firms to shut down production of electricity, if the situation demands, and stop participating in the market for a period of time. However, if such a thing happens, then the costs incurred from remaining idle at that period of time must be over compensated by the profits earned by the firms when they remain active and participate in the production of electricity.
- i) We do not take into account externalities such as pilferage of electricity. A country may rely on various sources of electricity – non renewable and renewable. We do not incorporate these differences into our model.

Technical Details of the Model

We assume the following with respect to the power generation plant -

- a) The plant has a capacity restriction which is given by the maximum power that can be generated at any point. The ratio of the actual power generated and the maximum possible power generated is the plant load factor (PLF).
- b) Some small percentage of the power generated is lost during transmission.
- c) Production of electricity, transmission of electricity and the consumption of electricity (in terms of demand price equation) can be captured by a linear deterministic system.

The economic rationale for TOU pricing of electricity is as follows. Figure 1 shows the case of uniform pricing. The three demand curves represent the peak, shoulder and the off-peak periods. In case of uniform pricing, a constant price \bar{P} is charged in all the periods. The resulting quantity

demanded is equal to q_1 , q_2 , and q_3 respectively. However as seen from the diagram, during the peak period, there is an excess demand, given by CD. In case of the shoulder period and the off peak period, there is an excess supply given by BC and AC respectively. Thus there is an imbalance between the demand and supply of electricity due to uniform pricing.

[Figure 1: The Case of Flat Rate Pricing]

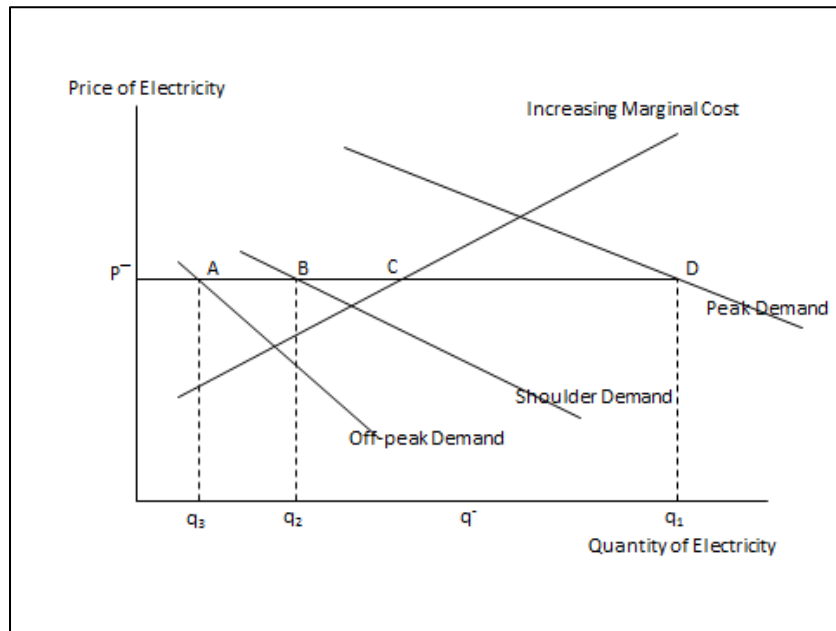
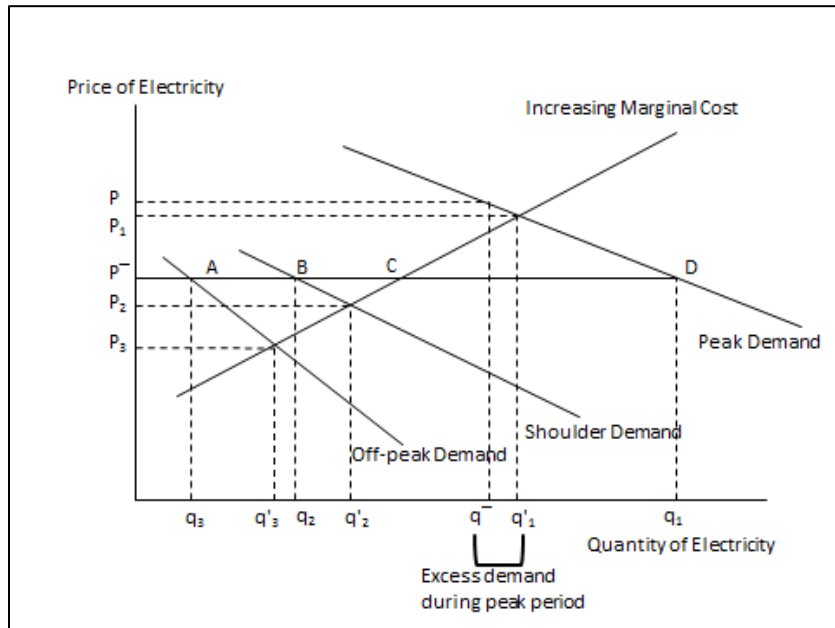


Figure 2 depicts the case of TOU pricing. The main idea is to charge different prices at different periods, which reflects the true cost of electricity at that time. The price-quantity combinations are given by (P_1, q'_1) , (P_2, q'_2) , and (P_3, q'_3) respectively. Suppose the capacity of the firm is given by \bar{q} , we observe that in the peak period, the quantity q'_1 exceeds the capacity. The gap by which this quantity exceeds capacity is smaller compared to the excess demand in case of flat-rate pricing. A price of P must be charged in the peak period to mitigate the excess demand. Thus TOU pricing helps to mitigate the problem of demand-supply imbalance.

[Figure 2: The Case of Time-of-Use Pricing]



4. MATHEMATICAL MODEL FOR A MONOPOLIST

We now look at the dynamics of the model in the following cases. Section 4.1 discusses the case of a monopolist when it charges the same price in all the three periods. We first assume the marginal costs to be constant in all the three periods, and later relax this assumption to decrease the marginal cost as production gets closer to the capacity. Section 4.2 discusses the monopolists' case when it charges different prices in different time periods, both under the assumption of constant marginal cost, and decreasing marginal cost, as the load increases. A comparison of flat rate pricing and time-of-use pricing is done in section 4.3. We verify the model by estimating three demand functions for peak, off-peak and shoulder periods each, and optimising it using AMPL software. The computational results are discussed under section 4.4.

For our model, we divide each day into peak period, shoulder period, and off-peak period, denoted by a suffix t , where $t=1$, for peak period, $t=2$, for shoulder period, and $t=3$, for off-peak period. The number of hours in a day is assumed to be divided into n_1 hours of peak period, n_2 hours of shoulder and n_3 hours of off-peak period.

The definitions used in the model are as follows:

q_t =Energy demanded per hour in time period t (*KW Hr*)

z_t = Energy generated per hour in time period t (*KW Hr*)

α_t =Intercept term of the demand function at time period t

β_t =Slope of the demand curve at time period t

p_t =Price charged by the monopolist at time period t in case of time-of-use pricing (*Rs./KW Hr*)

p =Price charged by the monopolist in case of uniform pricing (*Rs./KW Hr*)

C = Capacity constraint of the plant (*MW*)

F =Fixed Cost associated with generation and distribution of electricity (*Rs.*)

δ_t =Marginal Cost associated with an extra unit of electricity at time period t(*Rs./KW Hr*)

π =Total Profit earned by the monopolist (*Rs.*)

$T_l\%$ = Percentage of energy generated that is lost while transmission⁴

$\therefore (1 - T_l\%)z_t = q_t$ = Energy consumed/ demanded per hour for time period t

If $k=(1 - T_l\%)$, $kz_t = q_t$

Model Specification:

The firm faces a linear demand curve as given by:

$$q_t = \alpha_t - \beta_t p_t \forall t \in 1, 2, 3 \dots \dots \dots (1)$$

Here, we assume that, $\alpha_1 > \alpha_2 > \alpha_3$ and $\beta_t > 0$. β_t represents the responsiveness of consumers in the period t, to a unit change in prices. We also assume that this responsiveness is the lowest in the peak period followed by the shoulder and then off-peak period. Thus, $\beta_3 > \beta_2 > \beta_1$. Since the energy generated in each period cannot exceed the capacity constraint, $z_t < 1000C, \forall t \in 1, 2, 3$.⁵

All the production, operation, transmission, and distribution costs are together termed as total cost denoted by TC . We assume that the firm faces a linear cost function of the form given by:

⁴This percentage is assumed to be constant throughout the day.

⁵The energy generated/consumed is in *KW Hr* and the capacity of a generator is in *MW*, hence, we multiply C by 1000 to convert *MW* into *KW*.

$$TC = F + \sum_{t=1}^3 n_t \delta_t z_t \dots\dots\dots (2)$$

The monopolist is assumed to be a profit maximizing agent. Thus, the monopolist’s problem can be stated as follows:

Maximize $\pi =$ Total Revenue- Total Costs, subject to the necessary constraints.

4.1 The Monopolist Charges the Same Price in all the Three Periods

a. Constant Marginal Costs

If we assume constant marginal cost in all time periods, that is, δ , the profit function is

$$\pi = n_1 p q_1 + n_2 p q_2 + n_3 p q_3 - \delta(n_1 z_1 + n_2 z_2 + n_3 z_3) - F$$

$$\pi = n_1 p q_1 + n_2 p q_2 + n_3 p q_3 - \frac{\delta}{k}(n_1 q_1 + n_2 q_2 + n_3 q_3) - F \dots\dots\dots (3)$$

We maximise this

subject to: $z_t \leq C \times 1000, \forall t \in 1, 2, 3$

or $q_t \leq C \times 1000 \times k, \forall t \in 1, 2, 3$

The optimisation problem thus becomes max,

$$\pi = n_1 p(\alpha_1 - \beta_1 p) + n_2 p(\alpha_2 - \beta_2 p) + n_3 p(\alpha_3 - \beta_3 p) - F - \frac{\delta}{k}[n_1(\alpha_1 - \beta_1 p) + n_2(\alpha_2 - \beta_2 p) + n_3(\alpha_3 - \beta_3 p)] \dots\dots\dots (4)$$

subject to,

(i) $\alpha_1 - \beta_1 p \leq 1000 C k$

(ii) $\alpha_2 - \beta_2 p \leq 1000 C k$

(iii) $\alpha_3 - \beta_3 p \leq 1000 C k$

Taking a Lagrangian function \mathcal{L} , we solve the constrained optimisation problem as follows:

$$\mathcal{L} = n_1 p(\alpha_1 - \beta_1 p) + n_2 p(\alpha_2 - \beta_2 p) + n_3 p(\alpha_3 - \beta_3 p) - F - \frac{\delta}{k}[n_1(\alpha_1 - \beta_1 p) + n_2(\alpha_2 - \beta_2 p) + n_3(\alpha_3 - \beta_3 p)] + \lambda_1[1000 C k - (\alpha_1 - \beta_1 p)] + \lambda_2[1000 C k - (\alpha_2 - \beta_2 p)] +$$

$$\lambda_3[1000Ck - (\alpha_3 - \beta_3 p)] \dots \dots \dots (5)$$

Where $\lambda_1, \lambda_2, \lambda_3$ are Lagrange Multipliers.

The First Order Conditions give us the following results:

$$\frac{\partial \mathcal{L}}{\partial p} = n_1 \alpha_1 + n_2 \alpha_2 + n_3 \alpha_3 - 2\beta_1 n_1 p - 2\beta_2 n_2 p - 2\beta_3 n_3 p + \frac{\delta}{k} (n_1 \beta_1 + n_2 \beta_2 + n_3 \beta_3) - \lambda_1 \beta_1 - \lambda_2 \beta_2 - \lambda_3 \beta_3 = 0 \dots \dots \dots (6)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 1000Ck - (\alpha_t - \beta_t p) \geq 0 \forall t \in 1, 2, 3 \dots \dots \dots (7)$$

We assume that it is possible for the generator to operate at its full capacity in the peak period but lower demand in the off-peak and shoulder period does not require the generator to operate at full capacity. Thus we take two cases for each scenario in line with the Karush-Kuhn-Tucker (KKT) conditions. First, full capacity utilisation does not take place in any of the three periods and second, there is full capacity utilisation in the peak period but not in the shoulder or off-peak periods.

For $t=2,3$ constraint (ii) and (iii) hold as strict inequality, hence $\lambda_2, \lambda_3 = 0$ for each case.

Case 1: $1000Ck > \alpha_1 - \beta_1 p$ (the capacity constraint is slack for all three periods).

$$\therefore \lambda_1 = 0$$

Putting $\lambda_1 = \lambda_2 = \lambda_3 = 0$ in equation (6),

$$n_1 \alpha_1 + n_2 \alpha_2 + n_3 \alpha_3 - 2p(n_1 \beta_1 + n_2 \beta_2 + n_3 \beta_3) + \frac{\delta}{k} (n_1 \beta_1 + n_2 \beta_2 + n_3 \beta_3) = 0 \dots \dots \dots (8)$$

Solving this for the price, we get

$$p = \frac{\sum_{t=1}^3 n_t \alpha_t + \frac{\delta}{k} \sum_{t=1}^3 n_t \beta_t}{2 \sum_{t=1}^3 n_t \beta_t}, \text{ and } q_t = \alpha_t - \beta_t p = \alpha_t - \beta_t \left[\frac{\sum_{t=1}^3 n_t \alpha_t + \frac{\delta}{k} \sum_{t=1}^3 n_t \beta_t}{2 \sum_{t=1}^3 n_t \beta_t} \right], \forall t \in 1, 2, 3 \dots \dots \dots (9)$$

The total quantity produced is the sum of quantity supplied in each of the three periods, that is,

$$Q = n_1 q_1 + n_2 q_2 + n_3 q_3.$$

This equals,

$$Q = \sum_{t=1}^3 n_t \alpha_t - \left[\frac{\sum_{t=1}^3 n_t \alpha_t + \frac{\delta}{k} \sum_{t=1}^3 n_t \beta_t}{2 \sum_{t=1}^3 n_t \beta_t} \right] \sum_{t=1}^3 n_t \beta_t \dots\dots\dots (10)$$

Thus, the total profit of the monopolist is as follows:

$$\pi = p \left[\sum_{t=1}^3 n_t \alpha_t + \frac{\delta}{k_1} \sum_{t=1}^3 n_t \beta_t - p \sum_{t=1}^3 n_t \beta_t \right] - F - \frac{\delta}{k_1} \sum_{t=1}^3 n_t \alpha_t,$$

$$\text{i.e. } \pi = \frac{[\sum_{t=1}^3 n_t \alpha_t + \frac{\delta}{k_1} \sum_{t=1}^3 n_t \beta_t]^2}{4 \sum_{t=1}^3 n_t \beta_t} - F -$$

$$\frac{\delta}{k_1} \sum_{t=1}^3 n_t \alpha_t \dots\dots\dots (11)$$

Case 2: $1000Ck = \alpha_1 - \beta_1 p$ (the capacity constraint for the peak period is tight and slack for off-peak and shoulder periods).

$$\therefore \lambda_1 \geq 0$$

$$p = \frac{\alpha_1 - 1000Ck}{\beta_1} \text{ and } q_t = \alpha_t - \beta_t \left[\frac{\alpha_1 - 1000Ck}{\beta_1} \right] \dots\dots\dots (12)$$

The total quantity produced will be the sum of quantities supplied in the three periods,

$$Q = \sum_{t=1}^3 n_t \alpha_t - \left[\frac{\alpha_1 - 1000Ck}{\beta_1} \right] \sum_{t=1}^3 n_t \beta_t \dots\dots\dots (13)$$

The total profit of the monopolist in this case is as follows,

$$\pi = p \left[\sum_{t=1}^3 n_t \alpha_t + \frac{\delta}{k} \sum_{t=1}^3 n_t \beta_t - p \sum_{t=1}^3 n_t \beta_t \right] - F - \frac{\delta}{k} \sum_{t=1}^3 n_t \alpha_t \dots\dots\dots (14)$$

Using (12), the profit in this case is:

$$\pi = \frac{\alpha_1 - 1000Ck}{\beta_1^2} \left[\beta_1 \sum_{t=1}^3 n_t \alpha_t + \left(\frac{\beta_1 \delta}{k} - (\alpha_1 - 1000Ck) \right) \sum_{t=1}^3 n_t \beta_t \right] - F - \frac{\delta}{k} \sum_{t=1}^3 n_t \alpha_t \dots\dots\dots (15)$$

b. Decreasing Marginal Costs

The marginal cost of electricity generation decreases as more units are put into use. Thus, we assume the marginal cost of producing electricity to be decreasing with the higher load, so if the per unit electricity cost varies across periods, that is, δ_t with $\delta_1 < \delta_2 < \delta_3$ then the profit function becomes:

$$\pi = n_1 p q_1 + n_2 p q_2 + n_3 p q_3 - n_1 \delta_1 z_1 - n_2 \delta_2 z_2 - n_3 \delta_3 z_3 - F$$

$$\pi = n_1 p q_1 + n_2 p q_2 + n_3 p q_3 - \frac{1}{k_1} (n_1 q_1 \delta_1 + n_2 q_2 \delta_2 + n_3 q_3 \delta_3) - F \dots\dots\dots (16)$$

subject to,

(i) $\alpha_1 - \beta_1 p \leq 1000Ck$

- (ii) $\alpha_2 - \beta_2 p \leq 1000Ck$
- (iii) $\alpha_3 - \beta_3 p \leq 1000Ck$

Taking a Lagrangian function \mathcal{L} , we solve the constrained optimisation problem as follows:

$$\mathcal{L} = n_1 p(\alpha_1 - \beta_1 p) + n_2 p(\alpha_2 - \beta_2 p) + n_3(\alpha_3 - \beta_3 p) - F - \frac{1}{k}[\delta_1 n_1(\alpha_1 - \beta_1 p) + \delta_2 n_2(\alpha_2 - \beta_2 p) + \delta_3 n_3(\alpha_3 - \beta_3 p)] + \lambda_1[1000Ck - (\alpha_1 - \beta_1 p)] + \lambda_2[1000Ck - (\alpha_2 - \beta_2 p)] + \lambda_3[1000Ck - (\alpha_3 - \beta_3 p)] \dots (17)$$

Where $\lambda_1, \lambda_2, \lambda_3$ are Lagrange Multipliers.

The First Order Conditions give us the following results:

$$\frac{\partial \mathcal{L}}{\partial p} = n_1 \alpha_1 + n_2 \alpha_2 + n_3 \alpha_3 - 2\beta_1 n_1 p - 2\beta_2 n_2 p - 2\beta_3 n_3 p + \frac{1}{k}[n_1 \delta_1 \beta_1 + n_2 \delta_2 \beta_2 + n_3 \delta_3 \beta_3] + \lambda_1 \beta_1 + \lambda_2 \beta_2 + \lambda_3 \beta_3 = 0 \dots (18)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 1000Ck - (\alpha_t - \beta_t p) \geq 0 \quad \forall t \in 1, 2, 3 \dots (19)$$

For $t=2,3$ the constraints (ii) and (iii) always hold with strict inequality, hence in each case $\lambda_2 = \lambda_3 = 0$.

Case 1: $1000Ck > \alpha_1 - \beta_1 p$ (the capacity constraint is slack in all three periods).

$$\therefore \lambda_1 = 0$$

Putting $\lambda_1 = \lambda_2 = \lambda_3 = 0$ in (17)

$$n_1 \alpha_1 + n_2 \alpha_2 + n_3 \alpha_3 + \frac{1}{k}[n_1 \delta_1 \beta_1 + n_2 \delta_2 \beta_2 + n_3 \delta_3 \beta_3] - 2n_1 \beta_1 p - 2n_2 \beta_2 p - 2n_3 \beta_3 p = 0 \dots (20)$$

$$p = \frac{\sum_{t=1}^3 n_t \alpha_t + \frac{1}{k} \sum_{t=1}^3 n_t \delta_t \beta_t}{2 \sum_{t=1}^3 n_t \beta_t} \text{ and } q_t = \alpha_t - \beta_t \left[\frac{\sum_{t=1}^3 n_t \alpha_t + \frac{1}{k} \sum_{t=1}^3 n_t \delta_t \beta_t}{2 \sum_{t=1}^3 n_t \beta_t} \right], \forall t \in 1, 2, 3 \dots (21)$$

The total quantity will be the sum of quantities produced in the three periods,

$$Q = \sum_{t=1}^3 n_t \alpha_t - \left[\frac{\sum_{t=1}^3 n_t \alpha_t + \frac{1}{k} \sum_{t=1}^3 n_t \delta_t \beta_t}{2 \sum_{t=1}^3 n_t \beta_t} \right] \sum_{t=1}^3 n_t \beta_t \dots\dots\dots (22)$$

The total profit of the monopolist is as given by:

$$\pi = p \left[\sum_{t=1}^3 n_t \alpha_t + \frac{1}{k} \sum_{t=1}^3 n_t \delta_t \beta_t - p \sum_{t=1}^3 n_t \beta_t \right] - F - \frac{1}{k} \sum_{t=1}^3 n_t \delta_t \alpha_t$$

$$\pi = \frac{\left(\sum_{t=1}^3 n_t \alpha_t + \frac{1}{k} \sum_{t=1}^3 n_t \beta_t \delta_t \right)^2}{4 \sum_{t=1}^3 n_t \beta_t} - \frac{1}{k} \sum_{t=1}^3 n_t \alpha_t \delta_t - F \dots\dots\dots (23)$$

Case 2: $1000Ck = \alpha_1 - \beta_1 p$ (the capacity constraint for the peak period is tight and slack for off-peak and shoulder periods).

$$\therefore \lambda_1 \geq 0$$

$$p = \frac{\alpha_1 - 1000Ck}{\beta_1} \text{ and } q_t = \alpha_t - \beta_t \left[\frac{\alpha_1 - 1000Ck}{\beta_1} \right] \dots\dots\dots (24)$$

The total quantity will be the sum of quantities produced in the three periods,

$$Q_t = \sum_{t=1}^3 \alpha_t - \left[\frac{\alpha_1 - 1000Ck}{\beta_1} \right] \sum_{t=1}^3 \beta_t \dots\dots\dots (25)$$

The total profit of the monopolist in this case is as follows

$$\pi = p \left[\sum_{t=1}^3 n_t \alpha_t + \frac{1}{k} \sum_{t=1}^3 n_t \delta_t \beta_t - p \sum_{t=1}^3 n_t \beta_t \right] - F - \frac{1}{k} \sum_{t=1}^3 n_t \delta_t \alpha_t$$

Thus,

$$\pi = \frac{\alpha_1 - 1000Ck}{\beta_1^2} \left[\beta_1 \sum_{t=1}^3 n_t \alpha_t + \frac{\beta_1}{k} \sum_{t=1}^3 \delta_t n_t \beta_t - (\alpha_1 - 1000Ck) \sum_{t=1}^3 n_t \beta_t \right] - F - \frac{1}{k} \sum_{t=1}^3 n_t \delta_t \alpha_t$$

..... (26)

4.2. The monopolist charges different prices in different periods

a. Constant Marginal Costs

If the monopolist charges different prices in different periods, the profit function is:

$$\pi = n_1 p_1 (\alpha_1 - \beta_1 p_1) + n_2 p_2 (\alpha_2 - \beta_2 p_2) + n_3 p_3 (\alpha_3 - \beta_3 p_3) - F - \frac{\delta}{k} [n_1 (\alpha_1 - \beta_1 p_1) + n_2 (\alpha_2 - \beta_2 p_2) + n_3 (\alpha_3 - \beta_3 p_3)] \dots\dots\dots (27)$$

Thus the optimisation problem becomes

Maximise $\pi = n_1 p_1 (\alpha_1 - \beta_1 p_1) + n_2 p_2 (\alpha_2 - \beta_2 p_2) + n_3 p_3 (\alpha_3 - \beta_3 p_3) - F - \frac{\delta}{k} [n_1 (\alpha_1 - \beta_1 p_1) + n_2 (\alpha_2 - \beta_2 p_2) + n_3 (\alpha_3 - \beta_3 p_3)],$

subject to,

- (i) $\alpha_1 - \beta_1 p_1 \leq 1000 Ck$
- (ii) $\alpha_2 - \beta_2 p_2 \leq 1000 Ck$
- (iii) $\alpha_3 - \beta_3 p_3 \leq 1000 Ck$

Taking Lagrangian function ψ , we solve the constrained optimisation problem as follows:

$$\psi = n_1 p_1 (\alpha_1 - \beta_1 p_1) + n_2 p_2 (\alpha_2 - \beta_2 p_2) + n_3 p_3 (\alpha_3 - \beta_3 p_3) - F - \frac{\delta}{k} [n_1 (\alpha_1 - \beta_1 p_1) + n_2 (\alpha_2 - \beta_2 p_2) + n_3 (\alpha_3 - \beta_3 p_3)] + \varphi_1 (1000 Ck - (\alpha_1 - \beta_1 p_1)) + \varphi_2 (1000 Ck - (\alpha_2 - \beta_2 p_2)) + \varphi_3 (1000 Ck - (\alpha_3 - \beta_3 p_3)) \dots\dots\dots (28)$$

where, $\varphi_1, \varphi_2, \varphi_3$ are the Lagrange Multipliers.

The First Order Conditions lead to the following results:

$$\frac{d\psi}{dp_t} = n_t \alpha_t - 2n_t \beta_t p_t + \frac{\delta}{k} n_t \beta_t + \beta_t \varphi_t = 0 \forall t \in 1, 2, 3 \dots\dots\dots (29)$$

$$\frac{d\psi}{d\varphi_t} = 1000 Ck - (\alpha_t - \beta_t p_t) \geq 0 \forall t \in 1, 2, 3 \dots\dots\dots (30)$$

The constraint (30) holds as strict inequality for $t=2,3$, hence $\varphi_2 = \varphi_3 = 0$ in all cases.

Case1: $1000 Ck > \alpha_1 - \beta_1 p_1$

$$\therefore \varphi_1 = 0$$

Putting $\varphi_1 = \varphi_2 = \varphi_3 = 0$ in equation (20)

$$n_t \alpha_t + \frac{\delta}{k} n_t \beta_t = 2n_t \beta_t p_t, \forall t \in 1, 2, 3 \dots\dots\dots (31)$$

$$p_t = \frac{n_t \alpha_t + \frac{\delta}{k} n_t \beta_t}{2n_t \beta_t}, \forall t \in 1, 2, 3 \text{ and } q_t = \frac{n_t \alpha_t - \frac{\delta}{k} \beta_t n_t}{2n_t}, \forall t \in 1, 2, 3 \dots\dots\dots (32)$$

The total profit of the monopolist is,

$$\begin{aligned}
\pi &= p_1 \left[n_1 \alpha_1 + \frac{\delta}{k} n_1 \beta_1 - p_1 n_1 \beta_1 \right] + p_2 \left[n_2 \alpha_2 + \frac{\delta}{k} n_2 \beta_2 - p_2 n_2 \beta_2 \right] \\
&\quad + p_3 \left[n_3 \alpha_3 + \frac{\delta}{k} n_3 \beta_3 - p_3 n_3 \beta_3 \right] - F - \frac{\delta}{k} \sum_{t=1}^3 n_t \alpha_t \\
\pi &= \sum_{t=1}^3 p_t \left[n_t \alpha_t + \frac{\delta n_t \beta_t}{k} - n_t \beta_t p_t \right] - F - \frac{\delta}{k} \sum_{t=1}^3 n_t \alpha_t \\
\pi &= \frac{n_t \alpha_t + \frac{\delta}{k} n_t \beta_t}{2 n_t \beta_t} \left[n_t \alpha_t + \frac{\delta}{k} n_t \beta_t - \frac{n_t \alpha_t + \frac{\delta}{k} n_t \beta_t}{2 n_t \beta_t} (n_t \beta_t) \right] - F - \frac{\delta}{k} \sum_{t=1}^3 n_t \alpha_t
\end{aligned}$$

Thus,
$$\pi = \sum_{t=1}^3 \frac{(n_t \alpha_t + \frac{\delta}{k} n_t \beta_t)^2}{4 n_t \beta_t} - F - \frac{\delta}{k_1} \sum_{t=1}^3 n_t \alpha_t \dots\dots\dots(33)$$

Case 2: $1000Ck_1 = \alpha_1 - \beta_1 p_1$ (the constraint for the peak period is tight, and slack for the off-peak and shoulder periods).

$$\therefore \varphi_1 \geq 0$$

$$\therefore p_1 = \frac{\alpha_1 - 1000Ck}{\beta_1} \text{ and } q_1 = 1000Ck \dots\dots (34)$$

Putting $\varphi_2 = \varphi_3 = 0$ in equation (19)

$$p_t = \frac{n_t \alpha_t + \frac{\delta}{k} n_t \beta_t}{2 n_t \beta_t} \forall t \in 2, 3 \text{ and } q_t = \frac{n_t \alpha_t - \frac{\delta}{k} \beta_t n_t}{2 n_t} \forall t \in 2, 3 \dots\dots\dots (35)$$

The total profit of the monopolist is,

$$\pi = \sum_{t=1}^2 \frac{(n_t \alpha_t - \frac{\delta}{k_1} n_t \beta_t)^2}{4 n_t \beta_t} - \sum_{t=1}^2 n_t \alpha_t - F + \left(\frac{\alpha_1 - 1000Ck}{\beta_1} - \frac{\delta}{k} \right) 1000Ck_1 n_1$$

Thus,
$$\pi = \frac{\alpha_1 - 1000Ck}{\beta_1} \left[\frac{\delta}{k} n_1 \beta_1 + 1000 n_1 Ck \right] + \sum_{t=1}^2 \frac{(n_t \alpha_t + \frac{\delta}{k} n_t \beta_t)^2}{4 n_t \beta_t} - F - \frac{\delta}{k} \sum_{t=1}^3 n_t \alpha_t \dots\dots\dots (36)$$

b. Decreasing Marginal Costs

If we assume that the marginal cost of producing electricity may decrease with the higher load, then the profit function is:

$$\pi = n_1 p_1 (\alpha_1 - \beta_1 p_1) + n_2 p_2 (\alpha_2 - \beta_2 p_2) + n_3 p_3 (\alpha_3 - \beta_3 p_3) - F - \frac{1}{k} [n_1 \delta_1 (\alpha_1 - \beta_1 p_1) - n_2 \delta_2 (\alpha_2 - \beta_2 p_2) - n_3 \delta_3 (\alpha_3 - \beta_3 p_3)]$$

Thus the optimisation problem becomes

$$\text{Max} = n_1 p_1 (\alpha_1 - \beta_1 p_1) + n_2 p_2 (\alpha_2 - \beta_2 p_2) + n_3 p_3 (\alpha_3 - \beta_3 p_3) - F - \frac{1}{k} [n_1 \delta_1 (\alpha_1 - \beta_1 p_1) - n_2 \delta_2 (\alpha_2 - \beta_2 p_2) - n_3 \delta_3 (\alpha_3 - \beta_3 p_3)] \dots\dots\dots (37)$$

subject to,

$$(i) \alpha_1 - \beta_1 p_1 \leq 1000Ck$$

$$(ii) \alpha_2 - \beta_2 p_2 \leq 1000Ck$$

$$(iii) \alpha_3 - \beta_3 p_3 \leq 1000Ck$$

Taking Lagrangian function ψ , we solve the constrained optimisation problem as follows:

$$\psi = n_1 p_1 (\alpha_1 - \beta_1 p_1) + n_2 p_2 (\alpha_2 - \beta_2 p_2) + n_3 p_3 (\alpha_3 - \beta_3 p_3) - F - \frac{1}{k} [n_1 \delta_1 (\alpha_1 - \beta_1 p_1) - n_2 \delta_2 (\alpha_2 - \beta_2 p_2) - n_3 \delta_3 (\alpha_3 - \beta_3 p_3)] + \varphi_1 (1000Ck - (\alpha_1 - \beta_1 p_1)) + \varphi_2 (1000Ck - (\alpha_2 - \beta_2 p_2)) + \varphi_3 (1000Ck - (\alpha_3 - \beta_3 p_3)) \dots\dots\dots (38)$$

Where $\varphi_1, \varphi_2, \varphi_3$ are Lagrange Multipliers.

The First Order Conditions lead to the following results:

$$\frac{d\psi}{dp_t} = n_t \alpha_t - 2n_t \beta_t p_t + \frac{n_t \delta_t \beta_t}{k} + \beta_t \varphi_t = 0 \forall t \in 1, 2, 3 \dots\dots\dots (39)$$

$$\frac{d\psi}{d\varphi_t} = 1000Ck - (\alpha_t - \beta_t p_t) \geq 0 \forall t \in 1, 2, 3 \dots\dots\dots (40)$$

The constraint (40) holds as strict inequality for $t=2,3$, hence $\varphi_2 = \varphi_3 = 0$ in each case.

Case1: $1000Ck_1 > \alpha_1 - \beta_1 p_1$ (the constraints for all three periods are slack).

$$\therefore \varphi_1 = 0$$

Putting $\varphi_1 = \varphi_2 = \varphi_3 = 0$ in equation (25)

$$n_t \alpha_t + \frac{n_t \delta_t \beta_t}{k} = 2n_t \beta_t p_t$$

Thus,

$$p_t = \frac{n_t \alpha_t + \frac{n_t \delta_t \beta_t}{k}}{2n_t \beta_t}, \forall t \in 1, 2, 3 \text{ and } q_t = \frac{n_t \alpha_t - \frac{n_t \delta_t \beta_t}{k}}{2n_t}, \forall t \in 1, 2, 3 \dots\dots\dots (41)$$

The total profit of the monopolist is given by,

$$\begin{aligned} \pi &= \sum_{t=1}^3 p_t \left[n_t \alpha_t + \frac{n_t \delta_t \beta_t}{k} - n_t \beta_t p_t \right] - F - \frac{1}{k} \sum_{t=1}^3 n_t \delta_t \alpha_t \\ \pi &= \sum_{t=1}^3 \frac{n_t \alpha_t + \frac{n_t \delta_t \beta_t}{k}}{2n_t \beta_t} \left[n_t \alpha_t + \frac{n_t \delta_t \beta_t}{k} - \frac{n_t \alpha_t + \frac{n_t \delta_t \beta_t}{k}}{2n_t \beta_t} (n_t \beta_t) \right] - F - \frac{1}{k} \sum_{t=1}^3 n_t \delta_t \alpha_t \\ \pi &= \sum_{t=1}^3 \frac{(n_t \alpha_t + \frac{n_t \delta_t \beta_t}{k})^2}{4n_t \beta_t} - F - \\ &\frac{1}{k} \sum_{t=1}^3 n_t \delta_t \alpha_t \dots \dots \dots (42) \end{aligned}$$

Case2: $1000Ck = \alpha_1 - \beta_1 p_1$ (the constraint is tight for the peak period but slack for the shoulder and off-peak periods)

$$\therefore \varphi_1 \geq 0$$

$$p_1 = \frac{\alpha_1 - 1000Ck}{\beta_1} \text{ and } q_1 = 1000Ck \dots (43)$$

$$p_t = \frac{n_t \alpha_t + \frac{n_t \delta_t \beta_t}{k}}{2n_t \beta_t} \forall t \in 2, 3 \text{ and } q_t = \frac{n_t \alpha_t - \frac{n_t \delta_t \beta_t}{k}}{2n_t} \forall t \in 2, 3 \dots \dots \dots (44)$$

The profit function then becomes,

$$\pi = \sum_{t=1}^2 \frac{(n_t \alpha_t + \frac{n_t \delta_t \beta_t}{k})^2}{4n_t \beta_t} - F - \frac{\delta}{k} \sum_{t=1}^2 n_t \alpha_t + \left(\frac{\alpha_1 - 1000Ck_1}{\beta_1} - \frac{\delta_1}{k} \right) 1000n_1 Ck_1$$

Thus,

$$\begin{aligned} \pi &= \frac{\alpha_1 - 1000Ck_1}{\beta_1} \left[\frac{n_1 \delta_1 \beta_1}{k} + 1000n_1 Ck \right] + \sum_{t=1}^2 \frac{(n_t \alpha_t + \frac{n_t \delta_t \beta_t}{k})^2}{4n_t \beta_t} - F - \\ &\frac{1}{k} \sum_{t=1}^3 n_t \delta_t \alpha_t \dots \dots \dots (45) \end{aligned}$$

4.3 Comparison of Flat Rate and Time-of-Use Pricing in Case of Monopoly

To summarise, the profits earned by the monopolist using constrained maximisation under various conditions are given below in Table 1 and 2.

Table 1 : Flat rate vs Time of Use Pricing when all 3 constraints are slack

	Flat Rate Pricing	Time of Use Pricing
Profit in case of Constant Marginal Costs	$\pi = \frac{[\sum_{t=1}^3 n_t \alpha_t + \frac{\delta}{k} \sum_{t=1}^3 n_t \beta_t]^2}{4 \sum_{t=1}^3 n_t \beta_t} - F$ $- \frac{\delta}{k} \sum_{t=1}^3 n_t \alpha_t$	$\pi = \sum_{t=1}^3 \frac{(n_t \alpha_t + \frac{\delta}{k} n_t \beta_t)^2}{4 n_t \beta_t} - F - \frac{\delta}{k} \sum_{t=1}^3 n_t \alpha_t$
Profit in case of Decreasing Marginal Costs	$\pi = \frac{(\sum_{t=1}^3 n_t \alpha_t + \frac{1}{k} \sum_{t=1}^3 n_t \beta_t \delta_t)^2}{4 \sum_{t=1}^3 n_t \beta_t}$ $- \frac{1}{k} \sum_{t=1}^3 n_t \alpha_t \delta_t - F$	$\pi = \sum_{t=1}^3 \frac{(n_t \alpha_t + \frac{\delta_t}{k} n_t \beta_t)^2}{4 n_t \beta_t} - F$ $- \frac{1}{k} \sum_{t=1}^3 n_t \delta_t \alpha_t$

Table 2 : Flat rate vs Time of Use Pricing when the peak period constraint is tight and the other 2 are slack

	Flat Rate Pricing	Time of Use Pricing
Profit in case of Constant Marginal Costs	$\pi = \frac{\alpha_1 - 1000Ck}{\beta_1^2} [\beta_1 \sum_{t=1}^3 n_t \alpha_t$ $+ \sum_{t=1}^3 n_t \beta_t (\frac{\beta_1 \delta}{k}$ $- (\alpha_1 - 1000Ck))] - F$ $- \frac{\delta}{k} \sum_{t=1}^3 n_t \alpha_t$	$\pi = \frac{\alpha_1 - 1000Ck}{\beta_1} \left[\frac{\delta}{k} n_1 \beta_1 + 1000 n_1 Ck \right]$ $+ \sum_{t=1}^2 \frac{(n_t \alpha_t + \frac{\delta}{k} n_t \beta_t)^2}{4 n_t \beta_t} - F$ $- \frac{\delta}{k} \sum_{t=1}^3 n_t \alpha_t$

Profit in case of Decreasing Marginal Costs	$\pi = \frac{\alpha_1 - 1000Ck}{\beta_1^2} \left[\beta_1 \sum_{t=1}^3 n_t \alpha_t + \frac{\beta_1}{k} \sum_{t=1}^3 \delta_t n_t \beta_t - (\alpha_1 - 1000Ck) \sum_{t=1}^3 n_t \beta_t \right] - F - \frac{1}{k_1} \sum_{t=1}^3 n_t \delta_t \alpha_t$	$\pi = \frac{\alpha_1 - 1000Ck}{\beta_1} \left[\frac{n_1 \delta_1 \beta_1}{k} + 1000n_1 Ck \right] + \sum_{t=1}^2 \frac{(n_t \alpha_t + \frac{n_t \delta_t \beta_t}{k})^2}{4n_t \beta_t} - F - \frac{\delta}{k} \sum_{t=1}^3 n_t \alpha_t$
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For there to be efficiency gains from practicing time-of-use pricing, the profit should be greater than that obtained in flat rate pricing. For the first case, that is, assuming constant marginal costs, the feasibility condition thus becomes:

$$\begin{aligned} \sum_{t=1}^3 \frac{(n_t \alpha_t + \frac{\delta}{k_1} n_t \beta_t)^2}{4n_t \beta_t} - F - \frac{\delta}{k} \sum_{t=1}^3 n_t \alpha_t &> \frac{[\sum_{t=1}^3 n_t \alpha_t + \frac{\delta}{k_1} \sum_{t=1}^3 n_t \beta_t]^2}{4 \sum_{t=1}^3 n_t \beta_t} - F - \frac{\delta}{k} \sum_{t=1}^3 n_t \alpha_t \\ &\Rightarrow \sum_{t=1}^3 \frac{(n_t \alpha_t + \frac{\delta}{k_1} n_t \beta_t)^2}{4n_t \beta_t} > \frac{[\sum_{t=1}^3 n_t \alpha_t + \frac{\delta}{k} \sum_{t=1}^3 n_t \beta_t]^2}{4 \sum_{t=1}^3 n_t \beta_t} \\ &\Rightarrow \frac{(n_1 \alpha_1 + \frac{n_1 \delta \beta_1}{k})^2}{4n_1 \beta_1} + \frac{(n_2 \alpha_2 + \frac{n_2 \delta \beta_2}{k})^2}{4n_2 \beta_2} + \frac{(n_3 \alpha_3 + \frac{n_3 \delta \beta_3}{k})^2}{4n_3 \beta_3} \\ &> [n_1 \alpha_1 + n_2 \alpha_2 + n_3 \alpha_3 + \frac{\delta}{k} (n_1 \beta_1 + n_2 \beta_2 + n_3 \beta_3)]^2 4(n_1 \beta_1 + n_2 \beta_2 + n_3 \beta_3) \\ &\Rightarrow \frac{n_1^2 \alpha_1^2}{4n_1 \beta_1} + \frac{n_2^2 \alpha_2^2}{4n_2 \beta_2} + \frac{n_3^2 \alpha_3^2}{4n_3 \beta_3} + \frac{\delta^2}{k^2} \left[\frac{n_1^2 \beta_1^2}{4n_1 \beta_1} + \frac{n_2^2 \beta_2^2}{4n_2 \beta_2} + \frac{n_3^2 \beta_3^2}{4n_3 \beta_3} \right] + \frac{\delta}{2k} [n_1 \alpha_1 + n_2 \alpha_2 + n_3 \alpha_3] \\ &> \frac{[n_1 \alpha_1 + n_2 \alpha_2 + n_3 \alpha_3]^2}{4(n_1 \beta_1 + n_2 \beta_2 + n_3 \beta_3)} + \frac{\frac{\delta^2}{k^2} [n_1 \beta_1 + n_2 \beta_2 + n_3 \beta_3]^2}{4(n_1 \beta_1 + n_2 \beta_2 + n_3 \beta_3)} \\ &+ \frac{2 \frac{\delta}{k} [n_1 \alpha_1 + n_2 \alpha_2 + n_3 \alpha_3] [n_1 \beta_1 + n_2 \beta_2 + n_3 \beta_3]}{4(n_1 \beta_1 + n_2 \beta_2 + n_3 \beta_3)} \end{aligned}$$

$$\begin{aligned}
&\Rightarrow \frac{n_1\alpha_1^2}{4\beta_1} + \frac{n_2\alpha_2^2}{4\beta_2} + \frac{n_3\alpha_3^2}{4\beta_3} + \frac{\delta^2(n_1\beta_1 + n_2\beta_2 + n_3\beta_3)}{4k^2} + \frac{\delta(n_1\alpha_1 + n_2\alpha_2 + n_3\alpha_3)}{2k} \\
&> \frac{[n_1\alpha_1 + n_2\alpha_2 + n_3\alpha_3]^2}{4(n_1\beta_1 + n_2\beta_2 + n_3\beta_3)} + \frac{\delta^2(n_1\beta_1 + n_2\beta_2 + n_3\beta_3)}{4k^2} \\
&\quad + \frac{\delta(n_1\alpha_1 + n_2\alpha_2 + n_3\alpha_3)}{2k} \\
&\Rightarrow \frac{n_1\alpha_1^2}{\beta_1} + \frac{n_2\alpha_2^2}{\beta_2} + \frac{n_3\alpha_3^2}{\beta_3} > \frac{[n_1\alpha_1 + n_2\alpha_2 + n_3\alpha_3]^2}{n_1\beta_1 + n_2\beta_2 + n_3\beta_3}
\end{aligned}$$

Similarly, we can derive the feasibility condition for the second case, that is, with increasing marginal costs⁶.

3.4 Computational Results

We illustrate the proposed model by collecting demand data from a local supplier of electricity and by modelling a 500MW generator in a power plant in Central India. The elasticity ranges derived from the demand data were used to create demand functions for the three periods – off-peak, shoulder and peak. For the peak period, the elasticity ranges from -1.51 to -1.78, for the shoulder period, the elasticity ranges from -1.39 to -1.93, and for the off-peak period, the elasticity ranges from -1.5 to -2.13. Each day is divided into 9 hours of off-peak period, 8 hours of shoulder period and 7 hours of peak period in line with the assumptions made by Cellibi and Fuller (2007). The assumptions for the cost function are based on data taken from the report on Performance of State Power Utilities for the years 2011-12 to 2013-14 by Power Finance Corporation Limited (2015). The parameters for the demand function and cost functions are given in Table 3.

⁶Available on request.

Table 3: Parameters in the model

	Peak Period (t=1)	Shoulder Period (t=2)	Off-peak Period (t=3)
α (intercept)	1200000	1100000	1000000
β (slope)	80000	85000	96000
δ (constant variable cost) (Rs / KwHr)	3.24	3.24	3.24
δ_t (decreasing variable cost) (Rs / KwHr)	3.24	3.30	3.36
Fixed Cost (Rs / KwHr)	Rs. 2700000		
Transmission Loss	4%		

We use AMPL (Fourer et al.,1994)to derive the optimal prices under time-of-use and flat rate pricing, both for a constant and decreasing (with load) marginal cost structure. The resultant prices and demand in each of the three periods and the total profitability of the plant under the scenario of TOU pricing and flat rate pricing are given in Table 4.

Table 4: Computational Results

	Scenario 1: Constant Marginal Cost		Scenario 2: Decreasing Marginal Costs	
	Flat Rate Pricing	TOU Pricing	Flat Rate Pricing	TOU Pricing
<u>Demand</u>				
Peak Period	480000 KwHr	465000 KwHr	480000 KwHr	465000 KwHr
Shoulder Period	335000 KwHr	406563 KwHr	335000 KwHr	403906 KwHr
Off-peak Period	136000 KwHr	338000 KwHr	136000 KwHr	332000 KwHr
<u>Price</u>				
Peak Period	Rs. 9	Rs. 9.18	Rs. 8.99	Rs. 9.18
Shoulder Period	Rs. 9	Rs. 8.15	Rs. 8.99	Rs. 8.18
Off-peak Period	Rs. 9	Rs. 6.89	Rs. 8.99	Rs. 6.95
Total Revenue	Rs.65376000	Rs.77416654	Rs.65376064	Rs.77158646
Total Cost	Rs.27216000	Rs.34929597	Rs.27536533	Rs.35251081
Total Profit	Rs.38160000	Rs.42487057	Rs.37839525	Rs.41907565

The prices for the peak period are found to be the highest in the case of time-of-use pricing followed by shoulder and off-peak periods. The results indicate that the monopolist stands to gain under a time-of-use pricing scheme as compared to a flat rate scheme. A gain in profit of Rs. 4,327,057 per day in case of constant marginal operating costs and Rs. 4,068,040 per day in case of decreasing marginal cost structure is estimated in the event of a shift to time-of-use pricing from a flat rate pricing scheme. The peak demand is reduced and the shoulder and off peak demand increases under time-of-use pricing by allowing demand response by consumers in the wake of a lower off-peak and shoulder price, and a slightly increased peak price. This shows that the time of use pricing is preferable over flat rate pricing in both cases — constant and decreasing marginal costs.

5. MATHEMATICAL MODEL FOR OLIGOPOLISTIC FIRMS

Next, we look at time-of-use pricing in the case of an oligopoly. With increasing deregulation in the electricity market, we observe the emergence of oligopolistic firms strategically interacting among themselves. Here we first take up the case of two firms which participate in a quantity competition and then move into the general case for n number of firms, with an assumption that n should be sufficiently small so that the oligopolistic nature of the electricity market is preserved. How firms behave in an oligopolistic set up depends on the market demand. The case of the peak period is discussed in section 5.1, the case of shoulder period in section 5.2 and the case of off-peak period in section 5.3.

We denote the firms by the suffix i such that $i=1, 2, 3, \dots, N$. In case of duopoly, we assume that there are two firms. The index t is used to denote the time period, as earlier.

Some additional definitions for the oligopoly model are as follows:

Q_t = Hourly market demand curve for electricity at time period t ($KW Hr$)

q_{it} = Electricity from firm i consumed by consumers per hour at time period t ($KW Hr$)

q_{-it} = Electricity from all but firm i consumed per hour at time period t ($KW Hr$)

Z_t = Total electricity generated per hour at time period t

z_{it} = Electricity generated by firm i per hour at time period t (KWHr)

z_{-it} = Electricity generated by all but firm i per hour at time period t (KWHr)

F_{it} = Fixed Cost for generation and distribution of electricity incurred by firm i and time period t (Rs.)

δ_i = Variable Cost for an extra unit of electricity incurred by firm i (Rs.)

C_i = Power Generation Capacity of firm i (MW)

π_{it} = Profit earned by firm i , in the period t (Rs.)

$T_l\%$ =Percentage of energy generated that is lost during transmission.

$\therefore (1 - T_l\%)z_t = q_t =$ Energy consumed/ demanded per hour for time period t .

$$\therefore kz_{it} = q_{it}$$

where $k = (1 - T_l\%)$.

Since k is assumed to be constant across firms and in all time periods, summing over the electricity generated by all firms and the electricity consumed by consumers from all firms gives us $kZ_t = Q_t$.

Model Formulations

Let the linear market demand curve be denoted by

$$Q_t = \alpha_t - \beta_t P_t \forall t \in 1, 2, 3 \dots \dots \dots (46)$$

Again, we assume that, $\alpha_1 > \alpha_2 > \alpha_3$ and $\beta_t > 0$.

In case of N firms,

$$Q_t = \sum_{i=1}^n q_{it} = q_{it} + q_{-it}, \forall t \in 1, 2, 3 \dots \dots \dots (47)$$

where, $q_{it} < 1000C_i k, \forall t \in 1, 2, 3$.

The unit of study in this case is 1 hour and we use the relevant conversions to bring the parameters into the same unit.

In case of a duopoly, the total market demand during period t, Q_t can be catered by the two firms.

$$Q_t = q_{1t} + q_{2t}, \forall t \in 1, 2, 3 \dots \dots \dots (48)$$

$$\text{Thus, } q_{1t} + q_{2t} = \alpha_t - \beta_t P_t \Rightarrow P_t = \frac{\alpha_t - q_{1t} - q_{2t}}{\beta_t}, \forall t \in 1, 2, 3 \dots \dots \dots (49)$$

$$\Rightarrow P_t = \frac{\alpha_t - kz_{1t} - kz_{2t}}{\beta_t}, \forall t \in 1, 2, 3$$

The Cost structure faced by firm i, is given as follows:

$$TC_i = \sum_{t=1}^3 F_{it} + \sum_{t=1}^3 n_t \delta_i z_{it} \dots \dots \dots (50)$$

In the oligopoly model, for each firm, we assume the marginal costs to be constant in all the three periods. This assumption can be relaxed to a decrease from the off-peak period to the shoulder period to the peak period, and is left as an extension to this study. We also assumed that the Marginal Cost for producing an extra unit of electricity is greater for firm 2 than for firm 1, that is, $\delta_2 > \delta_1$. In other words, Firm 1 is more efficient than Firm 2 in terms of cost advantage.

5.1 The case of Peak period

In the peak period, we assume that the market demand for electricity is high enough to exceed the capacity of the individual firms. Hence no single firm can cater to the entire market demand on its own. The two firms engage in quantity competition, of the type specified by Cournot. The two firms are assumed to participate in a one stage game and they choose the amount of electricity to be supplied simultaneously. Each firm tries to maximise its profit, observing the actions of its rival firm. We obtain the reaction functions of both the firms and then solve them to obtain the respective amounts of electricity that each firm generates.

Firm 1's optimisation problem can be stated as follows:

$$\text{Max } \pi_{11} = n_1 P_1 q_{11} - TC_{11} = n_1 \left(\frac{\alpha_1 - q_{11} - q_{21}}{\beta_1} \right) q_{11} - F_{11} - n_1 \delta_1 z_{11} \dots \dots \dots (51)$$

$$\pi_{11} = n_1 \left[\frac{\alpha_1 - kz_{11} - kz_{21}}{\beta_1} \right] kz_{11} - F_{11} - n_1 \delta_1 z_{11}$$

The First order conditions of this optimisation problem yield the following results:

$$\frac{d\pi_{11}}{dz_{11}} = n_1 k \left(\frac{\alpha_1 - 2kz_{11} - kz_{21}}{\beta_1} \right) - n_1 \delta_1 \dots \dots \dots (52)$$

Equating this to zero, we obtain the Reaction Function of Firm 1 as

$$z_{11} = \left(\frac{k\alpha_1 - \delta_1 \beta_1 - k^2 z_{21}}{2k^2} \right) \dots \dots \dots (53)$$

The Reaction Function of firm 1 can be defined as the best response of the firm, given that firm 2 chooses to generate z_{21} in the motive of profit maximisation. Similarly, firm 2's reaction function is as follows:

$$z_{21} = \left(\frac{k\alpha_1 - \delta_2\beta_1 - k^2z_{11}}{2k^2} \right) \dots\dots\dots (54)$$

Solving the two reaction functions in (30) and (31), we get

$$z_{11}^* = \left(\frac{k\alpha_1 - \beta_1(2\delta_1 - \delta_2)}{3k^2} \right) \text{ and } z_{21}^* = \left(\frac{k\alpha_1 - \beta_1(2\delta_2 - \delta_1)}{3k^2} \right) \dots\dots\dots (55)$$

And the quantities consumed from firm 1 and firm 2 respectively are,

$$q_{11}^* = \left(\frac{k\alpha_1 - \beta_1(2\delta_1 - \delta_2)}{3k} \right) \text{ and } q_{21}^* = \left(\frac{k\alpha_1 - \beta_1(2\delta_2 - \delta_1)}{3k} \right) \dots\dots\dots (56)$$

Substituting these values in (26), we get:

$$P_1^* = \frac{\alpha_1 - q_{11} - q_{21}}{\beta_1} = \frac{k\alpha_1 + \beta_1(\delta_1 + \delta_2)}{3k\beta_1} \dots\dots\dots (57)$$

The profits earned by the duopolists are:

$$\pi_{11} = n_1 z_{11} [kP_1 - \delta_1] - F_{11}$$

Substituting the values of z_{11} and P_1 into (34)

$$\pi_{11}^* = \frac{n_1}{9k^2\beta_1} [k\alpha_1 - \beta_1(2\delta_1 - \delta_2)]^2 - F_{11} \text{ and } \pi_{21}^* = \frac{n_1}{9k^2\beta_1} [k\alpha_1 - \beta_1(2\delta_2 - \delta_1)]^2 - F_{21} \dots\dots\dots (58)$$

Generalisation to the case of N firms

Here, the problem of each of the firms can be stated as follows:

$$\begin{aligned} \text{Maximize } \pi_{i1} &= n_1 P_1 q_{i1} - F_{i1} - n_1 \delta_i z_{i1} \\ &= n_1 \left[\frac{\alpha_1 - kz_{i1} - kz_{-i1}}{\beta_1} \right] kz_{i1} - F_{i1} - n_1 \delta_i z_{i1} \dots\dots (59) \end{aligned}$$

The first order conditions for profit maximisation lead us the following results:

$$\begin{aligned} \frac{d\pi_{i1}}{dz_{i1}} &= \frac{n_1 k \alpha_1 - 2n_1 k^2 z_{i1} - n_1 k^2 z_{-i1} - n_1 \delta_i \beta_1}{\beta_1} = 0 \\ &= n_1 k \alpha_1 - 2n_1 k^2 z_{i1} - n_1 k^2 (Z_1 - z_{i1}) - n_1 \delta_i \beta_1 = 0, \quad \text{since } z_{-i1} = Z_1 - z_{i1} \dots\dots\dots (60) \end{aligned}$$

Summing over the equations from 1 to N,

$$Nn_1k\alpha_1 - 2n_1k^2Z_1 - n_1k^2(N-1)Z_1 - n_1\beta_1\bar{\delta} = 0, \text{ where } \bar{\delta} \text{ is } \sum_{i=1}^N \delta_i$$

$$\Rightarrow Z_1 = \frac{Nk\alpha_1 - \beta_1\bar{\delta}}{k^2(N+1)} \dots\dots\dots(61)$$

Substituting the value of Z_1 in equation (48)

$$\Rightarrow n_1k\alpha_1 - n_1\delta_i\beta_1 - n_1 \left[\frac{Nk\alpha_1 - \beta_1\bar{\delta}}{1+N} \right] = n_1k^2Z_{i1}$$

Thus, the electricity generated by the i^{th} firm is given by:

$$Z_{i1}^* = \frac{k\alpha_1 - \beta_1[\delta_i(1+N) - \bar{\delta}]}{k^2(1+N)} \dots\dots\dots(62)$$

The quantity supplied by the i^{th} firm is,

$$q_{i1}^* = \frac{k\alpha_1 - \beta_1[\delta_i(1+N) - \bar{\delta}]}{k(1+N)} \dots\dots\dots(63)$$

The total quantity of electricity supplied by all the firms taken together,

$$Q_1 = kZ_1 = \frac{Nk\alpha_1 - \beta_1\bar{\delta}}{k(1+N)} \dots\dots\dots(64)$$

The price of electricity and the profits of the respective firms in general are given as below:

$$P_1^* = \frac{k\alpha_1 + \beta_1\bar{\delta}}{k\beta_1(1+N)} \dots\dots\dots(65)$$

$$\pi_{i1}^* = \frac{n_1\{k\alpha_1 - \beta_1[(1+N)\delta_i - \bar{\delta}]\}^2}{(N+1)^2k^2\beta_1} - F_{i1} \dots\dots\dots(66)$$

5.2 The case of the Shoulder period

In the shoulder period, though the demand for electricity is sufficiently high, it is not as high as compared to the peak period. There are three possible cases. In the first case, the capacity constraints are binding for both the firms. Since no firm can cater to the entire market alone, they strategically interact according to Cournot. In the second case and third case, the capacity constraints are non-binding. In such a situation, either only the most efficient firm operates (case ii), or both firms strategically interact according to the Stackleberg competition (case iii). These

cases are described below. The model is first built for a duopoly and then generalised to an N firm case.

Case (i): Capacity Constraints are binding for both firms

One possibility is that the capacity constraints for each of the two firms becomes binding. Hence, no firms can cater to the entire market demand all alone, and the two firms strategically interact between themselves and compete in quantities, according to Cournot. In this case, the problem is similar to that described above for the peak period. Using the same optimisation model, we get the electricity generated by both the firms as:

$$z_{11}^* = \left(\frac{k\alpha_2 - \beta_2(2\delta_1 - \delta_2)}{3k^2}\right) \text{ and } z_{21}^* = \left(\frac{k\alpha_2 - \beta_2(2\delta_2 - \delta_1)}{3k^2}\right) \dots\dots\dots(67)$$

The quantity demanded from each firm is,

$$q_{12}^* = \left(\frac{k\alpha_2 - \beta_2(2\delta_1 - \delta_2)}{3k}\right) \text{ and } q_{22}^* = \left(\frac{k\alpha_2 - \beta_2(2\delta_2 - \delta_1)}{3k}\right) \dots\dots\dots(68)$$

The price charged in the shoulder period will be as follows:

$$P_2^* = \frac{k\alpha_2 + \beta_2(\delta_1 + \delta_2)}{3k\beta_2} \dots\dots\dots(69)$$

And the respective profits earned by the duopolists are as follows:

$$\pi_{12}^* = \frac{n_2}{9k^2\beta_2} [k\alpha_2 - \beta_2(2\delta_1 - \delta_2)]^2 - F_{12} \text{ and } \pi_{21}^* = \frac{n_2}{9k^2\beta_2} [k\alpha_2 - \beta_2(2\delta_2 - \delta_1)]^2 - F_{22} \dots\dots\dots(70)$$

Generalising to N firms

Since the capacity constraints are binding, the firms interact based on Cournot competition, similar to the case of peak period. Thus, in the case of N firms, the electricity generated by the i^{th} firm and the total electricity generated by all the firms taken together are given as:

$$z_{i1}^* = \frac{k\alpha_1 - \beta_1[\delta_i(1+N) - \bar{\delta}]}{k^2(1+N)} \dots\dots\dots(71)$$

$$Z_2^* = \frac{Nk\alpha_2 - \beta_2\bar{\delta}}{k^2(N+1)}, \text{ where } \bar{\delta} = \sum_{i=1}^N \delta_i \text{ and } Z_2 = z_{i2} + z_{-i2} \dots\dots\dots(72)$$

The quantity demanded from the i^{th} firm and the total quantity demanded from all the firms taken together are respectively,

$$q_{i1}^* = \frac{k\alpha_2 - \beta_2[\delta_i(1+N) - \bar{\delta}]}{k(1+N)} \text{ and } Q_2^* = \frac{Nk\alpha_2 - \beta_2\bar{\delta}}{k(1+N)} \dots\dots\dots(73)$$

The price of electricity and the profits of the respective firms in general are given as below:

$$P_2^* = \frac{k\alpha_2 + \beta_2\bar{\delta}}{k\beta_2(1+N)} \dots\dots\dots (74)$$

$$\pi_{i1}^* = \frac{n_2\{k\alpha_2 - \beta_2[(1+N)\delta_i - \bar{\delta}]\}^2}{(N+1)^2k^2\beta_2} - F_{i2} \dots\dots\dots (75)$$

Case (ii): Capacity Constraints are non-binding: Only the more efficient firm operates

Another possibility is that the capacity constraints become non-binding for both the firms, that is, the capacity of each firm is sufficiently more than the demand for electricity. Hence, the most efficient firm generates and distributes electricity, while the competent firm does not produce anything because of its cost disadvantage. Thus, it is technologically feasible for firm 2 (the less efficient firm) to stop generation of electricity during this time on the assumption that the profits earned by firm 2 in the peak period are sufficiently high to compensate for the loss from not operating in this period. With $z_{22} = q_{22} = 0$, we modify the demand curve as follows:

$$P_2 = \left(\frac{\alpha_2 - q_{12}}{\beta_2}\right) = \left(\frac{\alpha_2 - kz_{12}}{\beta_2}\right) \dots\dots\dots (76)$$

The problem of firm 1 is as follows:

$$\text{Max } \pi_{12} = n_2P_2q_{12} - C_{12} = kn_2\left(\frac{\alpha_2 - kz_{12}}{\beta_2}\right)z_{12} - F_{12} - n_2\delta_1z_{12} \dots\dots\dots (77)$$

The first order conditions for profit maximization becomes:

$$\frac{d\pi_{12}}{dz_{12}} = \frac{kn_2\alpha_2 - 2k^2n_2z_{12} - n_2\delta_1\beta_2}{\beta_2} = 0 \dots\dots\dots (78)$$

$$\Rightarrow z_{12}^* = \frac{k\alpha_2 - \delta_1\beta_2}{2k^2} \text{ and } z_{22}^* = 0 \dots\dots\dots (79)$$

The quantity demanded from the first firm would be,

$$\Rightarrow q_{12}^* = \frac{(k\alpha_2 - \delta_1\beta_2)}{2k} \text{ and } q_{22}^* = 0 \dots\dots\dots (80)$$

Substituting this in (54), the price charged for electricity is given by:

$$P_2^* = \frac{(k\alpha_2 + \beta_2\delta_1)}{2k\beta_2} \dots\dots\dots (81)$$

The profits earned by firm 1 and 2 are:

$$\pi_{12}^* = \frac{n_2[k\alpha_2 - \delta_1\beta_2]^2}{4\beta_2k^2} - F_{12} \text{ and } \pi_{22}^* = -F_{22} \dots\dots\dots (82)$$

Generalising to N firms

In this case, only the most efficient firm (Firm 1) operates, and the quantity supplied by all the other firms is zero (i.e. $z_{i2} = q_{i2} = 0$ for all firms except $i=1$). Thus, the problem of firm 1 is reduced to what has been discussed in case (ii) for a duopoly. The electricity generated by firm 1 is given by:

$$z_{12}^* = \frac{k\alpha_2 - \delta_1\beta_2}{2k_1^2} \dots\dots\dots (83)$$

$$q_{12}^* = \left(\frac{k\alpha_2 - \delta_1\beta_2}{2k} \right) \dots\dots\dots (84)$$

The price of electricity is given by:

$$P_2^* = \frac{\alpha_2 - q_{12}}{\beta_2} = \frac{k\alpha_2 + \delta_1\beta_2}{2k\beta_2} \dots\dots\dots (85)$$

The profits earned by the firms are:

$$\pi_{12}^* = \frac{n_2[k\alpha_1 - \delta_1\beta_2]^2}{4\beta_2k^2} - F_{12} \text{ and } \pi_{i2}^* = -F_{i2} \text{ for all firms except } i = 1 \dots\dots\dots (86)$$

Case (iii): Capacity Constraints are nonbinding, but both firms participate

In this case, both firms participate in the market but they strategically interact between themselves sequentially, in a manner which has close proximity to Stackelberg’s oligopoly model. This is essentially a 2-stage game, where the more efficient firm (Leader) chooses its quantity of electricity to be supplied initially, and then in the next stage of the game, the rival firm (Follower), chooses its quantity, having observed the quantity chosen by the leader. In the Cournot Model, on the other hand, we saw a one period game where both firms choose the quantity of electricity to be supplied simultaneously.

In Stage 2, Firm 2’s problem is:

$$\begin{aligned} \text{Max } \pi_{22} &= n_2P_2q_{22} - C_{22} = kn_2P_2z_{22} - F_{22} - n_2\delta_2z_{22} \dots\dots\dots (87) \\ &= kn_2 \left[\frac{\alpha_2 - kz_{12} - kz_{22}}{\beta_2} \right] z_{22} - F_{22} - n_2\delta_2z_{22} \end{aligned}$$

The first order conditions yield the reaction function of Firm 2 as:

$$z_{22} = \left(\frac{k\alpha_2 - \delta_2\beta_2 - k^2z_{12}}{2k^2} \right) \dots\dots\dots (88)$$

In the first stage, firm 1 chooses its output, knowing that firm 2 will react to it in the second period according to its reaction function given by (88). Here firm 1's problem is:

$$\begin{aligned} \text{Max } \pi_{12} &= n_2 P_2 q_{12} - C_{12}, = kn_2 P_2 z_{12} - F_{12} - n_2 \delta_1 z_{12} \\ &= kn_2 \left(\frac{\alpha_2 - kz_{12} - \left(\frac{k\alpha_2 - \delta_2 \beta_2 - k^2 z_{12}}{2k} \right)}{\beta_2} \right) z_{12} - F_{12} - n_2 \delta_1 z_{12} \dots\dots\dots (89) \end{aligned}$$

since $P_2 = \left(\frac{\alpha_2 - q_{12} - q_{22}}{\beta_2} \right)$.

The first order conditions of profit maximisation are given by:

$$\frac{d\pi_{12}}{dq_{12}} = \left(\frac{kn_2 \alpha_2 - 2n_2 k^2 z_{12} + n_2 \beta_2 \delta_2}{2\beta_2} \right) - n_2 \delta_1 = 0 \dots\dots\dots (90)$$

Thus, the quantity of electricity generated by the Leader firm 1 is:

$$z_{12}^L = \left(\frac{k\alpha_2 - \beta_2(2\delta_1 - \delta_2)}{2k^2} \right) \text{ and } q_{12}^L = \left(\frac{k\alpha_2 - \beta_2(2\delta_1 - \delta_2)}{2k} \right) \dots\dots\dots (91)$$

Substituting this in the reaction function of firm 2 given in (52), we get:

$$z_{22}^F = \left(\frac{k\alpha_2 - \beta_2(3\delta_2 - 2\delta_1)}{4k^2} \right) \text{ and } q_{22}^F = \left(\frac{k\alpha_2 - \beta_2(3\delta_2 - 2\delta_1)}{4k} \right) \dots\dots\dots (92)$$

The respective price of electricity and the profits of the two firms may be obtained as follows:

$$P_2^* = \frac{k\alpha_2 + \beta_2(\delta_2 + 2\delta_1)}{4k\beta_2} \dots\dots\dots (93)$$

The respective profits are then as follows:

$$\pi_{12}^L = \frac{n_2 \{k\alpha_2 - \beta_2(2\delta_1 - \delta_2)\}^2}{8\beta_2 k^2} - F_{12} \dots\dots\dots (94)$$

$$\pi_{22}^F = \frac{n_2 \{k\alpha_2 - \beta_2(3\delta_2 - 2\delta_1)\}^2}{16\beta_2 k^2} - F_{22} \dots\dots\dots (95)$$

Generalising to N firms

In this case, the dynamics of the Stackelberg game can be solved by backward induction, which would result in the emergence of leader firms and follower firms in the market. The mathematical modelling is left as an extension for further work.

5.3. The case of off-peak period

We assume that the demand for electricity is in fact less than the capacity of the individual firms. Both the firms can therefore cater to the entire market demand individually, without strategically interacting with each other. The most efficient firm, due to its cost advantage as compared to its competitor, can actually charge a price which is less than the marginal cost of its competitor. Hence, the other firm cannot participate, as charging such a price would result in a loss. As a result, only the most efficient firm participates in the market, caters to the entire market demand and extracts the entire profit. The problem thus reduces to what has been discussed in Section 5.2, case (ii). As the entire market demand is catered by the first firm, i.e., $q_{23} = 0$, the required electricity generated by and demanded from firm 1, using the same optimisation model, is:

$$z_{13}^* = \frac{k\alpha_3 - \delta_1\beta_3}{2k^2} \text{ and } q_{13}^* = \frac{(k\alpha_3 - \beta_3\delta_1)}{2k} \dots\dots\dots (96)$$

The price charged for electricity is:

$$P_3^* = \frac{(k\alpha_3 + \beta_3\delta_1)}{2k\beta_3} \dots\dots\dots (97)$$

And the required profits for firm 1 and 2 are as follows:

$$\pi_{13}^* = \frac{n_3[k\alpha_3 - \delta_1\beta_3]^2}{4\beta_3k^2} - F_{13} \text{ and } \pi_{23}^* = -F_{23} \dots\dots\dots (98)$$

However, one important question is why the rival firm would choose to remain idle. A possible solution could be the payment of some side payments by the efficient firm to its rival, as a compensation for not producing. Also, as already discussed, the profits earned by the less efficient firm during the peak period and the shoulder period is sufficiently more than the loss from not producing in the off-peak period.

Generalization for n firms

Here, the dynamics of the model will be exactly same as that discussed in the case of a duopoly where the most efficient firm produces, while the other firms remain idle.

Thus, in the off-peak period, an oligopolistic / duopolistic structure breaks down into a monopolistic one.

To summarise the various cases, the profits earned by the oligopolists are presented in Table 5.

Table 5: Summary: Duopolist's Profits under various Demand Conditions

	Profit of Firm 1 (More Efficient Firm)	Profit of Firm 2 (Less Efficient Firm)
<u>Peak Period</u>		
High Demand leading to Cournot Competition	$\pi_{11}^* = \frac{n_1}{9k^2\beta_1} [k\alpha_1 - \beta_1(2\delta_1 - \delta_2)]^2 - F_{11}$	$\pi_{21}^* = \frac{n_1}{9k^2\beta_1} [k\alpha_1 - \beta_1(2\delta_2 - \delta_1)]^2 - F_{21}$
<u>Shoulder Period</u>		
Case (i): High Demand leading to Cournot Competition	$\pi_{12}^* = \frac{n_2}{9k^2\beta_2} [k\alpha_2 - \beta_2(2\delta_1 - \delta_2)]^2 - F_{12}$	$\pi_{22}^* = \frac{n_2}{9k^2\beta_2} [k\alpha_2 - \beta_2(2\delta_2 - \delta_1)]^2 - F_{22}$
Case (ii): Low Demand resulting in only the most efficient firm operating, resulting in a Monopoly	$\pi_{12}^* = \frac{n_2[k\alpha_1 - \delta_1\beta_2]^2}{4\beta_2k^2} - F_{12}$	$\pi_{22}^* = -F_{22}$
Case (iii): Low Demand resulting in Stackelberg type Competition	$\pi_{12}^L = \frac{n_2\{k\alpha_2 - \beta_2(2\delta_1 - \delta_2)\}^2}{8\beta_2k^2} - F_{12}$	$\pi_{22}^F = \frac{n_2\{k\alpha_2 - \beta_2(3\delta_2 - 2\delta_1)\}^2}{16\beta_2k^2} - F_{22}$
<u>Off Peak Period</u>		
Low Demand resulting in only the most efficient firm operating, resulting in a Monopoly	$\pi_{13}^* = \frac{n_3[k\alpha_3 - \delta_1\beta_3]^2}{4\beta_3k^2} - F_{13}$	$\pi_{23}^* = -F_{23}$

6. Conclusion & Implications for Practitioners

Time of Use pricing not only brings about economic efficiency in the market for electricity, the reduction in the peak demand may lessen the power generation, transmission and distribution costs by reducing the demand-supply imbalance. It may also lead to avoidance of costs associated with building new capacities.

However, time-of-use pricing affect producers depending on the market structure. In this study, we model time-of-use pricing in the context of a monopolistic set up and an oligopolistic set up.

In the presence of a single firm, that is, a monopoly, the firm operates using constrained optimisation techniques. We estimate the feasibility condition for there to be efficiency gains from practicing time-of-use pricing *vis-a-vis* flat rate pricing.

In an oligopolistic set up, the interaction between firms depends on the market demand. In the peak period, when the demand is very high, no single firm can cater to the demand of the entire market, and firms compete on the basis of quantity, that is, Cournot Competition. In the off shoulder period, the demand may be high enough to encourage Cournot Competition. However, if the demand is low, either only the most efficient firm operates, or there is an emergence of leaders and followers and the situation becomes similar to the Stackelberg Model. In the off-peak period, only the most efficient firm operates in the market, thus there is a breaking down of the market structure from an oligopoly to a monopoly.

The electricity sector is characterised by imperfect competition, strategic interaction, collective learning, asymmetric information and the occurrence of multiple equilibria. As suggested by the study, there is possibility of significant market power among the electricity suppliers, both in a monopolistic and an oligopolistic set up. Large firms, by increasing their production, lowering prices, and using different strategic interaction techniques can foreclose competition, and hence exercise market power. Standard economic theory suggests that less elastic the demand, more

market power is observed. Thus, implementing policies which promote both the consumer and the producer's responsiveness to short run fluctuations in prices can reduce market power.

This paper can be extended in several directions. Firstly, we assume that customers are homogeneous. By segmenting the customers into different groups, say residential users and commercial users, we can derive the models considering that each group has demand curves of different elasticities, and thus varying consumer surpluses. Secondly, we can derive the optimality conditions when we move from a linear demand price relationship as assumed in this paper, to a non-linear demand price relationship, for example, the case of a constant elasticity demand curve represented by a rectangular hyperbola. We can also examine the case of non-linear (cubic) cost functions. Thirdly, as discussed in this paper, in the shoulder period, when the demand is lower, it may be possible for one firm to entirely cater to the market - the most efficient firm, or the firms to get into Stackleberg competition. Here, there is an emergence of leaders and followers, and the equilibrium can be found using a game theory approach (sequential game) and can be solved by backward induction. Modelling this for n firms could be a possible extension. Fourthly, we can take into account various externalities. Finally, in this paper, we divide the day into three periods with predetermined prices. An extension could be to look at the imperfectly competitive market structures when electricity pricing is fully dynamic, that is, it follows Real Time Pricing.

Acknowledgments

We thank the Research and Publications Office of IIM, Ahmedabad for financial support for this work.

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