# Transitions in currency denomination structure as supply disruption and demand distortion: Efficiency, Effectiveness and Bullwhip 

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# Transitions in currency denomination structure as SUpply disruption and demand distortion: Efficiency, Effectiveness and Bullwhip 

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#### Abstract

Transition from one currency denomination structure to another is infrequent but not rare. Central Banks may adopt such transition for various reasons like prevention of counterfeiting or combating hyperinflation and may include demonetisation of specific denominations or introduction of new denominations. We study transitions in currency denomination from a supply chain perspective. Currency as a product flows through a three-stage supply chain in which currency denominations are substitutable products. We show that demand for a specific denomination depends on the denomination structure and distribution of transactions in the economy. During a transition from one denomination structure to another, the demand for a specific denomination is affected due to change in step size. In addition, the demand may be distorted due to hoarding resulting from supply shortages. Such transaction related hoarding behaviour may occur for lower denominations, in contrast to wealth accumulation related hoarding of higher denominations known in the literature, and can lead to a Bullwhip Effect. We propose efficiency and effectiveness related measures for the remonetisation process and study the impact of prioritisation of supply of one denomination over another on demand distortion. In doing so, we extend the literature on efficient transactions by introducing an aggregate transaction efficiency measure considering the transaction distribution and show how this measure is sensitive to transaction slabs, denomination structures and transaction distributions. Such analysis may inform Central Banks about relative vulnerabilities of different denominations to a supply disruption which distorts currency demand.


Keywords: Demonetisation, currency transition, denomination structure, remonetisation, supply disruption, demand distortion

## 1 Introduction

Currency, as notes and coins, is available in pre-decided denominations. The complete assortment of these denominations is called the currency denomination structure. For instance, denomination structure for the US dollar(USD) consists of notes of denominations $1,2,5,10,20,50$ and 100 and coins of denomination $1,5,10,25,50$ cents and 1 USD $^{1}$ ( 1 USD $=100$ cents). Currency as a product in the form of bank notes starts its lifecycle with the procurement of raw materials like paper and ink of specific grades and their printing in specialised presses. It is then distributed through a network of various bank branches and Automated Teller Machines (ATMs) and circulated amongst end users. There also exists a reverse logistics channel as old, soiled notes are sent back to dedicated facilities of the central bank for disposal. The entire life cycle of the currency is managed amidst high security, which adds to the complexity and the costs involved with the process.

Central Banks may change the existing denomination structure by introducing new denominations or by withdrawing legal tender status of specific denominations, an act referred to as demonetisation. Demonetisation may happen for all denominations as a country moves to a new currency (e.g.

[^1]transition to a common currency like Euro) or for select denominations for reasons like prevention of counterfeiting, combating hyperinflation, to launch a new series of notes or to unearth stocks of black money. Transitions in currency denomination structure have the potential to disrupt the supply and distort the demand of currency unless properly managed. Lack of availability of specific denominations with end users may negatively affect economic activity. The process of remonetisation includes activities like printing, storage and distribution of existing and newly introduced currency to achieve normalcy of economic transactions. Our paper is motivated by the demonetisation of Indian Rupee (INR) in November 2016, when currency denominations of 500 and 1000, worth around $86 \%$ of the value of money in the economy, were scrapped and replaced by new series of 500 and a new denomination of 2000. Other examples of demonetisation include Australia in 1988 (to prevent counterfeiting), Brazil in 1990 and 1993 (to fight hyperinflation), Russia in 1991 and 1993 (to fight organised crime), Europe in 1999 (introduction of new currency), Singapore in 1999 and 2015 (to mitigate money laundering), and Greece in 2015 (to address fiscal crisis) ${ }^{2}$.

A transaction using currency is said to be efficient if it requires minimum number of notes possible for the given denomination structure. Choice of denomination structure is an important decision for a central bank as the requirement of currency notes is a function of the denomination structure. It has been studied in prior literature by Cramer (1983), Kohli (1988), Telser (1995), Van Hove (2001). Notably absent is any work dealing with the robustness of denomination structure to supply shocks. These supply shocks change denomination wise demand. Demonetisation can be seen as a supply disruption as demonetised denominations cease to be legal tender, potentially creating severe supply shortage if replacement currency is unavailable due to printing or disbursal bottlenecks. Such shortages of specific denominations may result in increased demand for close substitutes. Product substitution has been identified as a mitigation strategy for supply disruption in existing literature Lu et al. (2011), Shao (2012). We are one of the first papers to study transitions in currency denomination structure as a supply disruption induced demand distortion in a supply chain with the currency denominations as substitute products.

Remonetisation related decisions include the choice of new denomination structure and the prioritisation in introduction of denominations. We propose metrics for measurement of operational performance of the remonetisation related tasks of printing and disbursal and show why printing and disbursal efficiency may not guarantee effective remonetisation. The changes in denomination structure and the sequence of introduction of new denominations are identified as having an impact on the effectiveness of the remonetisation process. Step size of a denomination, which is defined as the ratio between the subsequent denomination and the given denomination, impacts its substitutability. Transition to a new denomination structure may affect all or some of the denomination step sizes and may thus impact the demand of denominations differentially. A sudden currency transition during demonetisation may disrupt supply and lead to a shift to other modes of transaction like digital payments, cheques, credit and even barter. While central banks may take demonetisation as an opportunity to increase the proportion of digital transactions in the economy, we focus only on cash transactions. The effectiveness of the remonetisation process is defined in terms of achieving normalcy of cash transactions, where normalcy is the state when the proportion of cash transactions undertaken efficiently reach pre-demonetisation levels. We show that prioritisation of higher value denominations in the remonetisation process may have an unintended impact on the overall requirement of currency for the transaction, which is counter-productive for the overall effectiveness of the process.

Existing literature on currency denominations has discussed the prevalence of hoarding of higher denominations to accumulate wealth. We hypothesise about the existence of a transaction related hoarding that can occur as the demand for a denomination increases due to the shortage of the substitute denomination. We explain transaction related hoarding as a substitutable product Bullwhip Effect, discussed in the supply chain literature by Duan et al. (2015). Such transaction related hoarding may occur for lower denominations, a finding that is empirically supported by several news reports during the demonetisation of Indian Rupee in November 2016, discussed later in the paper.

[^2]Earlier work on efficient denomination structures had not considered the distribution of transactions existing in an economy over a range of transaction values. We extend this body of work by constructing an aggregated measure of transaction efficiency that considers the probability distribution of transactions over a range of transaction values. Using this aggregated efficiency measure and performing a sensitivity analysis, we show that a currency supply shock impacts transaction slabs differently. We compare the performance of alternate denomination structures and show that denomination structure affects the robustness of a denomination which is the extent of vulnerability of specific denomination to currency transition. We repeat the procedure for several theoretical probability distributions to understand the sensitivity of aggregate transaction efficiency to changes in distribution choice and parameters.

The paper is arranged in the following sequence. In the next section, we provide a review of the literature on denomination structure choice and currency demand; transaction patterns; and demonetisation. We describe the currency supply chain, and study supply disruption and demand distortion in Section 3. The demonetisation of Indian Rupee is discussed in Section 4. In Section 5, we discuss the remonetisation process and develop stage wise measures of operational performance to come up with key hypotheses regarding decision choices for an effective remonetisation. Section 6 elaborates the mathematical model, computational analysis to evaluate the aggregate transaction efficiency under denomination structures and probability distributions choices and lists key findings. We provide our concluding remarks in the last section.

## 2 Literature Review

Literature in the area of denomination structure choice is sparse. Cramer (1983) was one of the first papers to analyse denomination-wise circulation of currency. He defined the principle of efficient payments according to which people try to transact in the minimum possible number of notes and coins. According to the author, step size restricts the usage of a particular denomination as it indicates the value after which the substitute denomination is available. Kohli (1988) illustrated that while denominations are considered as perfect substitutes to each other, they vary in characteristics. Smaller denominations offer liquidity for transactions while larger denominations provide an additional utility for storage. Characteristics such as denomination value, physical size, colour and purchasing power may determine the share of each denomination in circulation. Step size impacts the demand for a denomination with higher step size resulting in higher demand. The increase in prices and income levels also impacts the demand for higher denominations positively. The author estimated that a shift from the denomination structure for Swiss Franc from 100, 500 and 1000 to 100 , 200 and 1000 would reduce the overall currency requirement by around 9 million notes.

Several papers compare payment transactions with the Bachet's Problem, a classic problem in number theory, discussed by Hardy and Wright (1979). This problem deals with weighing all integral weights in a range and finds that if weights are in powers of 3 , we can weigh any integral value from 1 to an upper bound. Telser (1995) referred to the Bachet's Problem and explained that for a currency system to optimise the number of notes used, step size should be close to 3 . The author considered transactions to be uniformly distributed between 1 and the upper bound. Van Hove and Heyndels (1996) compared the currency systems with step sizes of 2 and 3 and found the step size of 2 to be more efficient. Van Hove (2001) highlighted that each weight is used only once in a weighing transaction in the Bachet's Problem while a currency denomination can be used multiple times in a currency transaction. The author used multi-criteria optimisation with objectives of achieving efficient payments, compatibility with the decimal system and avoiding too much variety, giving highest weight to the criteria of efficient payments. Binary-decimal triplets ( 1-2-5; 10-20-50 etc.) were recommended as a denomination structure.

Few papers have modelled currency in circulation using optimisation models. Massoud (2005) provided a dynamic programming formulation for optimal bank note inventory to be maintained by
central banks minimising the costs of production and distributions and ensuring liquidity to manage shocks. Lee et al. (2005) provided a theoretical model for selecting a denomination structure considering the distribution of transactions and denominations held by consumer endogenous to the model. Ghezavati (2012) used mixed integer multi-objective model to optimise denomination choices minimising overall publication costs, notes transacted and return of cash in case of over payments.

Kippers et al. (2003)) conducted an empirical study to show that the content of wallet places a constraint on efficient payments. Comparing data on the circulation of the larger denominations with the average usage of these denominations for cash transactions, authors concluded that higher denominations were used for purposes beyond cash transactions. Demand for higher denominations was mostly for savings (estimated at $70 \%$ of the notes in circulation). Another study by Franses and Kippers (2007) showed that principle of efficient payment does not hold for many cases due to the computational efforts required, time pressures and availability of currency in the wallets. In their data set, only about $60 \%$ of the transactions were done in an efficient manner. Klee (2008) used a discrete choice model to determine that factors like transaction and handling costs, and product characteristics impacted the preferred choice of transaction method. Author used transaction data from the grocery shops as retail grocery outlets provide a holistic representation of the population for transaction choice modelling.

There have been very few studies exploring demonetisation. Dzokoto and Mensah (2010) analysed the impact of redenomination of currency in Ghana through an exploratory study. Their study confirmed Bank of Ghana's claim regarding the benefits of redenomination like security and portability. They also found that the new denomination structure had negative consequences like confusion amongst people on the conversion between old currency and the new one. This confusion occurred due to the counting efforts involved and uncertainty over value of money to be withdrawn as purchasing power was unclear. Few others studies like the one by Ofonagoro (1979), studied the currency change, which happened in Africa as European powers gained control around the early twentieth century. In Nigeria, pre-colonial currencies were denied status as legal tender without proper exchange being offered, resulting in a financial distress on the natives. Interestingly, these old currencies continued circulation even years after their demonetisation suggesting how communities can sustain the idea of money without institutional support through the exchange.

## 3 Currency Supply Chain

Central bank, banks and the end user form a three-stage supply chain with currency as the product. Currency notes of different denominations represent substitutable products. Substitutability of a denomination is a function of step size, which is the ratio between the subsequent denomination and the given denomination. Higher is the step-size, lower is the substitutability. Central Bank, as the producer of the currency, is the upstream member of this supply chain. It procures raw materials, manages the production facilities, and controls the overall currency in circulation. After production, currency is moved to currency chests at different locations and is provided to banks for further distribution through the bank branches and ATMs. Banks and ATMs are the next stage of this supply chain and provide the function of storage and distribution of money. People withdraw money from banks and ATMs for end use which includes transaction demand as well as storage for speculative demand and for wealth accumulation. End user represents the downstream stage of the supply chain. These end users may further have buyer-seller transactions between themselves representing sub-stages within the third stage; in addition to the withdrawal and deposit transactions with banks. This circular nature of the currency supply chain is in contrast with the unidirectional flow of products in traditional supply chains. Further, a reverse supply chain channel exists for return and disposal of soiled currency notes once they become unfit for circulation. Figure 1 provides a representation of the currency supply chain.


Figure 1: Three-stage supply chain for currency

### 3.1 Disruption of Currency Supply

Transition from one denomination structure to another has the potential to disrupt the supply chain of currency as the availability of denomination(s) is suddenly curtailed. Supply chain disruption and risk mitigation has been widely studied in the literature. Many papers have analysed it from an inventory management perspective by evaluating choices of ordering policies in the event of supply interruptions ( Parlar (1997), Moinzadeh and Aggarwal (1997), Arreola-Risa and DeCroix (1998), Sargut and Qi (2012)). Amongst the papers on broader risk mitigation strategies, Kleindorfer and Saad (2005) identifed risks arising from disruptions like natural disasters, strikes, economic disruptions and terrorist attacks to provide a framework for risk assessment and mitigation. Tomlin (2006) analysed the risk mitigation strategy for a firm as it makes a choice between two suppliers with different levels of reliability for a single product supply chain. Oke and Gopalakrishnan (2009) studied retail supply chains to identify key disruption risks involved as either supply or demand risks. For each of this risks, a classification was made based on the likelihood and impact of risks and mitigation strategies were recommended.

Similar to the other supply chains, currency supply chain is vulnerable to supply risks. We analyse a specific type of supply disruption caused by the transition from one denomination structure to another. It impacts availability of certain denominations as they are no longer a legal tender and thus their demand is shifted to other substitute denominations. Product substitution has been studied in existing literature as a mitigation strategy for supply chain disruption. Lu et al. (2011) studied this in a two-product setup. Shao (2012) compared impact of substitution of a lower value product as well as a higher value product, as each of the product's supply is disrupted. Currency supply chain is unique as the supply disruption due to currency transition is a planned disruption and therefore can be carried out in phased manner which allows planning of mitigation strategies. Additionally, unlike other supply chains, all existing substitutes are provided by the same firm, the central bank. Thus, the supply of one substitute can impact the production of the other due to the constraints in production capacities and distribution. End users are in need for currency for carrying
out transactions in the economy and therefore a shortage impacts transactions of other goods and also leads to a shift to alternate modes of payment.

### 3.2 Distortion in Currency Demand

Transitions in denomination structure may impact availability of certain denominations that in turn changes the demand of substitute denominations. This structural change leads to a behavioural change, as due to the perceived shortage, hoarding of high demand denominations ensues, which further amplifies the demand. Thus, distortion in currency demand can be of two types (i) Denomination structure change led distortion which we call as transition volatility and (ii) Shortage gaming led demand distortion which we call as hoarding volatility.

Transition volatility quantifies how requirement of number of notes for efficient transaction varies with the change in denomination structure. We find transition volatility by calculating average weighted requirement of notes $W$ for various denomination structure choices. Change in the average requirement of notes between denomination structures provides a measure for transition volatility. Hoarding or accumulation of currency more than demand can occur with an objective of wealth accumulation and also as a reaction to perceived shortage. Transition volatility and hoarding volatility combined is the overall demand distortion seen in the currency demand or the total volatility.

Hoarding for wealth accumulation, which occurs for higher denominations is well documented in the literature. Ghezavati (2012) provides a review of research in this area. Kohli (1988) argue that higher denominations have additional utility for storage of value. Higher denominations are hoarded as they are used for storage of value rather than transactional purposes. Kippers et al. (2003) compare notes in circulation with their use for transaction to conclude that more than 50 percent of the higher denominations are used for storage. Boeschoten and Fase (1992) empirically show that 60-70 percent of the Fl. 1000, the highest denomination in Netherlands, was used for hoarding.

Hoarding also exists when supply chain members accumulate more than the demand in case of perceived shortage. Supplier rations supplies between the downstream users, thereby creating a perception of shortage, reinforcing hoarding behavior. This hoarding can be for any denomination experiencing a shortage. Users experience difficulty in making a transaction due to unavailability of denominations required for efficient transactions and orders or draws more currency than needed. This phenomenon of the order being higher than the demand has been explained in supply chain literature by Lee et al. (2004) as the Bullwhip Effect. It is described as the amplification of variability of demand as one moves upstream from retailer to wholesaler to manufacturer in a multi-stage supply chain. Bullwhip Effect derives its name from similarity to a whip whose one end is vibrated. As in the case of such a whip, the other free end witnesses amplified vibration. In the currency supply chain, it is the downstream user who withdraws currency from banks and ATMs, more than the requirement, resulting in an amplification of upstream demand. This demand gets further amplified when banks place an order on the central bank.

Lee et al. (2004) in their paper have identified demand signal processing, rationing game, order batching, and price variation as the causes of Bullwhip Effect. Duan et al. (2015) extend the bullwhip literature by demonstrating occurrence of the Bullwhip Effect with changes in the substitute product's price and demand. Their study is in a multi-product context with competing products as substitutes and it provides empirical evidence to affirm that substitute product's stock outs and price changes influence the intensity of Bullwhip Effect. Due to change in the denomination structure, demand for certain denominations go up as the efficient way for doing a transaction changes.

Bullwhip Effect occurs when there is an upstream amplification of the fluctuation in downstream demand. However, there can be a situation when demand remains same or varies as before while the supply is suddenly disrupted. Rong et al. (2008) analyse the impact of the order variability in the case of supply disruption and find that variability increases downstream instead of upstream
as explained by the Bullwhip Effect. They call this phenomenon, the Reverse Bullwhip Effect. According to the authors, over reaction to the supply shock leads to amplification of demand at the end customer. Reverse Bullwhip Effect, in terms of the whip analogy, has an upstream fluctuation and it travels downstream with an amplification. Thus, it is the central bank, which generates the supply shock which travels downstream and causes amplification of currency demand with the end customer. As a result, demand anticipated by the end customer is higher than that by the banks and the central bank.

Rong et al. (2008) further show that both effects can occur simultaneously if there is a supply shock and as a result of the perceived shortage, ordering is also impacted. This can be thought of like a whip, which has both ends tied up and is then is vibrated from both sides resulting in maximum amplitude in the middle. In the presence of both effects, it is the middle member of the supply chain, which sees the highest amplification of vibration. The three-stage supply chain of money experiences a Bullwhip Effect as well as a Reverse Bullwhip Effect. Bullwhip Effect results in an over-amplification of orders from end-users to banks and from banks to the central bank while Reverse Bullwhip Effect causes demand amplification in the opposite direction. However, in this case, the central bank has complete information on total supply of notes (printing and circulation) and therefore it is anticipated that a dampening of this demand amplification is done by them for planning further production. However, as suggested by Rong et al. (2008), this may result in the middle member of currency supply chain, the banks, experiencing maximum demand distortion.

## 4 Currency Demonetisation in India

The denomination structure of Indian Rupee (INR) included denominations of $1,2,5,10,20,50$, 100,500 and 1000 in currency notes and coins of denominations 0.5 ( 50 paise), $1,2,5$ and $10 \mathrm{INR}^{3}$. Coins of denominations lower than 50 paisa were legal tender but are no longer in circulation due to inflation. On 8th November 2016, the Indian government made an announcement to immediately (i) scrap existing currency notes of denomination 500 and launch a new series as a replacement (ii) scrap existing currency notes of denomination 1000 without any replacement and (iii) introduce a new denomination of 2000. Provisions were made to allow for replacement or deposit of old currency notes in banks till 30th December ${ }^{4}$. Additionally, limited usage of the old currency notes was allowed for essential utilities like transactions at government hospitals, ticket booking counters for travel, and fuel-refilling stations till 24 th November ${ }^{5}$.

Historically, few countries have demonetised currency. In most of the instances, demonetisation was planned and done gradually in a phased manner and only a few had an element of suddenness or shock. Amongst the planned events, Europe went through one of the biggest demonetisation exercise as it moved to a single currency, Euro, in 1999. Australia in 1988 demonetised the existing paper currency and launched polymer notes with a longer life to tackle counterfeiting. Canada adopted similar measures in 2011. Notable amongst the sudden demonetisations were initiatives in Brazil to move to new currency in 1990 and 1993 to fight hyperinflation, and in erstwhile Soviet Union, where in 1991 denominations 50 and 100 were withdrawn suddenly to counter organised crime ${ }^{6}$.

India had implemented demonetisation on two previous occasions. In 1946, currency notes of 1000 and 10000 were demonetised while 1000,5000 and 10000 were demonetised in 1978. The current instance of demonetisation was unprecedented as it was planned to be a shock event. It was managed with utmost secrecy to meet government's stated objectives of black money eradication and curtailing the usage of fake currency for sponsoring terrorism. Currency for replacement was not produced or distributed in advance to avoid divulgence of information ${ }^{6}$. This event was carried out at a large

[^3]scale and 14730 billion INR ( 15.7 billion notes of 500 and 6.8 billion of 1000 s) representing around $86 \%$ of the currency by value was scrapped ${ }^{7}$.

Demonetisation led to a disruption to the currency supply chain. The Indian central bank, RBI (Reserve Bank of India), had to plan substitute denominations for remonetisation and their prioritisation for production and distribution. Currency for replacement was printed at four dedicated printing presses. Two of the four presses at Nashik (Maharashtra) and Dewas (Madhya Pradesh) are controlled by the Government of India and accounted for 40 percent of the total printing capacity. The other two at Mysore in Karnataka and Salboni in West Bengal are owned by Reserve Bank of India and provided remaining printing capacity. The government owned presses had old machinery and were known to be inefficient. Monthly printing capacity for the four currency printing presses was around 2.2 billion units on a two shift basis. Assuming three shifts, it is estimated that the new notes could have been supplied at a rate of 3.3 billion notes per month. Choosing 2000 as a replacement of 1000 , reduced the requirement of currency notes to be printed as a replacement to half. Therefore, 6.8 billion notes of 1000 were required to be replaced with 3.4 billion notes of 2000 . Factoring this for estimating the reduced overall requirement, it is anticipated that a period of six months would be required for producing the equivalent value in around 19 million notes. The two high capacity and more efficient printing facilities at Mysore and Salboni, with a combined capacity of printing about 16 million notes per year on a two-shift basis, were dedicated to the 2000-rupee note production after demonetisation ${ }^{8}$.

On the date of the announcement, the RBI had an inventory of about 4940 billion in value with the denomination 2000 and no inventory for replacement of the denomination $500^{9}$. It has been reported that production of 500 started on November 23 , two weeks after demonetisation ${ }^{10}$. Even a month after demonetisation, many places in the country were yet to receive the first lot of 500 , a denomination that represented around 54 percent of total value of currency in circulation before demonetisation ${ }^{11}$. Prioritisation of 2000 over 500 in the remonetisation process was aimed at achieving a faster replacement of the value withdrawn from the economy. This prioritisation based on the value of currency was evident from the key decisions like allocation of the presses and assurances made by the policy makers ${ }^{12}{ }^{13}$. Deputy Governor of the RBI, responsible for currency management, attributed this decision of prioritisation to having sufficient value for exchange of old currency ${ }^{14}$.

Shortage after demonetisation led to withdrawal of currency by the end user in amounts that was more than the requirement. Newly released denomination of 2000 was in short supply as it had to cater to the demand of demonetised denominations of 500 and 1000 . This resulted in hoarding of $2000^{15}$, possibly both due to a reaction to the perceived shortage and for wealth accumulation. This perceived shortage resulted in amplification of the upstream demand. Banks witnessed a huge rush of people and operated overtime to maximise the number of people served. As explained using Bullwhip Effect in the previous section, a rationing behaviour was seen by the banks and the central bank as a reaction to the perceived shortage which further led to a demand amplification. A daily withdrawal limit of INR 2000 from ATMs and INR 2500 was set for bank branches to ration the supply of

[^4]the new currency ${ }^{16}$. Banks also preferred serving internal customers and hence prioritised cash disbursal through bank branches over ATMs. The extent of unavailability of currency varied across geographies and between urban and rural areas due to the significant difference in the penetration of ATMs and bank branches across regions. There was a disparity in refill rates in the urban areas and the rural areas. Rationing behaviour by the central bank resulted in a perception that private banks were getting prioritised over public banks in cash distribution ${ }^{17}$.

There was also surge in demand of 100-rupee note and of other smaller denominations. Absence of the pre-demonetisation denominations of 500 and 1000 and a very low availability of newly introduced 2000 further increased the demand of denomination 100 , due to absence of a substitute denomination of higher value ${ }^{18}$. Before demonetisation, any transaction above 500 required usage of 500 or 1000 for an optimal transaction. This needed to be substituted with the usage of multiple 100 rupee notes now, thus leading to a shortage. Amplification in the demand of 100 can be explained as a substitute product bullwhip as its demand shoots up due to stock outs of the pre-existing substitute denominations of 500 and 1000. Similarly, demand of 500 also shoots up as substitutability of 2000 is lower than that of 1000 .

ATMs usually had 2 to 5 cassettes for holding money, with each cassette calibrated for a specific currency note. New notes had a reduced size, and thus ATMs needed recalibration to be ready to dispense new currency. This resulted in 100 being the only denomination available for withdrawal from the ATMs. The cash carrying capacity of the ATMs was therefore, limited to the cassettes calibrated for carrying 100. ATM manufacturers and third-party cash management firms were faced with an enormous challenge to recalibrate ATMs, test software for the new currency and to simultaneously ensure more frequent refilling owing to lowered capacity ${ }^{19}$. This further amplified the severe shortage of 100 -rupee note, which resulted in banks recalling soiled notes for circulation again ${ }^{20}$. The resulting hoarding of this denomination led to RBI issuing a statement to urge people not to hoard smaller denominations ${ }^{21}$.

Restoring the normalcy of economic transactions was a key concern for the government and the central bank, as highlighted in the announcements by the policy makers focusing on the time duration for return to normalcy. The Economic Survey of India, 2016-17 also indicated speed or rate of remonetisation as the key operational objective ${ }^{22}$. In his statement announcing the demonetisation move, Prime Minister assured that the situation would improve in 50 days ${ }^{23}$. Due to this focus, public expectations were built on the promise of a quick remonetisation and thus we take time to return to normalcy as the benchmark for checking the effectiveness of remonetisation efforts.

## 5 Remonetisation Process

We take a supply chain perspective to understand the remonetisation process which involves printing and distribution of the currency in chosen denominations to achieve the desired output; normalcy of economic transactions. This normalcy has not been defined in the existing literature. We define normalcy of economic transactions as the state when we reach the pre-demonetisation levels for the percentage of cash transactions done efficiently in the economy. Efficient transaction is defined

[^5]in the literature as the one requiring a minimum number of notes for the existing denomination structure. Even when there is no constraint on currency availability, only a fraction of transactions are done in efficient manner, as availability of all denomination, with all people, at all time is not guaranteed. Content of the wallet acts as a capacity constraint for an individual user who may not be able to determine efficient payment option or may not act rationally. While no study is available in the Indian context; an empirical study by Franses and Kippers (2007) in the US has shown that under normal availability of currency, around $60 \%$ of the transactions are done optimally owing to the constraints on denomination wise availability of currency.

Demonetisation changes availability of various denominations as newly introduced denominations are in short supply and demonetised denominations are no longer considered a legal tender. Thus, the proportion of efficient transactions comes down and remonetisation efforts are directed towards restoring it to pre-demonetisation levels. Effectiveness of a process is defined in terms of the desired output, and therefore, effectiveness of the remonetisation process is a function of the speedy restoration of the proportion of efficient transactions to the pre-demonetisation levels. To achieve this, measures can be taken to increase the availability of the value demonetised and/or to decrease the overall requirement of currency notes which is shown to be dependent on the choice of denomination structure.

Efficiency measures look at reducing the resource usage to achieve high productivity. Remonetisation involves operational tasks at each of the three stages of supply chain of currency pertaining to efficient printing and disbursal of the new currency to ensure ease of transactions to the end user. It is known in the supply chain literature (Meindl and Chopra (2001)) that optimisation at some stages of a supply chain by improving the process efficiencies may lead to sub-optimal service to the end customer. In case of the currency supply chain, it implies that maximising printing or disbursal efficiency may not result in optimal availability of currency for transaction with the end users. Additionally, trade-offs are involved in the impact of measures taken for productivity improvement as decision taken for improving efficiency at one stage may adversely affect efficiency at another stage.

After demonetisation, central banks make several decisions for the currency transition. The choice of the denomination structure and the new denominations and prioritisation amongst these new denominations are the key decisions which impact efficiency and effectiveness of the remonetisation process. Change in denomination structure leads to a change in step size and hence substitutability of the denominations changes. Prioritisation amongst the newly added denomination creates a temporary denomination structure as non-prioritised denominations are not available. Denominations of different values impact printing, disbursal and transactions differently. We discuss these operational measures in detail in the following section.

### 5.1 Operational Performance Measures for the Remonetisation Process

We identify tasks undertaken at each stage of currency supply chain to identify performance measures for the remonetisation process. We define efficiency measures for the key tasks at each of the stages of the currency supply chain. At the first stage, production is the key task undertaken by the central bank and therefore measures are taken to achieve higher printing efficiency. As currency moves to the second stage with banks and ATMs, which perform the distribution activity, focus is to increase the disbursal efficiency to achieve high productivity. Currency is used for transactions by the end user who forms the last stage of the currency supply chain. An efficient transaction involves usage of minimum number of notes for the given denomination structure and thus transaction efficiency is the performance measure for the end user.

Printing efficiency can be measured in many different ways. It can be measured either as the total value or the number of notes produced per unit resource usage, with time and raw material like paper and ink and printing press as the key resources required for printing. In the Indian context, as
discussed in section 4, the policy makers' focus was on the value replenished and therefore printing efficiency was considered in terms of the value rather than the number of notes. Printing volumes were determined by the capacities of the printing presses which were run in three shifts on a war footing. Choice of a higher denomination of 2000 and its prioritisation achieved higher printing efficiency as it needed only half as many as 1000s and one fourth as many as 500 s to be printed for the same value. A sudden spurt in production volumes also put supply pressures on the raw materials specific grades of ink and paper are required. Reducing the size of the new currency notes improved the printing efficiency by reducing the raw material requirement.

Disbursal efficiency could be measured either as the number of people or the value disbursed per unit time per unit resource usage. The resources in case of distribution are working hours of banks, functional ATMs and manpower for undertaking distribution activity including refilling of ATMs and transactions at banks. Prioritisation of 2000 suggests a preference of value disbursed and not of the number of people the value is disbursed to. This prioritisation could have had a positive effect on the disbursal efficiency as more value was disbursed with lesser number of notes. This would be true if currency availability was not a constraint and the bottleneck was the process of withdrawal of cash through the cashier or ATMs. However, in the case of cash shortage, the bottleneck shifts from the dispensing activity to the availability of cash. Choice of 500 over 2000 would have provided better overall circulation as the same amount of money could be distributed and exchanged more. A value of 2000 can be with a single individual only in a note of denomination 2000 while it can be distributed amongst four people with the denomination of 500 .

Further, a move towards a currency note of smaller size improved printing efficiency owing to lesser raw material usage, but was counterproductive for disbursal efficiency, as ATMs needed recalibration due to the changed size of notes. With 100 being the only available denomination for disbursal, cash carrying capacity of the ATMs was lowered. Disbursal efficiency was also impacted by the uneven distribution of bank branches and ATMs across the country as it reduced the number of people who could be served and the cash availability varied across geographies.

Transactions and wealth accumulation are the two uses of currency as it moves to the third stage with the end user. Transactions of same value can be carried out using different number of notes. Principle of least effort establishes that people try to transact in minimum number of notes possible while several constraints limit such optimal transactions. Transaction efficiency is defined in terms of the number of transactions done per unit resource usage with the resource being currency notes. We calculate transaction efficiency as the reciprocal of average number of notes required per transaction. It is a function of denomination structure as well as denomination wise availability of currency. To illustrate the first point, take a transaction of 900. A denomination structure with 100 and 2000 as the available denominations would require nine notes of 100 to make an optimal transaction, whereas, a denomination structure with 100,500 and 1000 requires only two notes for the optimal choice, with one note of 1000 being paid and 100 being returned. Even a suboptimal transaction using one note of 500 and four notes of 100 (five notes in total) is more efficient than the optimal choice with the earlier denomination structure.

Additionally, a denomination structure that is less efficient for one transaction slab may be more efficient for another transaction slab. Consider a transaction of value 4100 and the same two denomination structures as considered before. This transaction can be done optimally using two notes of 2000 and one note of 100 (total of three notes) while a denomination structure with 100,500 and 1000 requires at least five notes for the same transaction. Transaction efficiency measure, while optimising number of notes required for a particular slab, ignores this slab wise difference. Therefore to evaluate the ease of doing a transaction after demonetisation, an aggregate measure of transaction efficiency is needed that is not restricted to a single transaction but considers the entire range of transactions.

We define an overall measure of transaction efficiency which minimises the requirement of currency notes across transaction slabs for a given distribution of transactions in the economy. We compare overall transaction efficiency of denomination structures using weighted average requirement of notes
for all transactions $W$. It is calculated by multiplying notes required for optimal transaction for a transaction slab with the proportion of transactions falling in the slab. Mathematically, if $I$ is the set of denominations and $J$ is the set of transaction slabs, weighted average requirement of notes $W$ is

$$
W=\sum_{\forall J}\left(\sum_{\forall I} y_{i j}\right) * p_{j}
$$

$y_{i j}$ is the number of notes of $i^{t h}$ denomination used in the $j^{t h}$ transaction slab and $p_{j}$ is the proportion of transactions in the $j^{\text {th }}$ transaction slab. Overall transaction efficiency or aggregate efficiency for a denomination structure is calculated by minimising the average requirement of notes $W$. This proposed measure incorporates slab wise differences, as a denomination structure may result in a highly efficient transaction for one slab and an inefficient one for another slab. Thus, it provides an extension to the existing work on the principle of efficient payment.

### 5.2 Managing the Currency Transition

It has been seen that choice of denomination structure and prioritisation for launch has an impact on the printing, disbursal and transaction efficiency and thus on the overall effectiveness of the remonetisation process. It also results in volatility in currency demand due to the transitions in currency structure and subsequent hoarding. This hoarding can be attributed to wealth accumulation and can also be a reaction to perceived shortage. We evaluate performance of the alternate denomination structures to find out whether the prioritisation of 2000 was justifiable.

Overall transaction efficiency is calculated to evaluate the performance of the denomination structures. Higher the weighted average number of notes; lower is the overall transaction efficiency. Effectiveness of remonetisation targets restoration of normalcy of economic transactions. It results in a push for more value and thus a prioritisation of higher denominations is seen. The new normal achieved in this process may be inefficient; owing to inferior optimal transactions of the chosen denomination structure. We analyse that if the prioritisation of 2000 is changed to 500 , will the overall transaction efficiency of the supply chain be better.

Change in demand of a denomination during a currency transition is not the same for all denominations. Susceptibility of a denomination to such disruption risks is dependent on the presence of other substitute denominations. A denomination can be said to be robust if its demand is not impacted by currency transitions and thus robustness of a denomination is inversely related to the change in demand experienced by a denomination during a currency transition. We evaluate change in denomination specific demand to find out the impact of prioritisation on the robustness of the existing denominations.

The impact of prioritisation of denominations after demonetisation can be modelled by using denomination structure choices as scenarios. Each scenario represents the alternate choice available with the policy makers. Prioritisation results in a temporary denomination structure. In the transient stage while currency is reintroduced, the prioritised denomination is made available first, with no availability of the non-prioritised denomination. We represent different types of transactions in the economy for various consumer segments and product categories through statistical distributions with different means. Our choice of statistical distributions is based on an empirical study and also includes inputs from the literature. We build our hypotheses to test the impact of various decision choices available with the policy makers on the overall transaction efficiency.

As discussed in section 4, after demonetising the existing currency notes of 500 and 1000 , it was decided to relaunch a new series of 500 and introduce 2000 as a new denomination. It was also decided to prioritise 2000 for introduction initially. This resulted in 100 and 2000 being the available high value denominations after demonetisation. We call denominations with value 100 and above as highvalue to differentiate the denominations that were directly impacted by demonetisation and these
denominations are the focus of this work. We anticipate that the presence of 100 and 500 as high value denominations is better suited as compared to 100 and 2000 for carrying transactions in the economy. Due to higher step size, the average number of notes required for most of the transactions is higher with 100 and 2000, barring the transactions that are close to an integral multiple of 2000 . As overall transaction efficiency is the reciprocal of this average weighted requirement of notes, therefore we hypothesise that

## Hypothesis 1 Prioritisation of 2000 over 500 decreases the overall transaction efficiency

As 2000 was prioritised, the step size between 100 and next denomination changed from the predemonetisation value of five to twenty. In the absence of a substitute denomination till 2000 , requirement of 100 increases as it compensates for the absence of the substitute denomination of 500. Demand for 100 is further increased due to the initial shortage of 2000 as transactions are done sub optimally even for the available denominations. We hypothesise that the demand of 100 increases and this increase would have been lower if 500 is prioritised.

Hypothesis 2 Demand for 100 increases due to prioritisation of 2000 over 500

The alternate option available with the same choice of denominations was to prioritise 500 over 2000. Demand for a denomination is dependent on the ratio with the next higher denomination, the step size. The next denomination is a more efficient substitute for a transaction with value higher than the value of the given denomination. Therefore, demand for 100 should remain unchanged if 500 is introduced first and is available. Even during the transient phase while 500 is not available fully, the impact on the 100 -rupee demand would be less as compared to that with prioritisation of 2000 . For any denomination choice with 100 and 500 as consecutive denominations, the demand for 100 should be same owing to same step size.

Hypothesis 3 Demand for 100 remains unchanged if 500 is available as the next higher denomination

If denomination 500 is introduced first, we anticipate that the demand for 100 remains unaffected, but there should be an impact on the demand of 500 compared to the pre-demonetisation levels, as there is no higher denomination for substitution available initially, while pre-demonetisation case had 1000 as the next higher denomination. Even when 2000 becomes available, step size for 500 increases from two to four. However, this increase of demand of 500 is lower than the increase of demand of 100 that happens with prioritisation of 2000 as discussed in hypothesis 2.

Hypothesis 4 Demand for 500 increases if it is introduced first but the increase is less compared to that on the demand of 100 if 2000 is introduced first

Hypothesis 4 predicts that the demand of 500 will go up initially, as it is the highest value denomination available and is therefore used for optimal transactions above 500. When 2000 is introduced first, it can be efficiently substituted for transactions around 2000 and its integral multiples as each note compensates for four notes of 500 and even more for lower value denominations. We anticipate that for such high value transactions, negative impact of high step size with 2000 on transaction efficiency will be offset as it substitutes a large number of low denomination notes. Thus, prioritisation of 2000 will lead to inefficient transactions for lower values but these will become more efficient as we approach higher transaction values of 2000 and more.

Hypothesis 5 Impact of prioritisation of 2000 on the overall transaction efficiency is higher for low-value transactions

In the above five hypotheses, we have evaluated transaction efficiency of various alternate demonetisation structures based on the options available with the policy makers once the new denominations were already decided. Through next hypothesis, we examine if a more efficient denomination structure exists. Existing literature of optimal denomination structure (Van Hove (2001)) suggests binary-decimal triplets of 1:2:5 to be optimal. INR has this structure for low value denominations from 1 till 100 . We calculate the overall transaction efficiency if the same denomination structure is retained for higher value denominations with 100,200 and 500 as the available denominations. We anticipate transaction efficiency to gain from the reduced step size with this structure.

Hypothesis 6 Denomination choice of 100:200:500 has the highest overall transaction efficiency amongst the options considered.

We assume that the triplet of 100,200 and 500 was chosen to be the new denomination structure post demonetisation. Thus, 2000 is replaced with 200 as the new denomination to be introduced. Prioritisation of a denomination was needed to meet operational objectives as in the case of prioritising 2000. We examine the case with 200 prioritised over 500 . Having 200 reduces the step size for 100 from twenty to two, as compared to the choice and prioritisation of 2000 , and thus its demand falls. Thus, overall currency demand should reduce with 200 as compared to the prioritisation of 2000. As plans to introduce 200 as a denomination are being discussed at the $\mathrm{RBI}^{24}$, evaluating it as an option for remonetisation gains further significance.

Hypothesis 7 Choice and prioritisation of 200 over 2000 results in higher transaction efficiency.

### 5.3 Scenarios

To test our hypotheses regarding the impact of available denominations on average currency demand and thus compare the transaction efficiency of denomination structures, we build scenarios representing denomination structure choices. These scenarios model decisions taken by the RBI which include the denominations chosen for demonetisation, the new denominations for remonetisation and the order in which these denominations are introduced back. For instance, having a scenario for denomination structure with only 100 as the available denomination represents the state just after demonetisation with 2000 and 500 unavailable. Each scenario assumes the chosen denominations for a particular scenario choice to be fully available. Transient states exist while a particular denomination is newly introduced and is not fully available. For modelling transient states, different scenarios can be used to represent the initial state and the final states. Scenarios vary in denomination structure for higher value denominations (100 and beyond) only and a similar structure is assumed for lower denominations as these are not affected by demonetisation. We also model a scenario with a hypothetical denomination structure to test hypothesis 6 regarding an efficient denomination structure choice. The scenarios considered are discussed in detail below:

Scenario 1: This represents the pre-demonetisation base case with 100,500 and 1000 as the available denominations and with each denomination sufficiently available.

Scenario 2: This represents the scenario after demonetisation with 500 and 2000 as the planned denominations with priority given to 2000 initially. Thus, 100 and 2000 are the available denominations with 2000 assumed to be fully available

Scenario 3: This scenario considers 500 and 2000 as the planned denominations with priority given to 500 initially. 100 and 500 are the available denominations in this scenario with 500 assumed to be fully available.

[^6]Scenario 4: This scenario considers 500 and 2000 as the planned denominations with no priority assigned between the choices of denominations making all three denominations of 100,500 and 2000 simultaneously available.

Scenario 5: In this scenario, we consider demonetisation without reintroduction of any new currency denomination making 100, the highest available denomination. This scenario helps us to model the situation immediately after the demonetisation exercise as both 2000 and 500 are gradually introduced. We use this scenario to extrapolate gradual the increase of availability of denominations 500 and 2000 to 100 percent as modelled with scenario 3 and 2 respectively.

Scenario 6: Literature on optimal step size suggests usage of binary-decimal triplets of 1-2-5 as optimal denomination structure. In this scenario, we explore this hypothesis by continuing with the same structure beyond 100 also and having 100,200 and 500 as the available denominations.

Scenario 7: This scenario extends the idea discussed in scenario 6. Here we consider the denominations of 100,200 and 500 , and additionally prioritise 200 over 500 for introduction, making 100 and 200 as the available denominations initially.

Table 1 summarises the existing denominations with each of the scenarios discussed above.
Table 1: Scenario wise available denominations

| Denominations | 100 | 200 | 500 | 1000 | 2000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Scenario 1 | $\checkmark$ |  | $\checkmark$ | $\checkmark$ |  |
| Scenario 2 | $\sqrt{ }$ |  |  |  | $\checkmark$ |
| Scenario 3 | $\checkmark$ |  | $\checkmark$ |  |  |
| Scenario 4 | $\sqrt{ }$ |  | $\checkmark$ |  | $\checkmark$ |
| Scenario 5 | $\sqrt{ }$ |  |  |  |  |
| Scenario 6 | $\sqrt{ }$ | $\checkmark$ | $\checkmark$ |  |  |
| Scenario 7 | $\checkmark$ | $\checkmark$ |  |  |  |

## 6 Analysis

We calculate the weighted average requirements of notes $W$ for each of the scenarios discussed. Higher average requirement indicates lower transaction efficiency. A mathematical model is developed to calculate the optimal transaction choice for a given denomination structure across transaction slabs. Transactions upto the value of 10000 are considered, as we envisage that the range of 1 to 10000 will contain the majority of the cash transactions in the economy. Krishnaswamy (2012) report that monthly per capita expenditure in India was INR 1774.83 in $2011^{25}$ and size of an average household, as per 2011 census, was $4.8^{26}$. Thus, the average household expenditure comes around INR 8515. Considering an annual growth rate of 8 percent annually ${ }^{27}$, the average household expenditure for 2017 should be around INR 13500. Even if we consider one transaction per day; average transaction value comes around 450. We expect transaction values to vary across geographies, across cities and rural areas, type of store and product category. Step size of 100 is chosen as a slab with an assumption that the transactions with lower denominations are carried out in a similar manner as lower denominations are unchanged. For instance, to make a payment of 250 efficiently, in each of the scenarios, we would be paying two notes of 100 and one note of 50 .

[^7]Additionally, in the Indian context, lower denominations acting as a substitute for 100 rupee was highly unlikely, owing to high relative share of denomination 100 in circulation. As per RBI study by Nachane et al. (2013), the ratio of currency in circulation for 100 with lower denominations of 50 , 20 and 10 was $14.8: 1.7: 0.6: 2.2$ in 2010-11. Therefore, we assume that a 100 -denomination note would not be substituted with two notes of 50 or ten notes of 10 . If substituted in case of shortage, it has a further detrimental impact on transaction efficiency.

### 6.1 Transaction size and distribution

Franses and Kippers (2007) argue that retail stores are the best holistic representation of the population for collecting transaction related data. The average value of transactions would vary for retail outlets based on the scale of operation and their location, amongst other factors. We collected daily transaction data for a small retail outlet located on a residential college campus stocking goods across categories like grocery, stationary and crockery. The overall average value of transaction was around 110 for this data. Summary statistics have been provided in table 2 . We collected data for weekdays as well as weekends; before and after the demonetisation exercise to account for any fluctuation between days. Additionally, sample daily transaction data from one of the online grocers, collected over the period from May 2016 to November 2016 has mean transaction value at 985 (485 observations). One of the biggest retail chains in India reports the average value of the transaction to be between 1100 and $1400^{28}$. These mean values are a representation of different consumer segments and product categories and hence we check impact of demonetisation on these segments by using a set of distributions mean values.

Table 2: Summary table of the number of daily transactions across slabs

| Transaction | Date wise observations |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Slabs | 6th November | 7th November | 13th November | 14th November |
| $1-100$ | 135 | 133 | 88 | 74 |
| $101-200$ | 53 | 28 | 25 | 32 |
| $201-300$ | 19 | 11 | 8 | 18 |
| $301-400$ | 9 | 3 | 1 | 7 |
| $401-500$ | 1 | 4 | 5 | 2 |
| $501-600$ | 2 | 2 | 0 | 2 |
| 601-700 | 0 | 1 | 1 | 0 |
| $\geq 700$ | 2 | 0 | 0 | 3 |
| Average | 116.53 | 91.78 | 95.91 | 134.91 |
| Standard Deviation | 170.76 | 109.97 | 105.46 | 159.48 |
| Minimum | 1 | 2 | 2 | 4 |
| Maximum | 1995 | 650 | 603 | 1108 |
| Range | 1994 | 648 | 601 | 1104 |
| Mode | 10 | 10 | 10 | 10 |

Very few papers have discussed probability distribution for the transactions in the economy. In the absence of any details about transaction patterns, some of the earlier works like Van Hove and Heyndels (1996), Telser (1995) assumed each transaction value within the range to be equally likely and used Uniform distribution. Boeschoten (2012) state using budget surveys in Netherlands that the distribution of transactions in an economy is Lognormal. Nachane et al. (2013) suggest using Lognormal distribution in absence of data. Our empirical study on the daily transactions data at the campus grocery store provides support for using Exponential distribution, Weibull distri-

[^8]bution, and the Lognormal distribution. Sample data from the online grocery store also provides support for using Exponential and Weibull distribution. We use Uniform distribution, Lognormal distribution (100,125), Weibull distribution (200,6), Exponential (100), Exponential (500) and, Exponential (1000) for our analysis. Additionally, Exponential distribution with higher mean values are considered to analyse the impact of demonetisation on different values of transaction and test our hypothesis about the range of transactions most impacted by demonetisation and the choice of denominations. We calculate transaction slab wise weights for calculating the weighted average requirement of notes $W$ from the cumulative probability distribution (cdf) of these underlying transaction distribution.

### 6.2 Model

The integer program minimises the weighted average requirement of notes $W$. This results in the choice of optimal transactions for each slab for a particular denomination structure. The formulation discussed below represents a particular scenario; the pre-demonetisation case with 100,500 and 1000 as the available high value denominations (Scenario 1). Taking a specific scenario in the formulation allows us to make the constraint clearer as it is dependent on the number of denominations in a scenario. We discuss an alternate generalised formulation for this model later.

## Sets:

$I: \quad$ set of denomination choices, indexed by $i ; i=1$ for $100, i=2$ for $500, i=3$ for 1000
$J: \quad$ set of transaction slabs, indexed by $j ; j=1$ for $1-100, j=2$ for $101-200, \ldots, j=100$ for 9901-10000
$K$ : set of denomination choices available for a transaction slab, indexed by $k ; K=1, \ldots, k=$ $2^{n}-1$, where $n$ is number of available denomination choices, $n=3$

## Decision Variables:

$y_{i j}: \quad$ number of notes of $i^{t h}$ denomination required in $j^{\text {th }}$ transaction slab; 300 variables; $y_{11}$, $y_{12}, . ., y_{1100}, y_{21}, . ., y_{2100}, y_{31}, \ldots, y_{3100}$
$x_{k}$ : binary variable for choice of denominations for a payment in a transaction slab; $x_{1}, x_{2}, \ldots, x_{7}$

## Parameters:

$p_{j}$ : weight for the $j^{\text {th }}$ transaction slab
$d_{i}$ : value of the $i^{t h}$ denomination $\left(d_{1}=100, d_{2}=500, d_{3}=1000\right)$
$T_{j}$ : Upper limit of the $j^{t h}$ transaction slab ( for $j=1, T_{j}=100 ; j=2, T_{j}=200, .$. )

## Objective:

$$
\operatorname{Minimize} \sum_{j=1}^{100}\left(\sum_{i=1}^{3} y_{i j}\right) p_{j}
$$

## Constraints:

$$
\begin{aligned}
& \text { 1. } T_{j-1} \leq \\
& \quad\left(x_{1}\right)\left(y_{1 j} d_{1}+y_{2 j} d_{2}+y_{3 j} d_{3}\right)+\left(x_{2}\right)\left(-y_{1 j} d_{1}+y_{2 j} d_{2}+y_{3 j} d_{3}\right)+ \\
& \quad\left(x_{3}\right)\left(y_{1 j} d_{1}-y_{2 j} d_{2}+y_{3 j} d_{3}\right)+\left(x_{4}\right)\left(-y_{1 j} d_{1}-y_{2 j} d_{2}+y_{3 j} d_{3}\right)+ \\
& \quad\left(x_{5}\right)\left(y_{1 j} d_{1}+y_{2 j} d_{2}-y_{3 j} d_{3}\right)+\left(x_{6}\right)\left(-y_{1 j} d_{1}+y_{2 j} d_{2}-y_{3 j} d_{3}\right)+ \\
& \quad \quad\left(x_{7}\right)\left(y_{1 j} d_{1}-y_{2 j} d_{2}-y_{3 j} d_{3}\right) \\
& \quad \leq T_{j} \quad \\
& \text { 2. } x_{1}+x_{2}+x_{3}+x_{4}+x_{5}+x_{6}+x_{7}=1 \\
& \text { 3. } y_{i j} \geq 0 \quad \forall i, j \text { (Integer variable) }
\end{aligned}
$$

In the above formulation, the objective function minimises the average requirement of notes over the entire range of transactions. As transactions may not be uniformly distributed over the entire range,
a weighted average is taken with $p_{j}$ as the weight for calculating the average. $p_{j}$ is the proportion of transactions in the $j^{t h}$ transaction slab. Constraint (1) ensures that the total payment is between the upper and the lower bounds of the transaction range. This constraint incorporates various payment choices available as the payment can be made directly by paying the desired sum or can also be made by overpayment and exchange of extra cash as the return. Using the binary variables $x_{k}$, we evaluate all possibilities of efficient payment, including the ones involving overpayment. Constraint (2) puts the sum of these binary variables as 1 to ensure that the model evaluates only one option at a time, through Constraint (2). Constraint (3) is the non-negativity constraint on the number of notes. Using this model, the weighted average requirement of notes $W$ can be calculated for each of the scenarios for transaction values ranging from 1 to 10000 in slabs of 100 . This formulation, however, results in evaluation of a large number of overpayment choices that increase exponentially with the number of denominations in the denomination structure.

A more generalised formulation for this problem has a non-linear objective function with an absolute value of the variable $y_{i j}$ being used. This allows us to do away with separate modelling of overpayment choices, as done in constraint 1. $y_{i j}$ now not restricted to non-negative values and thus return of cash is modelled as a negative value of $y_{i j}$. The sets and parameters are the same as the first model and additional binary variable $x_{k}$ isn't required in this case as each overpayment choice is not tested separately.

## Objective:

$$
\operatorname{Minimize} \sum_{\forall J}\left(\sum_{\forall I}\left|y_{i j}\right|\right) p_{j}
$$

## Constraints:

$$
T_{j-1} \leq \sum_{\forall I} y_{i j} d_{i} \leq T_{j} \quad \forall j
$$

### 6.3 Procedure

Optimisation model provided in the last sub-section is not used to solve the problem of finding the overall transaction efficiency due to an interesting property of optimal transactions when transaction slabs are arranged in an increasing manner. Cramer (1983) highlighted that a denomination's frequency in an efficient transaction follows a pattern, period of which is determined by the step size. Identifying this pattern allows us to generate a spreadsheet with the optimal transactions by finding the period of the pattern and repeating the sequence. Table 3 enlists the number of notes required for optimal transactions in Scenario 1, which represents the pre-demonetisation case. Transactions slabs till the upper limit 3000 are represented in the Table 3, which is extended till 10000 for our calculations. We choose optimal transactions and wherever more than one optimal choice exists, all such options are considered and are represented as options. For instance, in scenario 1 , a transaction between 700 and 800 could be done using a note of denomination 500 and two notes of denomination 100 along with lower denomination change or by using one note of denomination 1000 and getting in return two notes of denomination 100 along with lower denomination change. The two choices result in usage of three notes each and are optimal choices represented as options in Table 3. The same transaction can be done sub-optimally using seven notes of denomination 100.

For generating the pattern discussed above, we arrange transaction slabs sequentially in the order of increasing values as in Table 3. We find that the transaction frequency for the smallest denomination (100 in our case) follows a sequence, length of which is determined by the step size for the denomination, which is 5 . Sequence followed by 100-rupee usage for optimal choices is $1,2,1,0,0$ as seen in serial number 2 to 6 of Table 3 and this sequence is repeated. Similarly, usage of 500 alternates between zero and one with a period of five each (serial number 9 to 18). The pattern repeats for all denominations except for the largest one, where the number of notes used is increased by one at
every period. For the same transaction slabs from serial number 9 to 18 , usage of 1000 , the highest denomination increases by one after each period.

This periodicity can also be explained using the concept of substitute product. We know that the next higher denomination acts as a substitute for a denomination. As the size of transaction increases, usage of a denomination increases till we reach a point where usage of higher valued substitute denomination is better for transaction efficiency. Once replaced for a higher denomination substitute, the pattern for the denomination is again repeated as we move to next transaction slabs. Usage of highest denomination increases in a sequence due to the absence of a higher substitute denomination. We use this logic to complete the table with optimal payment choices, using this pattern identification approach till the transaction slab with 9901-10000 and repeat the procedure for other scenarios also.

To evaluate our hypothesis regarding the changed demand of 100 , we also calculate the usage of denomination 100 in the optimal choice for each transaction slab. In cases where there is more than one way to do the transaction optimally, we assume each optimal option to be equally likely. If such choices differ in the number of 100-rupee notes used, an average of the number of 100 rupee notes used in each choice is taken. However, in the presence of hoarding behaviour, higher weight can be put to the option using lesser number of the hoarded denomination. To test hypothesis 4 , we also calculate usage of denomination 500 for the scenarios where it is an available denomination.

### 6.4 Computational Results

Table 4 provides a summary of the weighted average number of notes required per transaction for each of the scenarios. We observe that the demand for the average number of notes required per transaction increases from Scenario 1 to Scenario 2, suggesting that transactions become less efficient post demonetisation, even when 2000 is assumed to be easily available. For Scenario 5, which represents the case just after demonetisation, increase from Scenario 1 is much higher for all distribution choices except Exponential (100) case, which explains the sudden hike in demand experienced just after demonetisation. Scenario 4, which represents complete remonetisation with both 500 and 2000 available, has higher average requirement as compared to Scenario 1 for most of the distribution choices except for the Uniform distribution and Exponential (1000), for which the requirement is almost similar. Thus, the pre-demonetisation denomination structure was a more efficient choice as compared to the post remonetisation one.

The increase in demand of notes between Scenario 1 and Scenario 2 varies with the choice of distribution. Exponential (100) represents the case when the majority of transactions are of value 100 and lesser and these transactions are not impacted by demonetisation directly. Uniform distribution consider all values of transaction till 10000 to be equally likely, which is implausible as the number of high-value transactions would always be lower than the number of low-value transactions. Scenario 3 , the case of 500-rupee note getting introduced first, fares better than Scenario 2 for all distribution choices except for the Uniform distribution. Thus, prioritisation of 500 is a more efficient choice and Hypothesis 1 which suggests that prioritisation of 2000 over 500 reduced the overall transaction efficiency is supported for all except one distribution choices.

In Table 5, we provide the weighted average 100-rupee demand across scenarios. Demand for denomination 100 increases after demonetisation. Comparing the average requirement of denomination 100 between Scenario 2 and 3, we find that Hypothesis 2 is supported for all distribution choices as presence of a higher denomination of 500 restricts usage of 100. For Exponential (1000), the increase in demand of 100 from Scenario 3 to Scenario 2 is 4.76 times. This increase completely offsets the gains in printing efficiency achieved through prioritisation of 2000 over 500 as more 100 rupee notes are printed. By printing 2000 instead of 500 , four times less number of notes were required. The demand of 100 was increased by upto 4.76 times and therefore printing of additional notes of 100 was required. This nullified the gains made in printing efficiency, while also making

Table 3: Slab wise Optimal Transactions

| Scenario 1: Pre-demonetisation Base Case |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S.No. | Transaction Value | Number of notes required |  |  |  |  |  |  |  |
|  |  | Option 1 |  |  | Option 2 |  |  | All |  |
|  |  | 100 | 500 | 1000 | 100 | 500 | 1000 | denominations | 100s |
| 1 | 1-100 | 0 | 0 | 0 |  |  |  | 0 | 0 |
| 2 | 100-200 | 1 | 0 | 0 |  |  |  | 1 | 1 |
| 3 | 200-300 | 2 | 0 | 0 |  |  |  | 2 | 2 |
| 4 | 300-400 | 1 | 1 | 0 |  |  |  | 2 | 1 |
| 5 | 400-500 | 0 | 1 | 0 |  |  |  | 1 | 0 |
| 6 | 500-600 | 0 | 1 | 0 |  |  |  | 1 | 0 |
| 7 | 600-700 | 1 | 1 | 0 |  |  |  | 2 | 1 |
| 8 | 700-800 | 2 | 1 | 0 | 2 | 0 | 1 | 3 | 2 |
| 9 | 800-900 | 1 | 0 | 1 |  |  |  | 2 | 1 |
| 10 | 900-1000 | 0 | 0 | 1 |  |  |  | 1 | 0 |
| 11 | 1000-1100 | 0 | 0 | 1 |  |  |  | 1 | 0 |
| 12 | 1100-1200 | 1 | 0 | 1 |  |  |  | 2 | 1 |
| 13 | 1200-1300 | 2 | 0 | 1 |  |  |  | 3 | 2 |
| 14 | 1300-1400 | 1 | 1 | 1 |  |  |  | 3 | 1 |
| 15 | 1400-1500 | 0 | 1 | 1 |  |  |  | 2 | 0 |
| 16 | 1500-1600 | 0 | 1 | 1 |  |  |  | 2 | 0 |
| 17 | 1600-1700 | 1 | 1 | 1 |  |  |  | 3 | 1 |
| 18 | 1700-1800 | 2 | 1 | 1 | 2 | 0 | 2 | 4 | 2 |
| 19 | 1800-1900 | 1 | 0 | 2 |  |  |  | 3 | 1 |
| 20 | 1900-2000 | 0 | 0 | 2 |  |  |  | 2 | 0 |
| 21 | 2000-2100 | 0 | 0 | 2 |  |  |  | 2 | 0 |
| 22 | 2100-2200 | 1 | 0 | 2 |  |  |  | 3 | 1 |
| 23 | 2200-2300 | 2 | 0 | 2 |  |  |  | 4 | 2 |
| 24 | 2300-2400 | 1 | 1 | 2 |  |  |  | 4 | 1 |
| 25 | 2400-2500 | 0 | 1 | 2 |  |  |  | 3 | 0 |
| 26 | 2500-2600 | 0 | 1 | 2 |  |  |  | 3 | 0 |
| 27 | 2600-2700 | 1 | 1 | 2 | 2 | 0 | 3 | 4 | 1 |
| 28 | 2700-2800 | 2 | 1 | 2 |  |  |  | 2 | 2 |
| 29 | 2800-2900 | 1 | 0 | 3 |  |  |  | 4 | 1 |
| 30 | 2900-3000 | 0 | 0 | 3 |  |  |  | 3 | 0 |

transactions inefficient simultaneously and hence resulting in poorer service to the end user. Demand for denomination of 100 is influenced by the availability of the next denomination (substitute denomination). For scenarios with 500 as the next available denomination, which are Scenarios 1,3 and 4, the demand for 100 remains same. This supports Hypothesis 3.

Table 6 captures the average requirement of denomination 500 for the scenarios where it is an available denomination. Demand of denomination 500 increases if it is prioritised to be introduced first (Scenario 3) or both 500 and 2000 are introduced simultaneously (Scenario 4), as compared to the base case (Scenario 1). Demonetising 1000 impacts the demand of 500 as 1000 is a substitute denomination with a lower step size of two as compared to 2000 with step size of four. This increase in demand of 500 from Scenario 1 to Scenario 3 however is less compared to the increase in the requirement of 100 from Scenario 1 to Scenario 2 except for the Uniform distribution case. Thus, Hypothesis 4 is supported for all other distribution choices. For Exponential (500), the increase in demand of 500 from scenario 1 to scenario 3 is 2.28 times, while the increase of demand of 100 from Scenario 1 to Scenario 2 is around 3.65 times. Comparing Scenario 3 with 2 for the total requirement, Scenario 3 with 500 performs better for all distributions except for the Uniform distribution. Scenario 2 has an additional demand for the denomination of 2000, apart from the demand of 100 and thus

Table 4: Weighted average requirement of notes per transaction

| Scenario <br> Number | Available <br> Denominations | Uniform | Lognormal <br> $(100,125)$ | Expo <br> $(100)$ | Expo <br> $(500)$ | Expo <br> $(1000)$ | Weibull <br> $(200,6)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $100,500,1000$ | 6.00 | 1.65 | 1.07 | 1.59 | 2.06 | 1.65 |
| 2 | 100,2000 | 7.01 | 3.20 | 1.21 | 3.64 | 4.65 | 4.71 |
| 3 | 100,500 | 10.71 | 1.97 | 1.12 | 1.83 | 2.77 | 1.82 |
| 4 | $100,500,2000$ | 4.11 | 1.66 | 1.12 | 1.69 | 2.05 | 1.81 |
| 5 | 100 | 49.50 | 5.14 | 0.58 | 4.52 | 9.50 | 4.82 |
| 6 | $100,200,500$ | 10.51 | 1.78 | 1.03 | 1.63 | 2.57 | 1.61 |
| 7 | 100,200 | 25.01 | 3.01 | 1.06 | 2.66 | 5.08 | 2.69 |

Table 5: Weighted average requirement of 100 rupee notes per transaction

| Scenario <br> Number | Available <br> Denominations | Uniform | Lognormal <br> $(100,125)$ | Expo <br> $(100)$ | Expo <br> $(500)$ | Expo <br> $(1000)$ | Weibull <br> $(200,6)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $100,500,1000$ | 0.81 | 0.99 | 1.07 | 0.96 | 0.89 | 0.87 |
| 2 | 100,2000 | 4.54 | 3.03 | 1.21 | 3.51 | 4.24 | 4.66 |
| 3 | 100,500 | 0.81 | 0.99 | 1.07 | 0.96 | 0.89 | 0.87 |
| 4 | $100,500,2000$ | 0.81 | 0.99 | 1.07 | 0.96 | 0.89 | 0.87 |
| 5 | 100 | 49.50 | 5.14 | 0.58 | 4.52 | 9.50 | 4.82 |
| 6 | $100,200,500$ | 0.21 | 0.41 | 0.77 | 0.38 | 0.29 | 0.23 |
| 7 | 100,200 | 0.26 | 0.44 | 0.77 | 0.41 | 0.33 | 0.28 |

Table 6: Weighted average requirement of 500 rupee notes per transaction

| Scenario <br> Number | Available <br> Denominations | Uniform | Lognormal <br> $(100,125)$ | Expo <br> $(100)$ | Expo <br> $(500)$ | Expo <br> $(1000)$ | Weibull <br> $(200,6)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $100,500,1000$ | 0.44 | 0.32 | 0.05 | 0.38 | 0.42 | 0.58 |
| 3 | 100,500 | 9.90 | 0.98 | 0.05 | 0.87 | 1.88 | 0.96 |
| 4 | $100,500,2000$ | 0.92 | 0.52 | 0.05 | 0.65 | 0.83 | 0.93 |
| 6 | $100,200,500$ | 9.80 | 0.91 | 0.03 | 0.79 | 1.79 | 0.85 |

the overall requirement is much higher. Having both 500 and 2000 , reduces the requirement of 500 significantly for distributions with higher mean value, as 2000 is available as a substitute for higher value transactions.

We also try to evaluate the range of transaction values for which the overall transaction efficiency shows maximum change. Figure 2 compares the weighted average requirement of notes for Scenario 2 and 3. We find that having the denomination of 2000 fares better for transactions with mean value of 2000 and above as discussed in Hypothesis 5. This range indicates the segments of the economy where transactions become more efficient post demonetisation and the product in this price range remain relatively unaffected. However, Hypothesis 5 is only partially supported as average requirement of notes does not decrease monotonically with distribution mean values as expected, but follows an inverted U shaped function. We find that transactions with mean between 500 and 1000 are impacted the most, while transaction below 100 also show no significant impact of prioritisation. We see in Figure 2 that for Exponential (100) case, the difference between the two scenario is only of


Figure 2: Ratio of average number of notes required between scenario 2 and 3

8 percent which increases upto a hundred percent change for Exponential(500). The reason for low value transactions showing less change is currency requirement is due to the fact that transactions below 100 can be carried out same as before demonetisation. These transactions are not affected by the change in denomination structure but can only be inefficient due unavailability of 100 because of the perceived shortage.

Hypothesis 6 evaluates the overall transaction efficiency for the denomination structure choice of 100,200 and 500 . It is seen to be the best-suited denomination structure only if the mean value of transactions is around 100 or lower. Scenario 1 outperforms Scenario 6 on overall transaction efficiency for all distribution choices except for Exponential (100). However, Table 5 also shows that Scenario 6 reduces the 100 -rupee requirement significantly as compared to the earlier scenarios. If we compare Scenario 6 as a post demonetisation choice as compared to Scenario 4, both choices are without 1000 as a denomination. Scenario 6 has lower average requirement of currency and thus, Hypothesis 6 is supported for distribution choices with mean values upto 500 . Scenario 6 is therefore a more viable post demonetisation option. 200 should be looked at as an additional denomination and not as a replacement for 1000. Hypothesis 7 extends the idea of Scenario 6 as the post-demonetisation choice and compares prioritisation of 2000 (Scenario 2) to prioritisation of 200 (Scenario 7). It is supported for most of the distribution choices as Scenario 7 has lower average requirement of notes except for Uniform distribution and Exponential (1000). As more and more transactions move away from cash to digital modes of payments, we can expect the mean value for cash transactions to come down and therefore utility of having 200 as a denomination would further increase.

### 6.5 Sensitivity Analysis

Volatility in demand can lead to both an amplification as well as a contraction of demand. As seen in Table 5, denomination 100 experiences a demand amplification from Scenario 1 to Scenario 2 , as demand increases for all distribution choices, highest being a 5.35 times increase. Similarly a demand contraction is seen for denomination 100 in a transition from Scenario 1 to Scenario 6. Here, demand of 100 decreases for all distribution choices, highest being a 3.86 times decrease. A demand amplification in case of currency supply chain may lead to a crisis as currency is hoarded, as explained through the Bullwhip Effect. A contraction in currency demand on the other hand doesn't
have similar impact on the supply chain as excess currency gets deposited back to the banks and to the central bank. Table 7 provides the change of currency requirement for each of the scenario as the percentage change with respect to the base case (Scenario 1 ). This percentage change represents the transition volatility experienced as each scenario is adopted as a remonetisation option. The volatility due to hoarding acts over and above transition volatility and thus overall volatility is even higher than the values indicated in Table 7. A percent change less than hundred reflects a contraction in the overall currency demand.

Table 7: Scenario wise percent change in requirement from base scenario

| Scenario <br> Number | Available <br> Denominations | Uniform | Lognormal <br> $(100,125)$ | Expo <br> $(100)$ | Expo <br> $(500)$ | Expo <br> $(1000)$ | Weibull <br> $(200,6)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $100,500,1000$ | 100 | 100 | 100 | 100 | 100 | 100 |
| 2 | 100,2000 | 117 | 194 | 113 | 229 | 225 | 285 |
| 3 | 100,500 | 179 | 119 | 105 | 115 | 134 | 110 |
| 4 | $100,500,2000$ | 69 | 100 | 105 | 106 | 100 | 110 |
| 5 | 100 | 825 | 312 | 54 | 284 | 461 | 292 |
| 6 | $100,200,500$ | 175 | 108 | 97 | 103 | 125 | 98 |
| 7 | 100,200 | 417 | 182 | 99 | 167 | 247 | 163 |

For Scenario 2, the overall demand of currency notes increases by close to 100 percent or more for all distribution choices except for Uniform distribution and Exponential (100), both of which have only about 10 percent increase between Scenario 1 and 2. Scenario 2 represents the case when 2000 is fully available and therefore to represent the changed currency demand just after demonetisation, we need to analyse the currency requirements in Scenario 5 also. The requirement just after demonetisation, as represented in Scenario 5, increases for all distribution choices except Exponential (100), in values varying from about 284 percent to about 825 percent, depending on the distribution choice. Requirement for higher denominations reduces if cash transactions are restricted to 100 and lower values. The large increase in demand after demonetisation explains the shortage and queues witnessed just after demonetisation. As 2000 is gradually introduced, we find that the requirement reduces a bit, more significantly with distributions with higher means. In Scenario 2, highest increase is in demand of close to 3 times is observed for Weibull(200,6). Exponential (500) as shows an increase of about 130 percent.

The values discussed above indicates that initially after demonetisation all cash transaction above 100 in value were impacted due to increased cash requirements. Later as supply of 2000 started, requirements reduced, though remaining significantly higher than Scenario 1. Requirement for transactions around 500 remained high despite the availability of 2000. This translates into observations about the product category wise impact of demonetisation. For instance, as 2000 is made available, a grocer with most transactions less than 1000 would face more currency shortage as compared to a consumer durable shop selling products in the higher price range. Similarly, hypermarkets which usually have bulk shopping should fare better as compared to stand alone stores for specific products as average transaction value is lower. Rural areas had lower penetration of banks and ATMs in addition to possibly lower average transaction values. Thus, the impact of demonetisation felt in rural areas should be more significant. Lesser impact on higher value transactions could have also led to a tendency of aggregation of purchases to reach values for which currency notes are available, like multiples of 2000 .

Transactions below 100 seem unaffected as the average requirement for these transactions remain same. These transactions were indirectly impacted as requirement for 100 went up to support higher value transactions and was therefore hoarded. Existence of denomination 200 as seen in Scenario 6 and its prioritisation after demonetisation as seen in Scenario 7, reduces the requirement of 100 . Though, overall currency requirement marginally increases with Scenario 6, as compared to Scenario

1 for some of the distribution choices, a decrease in requirement is seen for transactions with mean value close to 500 and lower. This suggests better suitability of the denomination choice of 100,200 and 500 for lower cash valued transactions and as a post demonetisation choice. It should be looked along with 1000 as a long term denomination choice option.


Figure 3: Comparison of average requirement of 100 between Scenario 2 and Scenario 1

We also analyse the denomination specific transition volatility for denomination of 100 , which shows maximum impact and this impact is explained due to changes in substitute denominations. Figure 3 compares the average requirement of 100-rupee note per transaction for exponential distribution with the mean value of transactions at $100,500,1000,1500,2000$ and 2500 . Requirement of 100 is same for denomination structures with 500 as the next denomination following 100 ( Scenario 1, 3 and 4), and therefore we compare average requirement of 100 in these scenarios with that of Scenario 2. We find that the ratio of the requirement of denomination 100 note between Scenario 2 and 1 is around 3.5 for transactions distributed exponentially with mean 500 and increases upto 5.55 for the distributions with the mean at 2500 . The gap between the two is less for transaction lower than 100 as seen with Exponential (100). The transaction ranges with large difference experience higher perceived shortage due to demand amplification as explained by the Bullwhip Effect. The impact is even more during the period when the newly introduced denomination of 2000 is not fully available. In the next section, we discuss the transient states while a denomination is not fully available.

## 7 Transient State Analysis

In the analysis till now, we have considered full availability of selected denominations for each scenario. A newly introduced denomination would be fully available after a span of time due to production and distribution capacity constraints. For the demonetisation instance discussed in this paper, a time span of 6 months was anticipated to restore adequate currency availability ${ }^{29}$. Government estimates suggested that by around 3 months after demonetisation, 7500 billion out of the 15440 billion notes had been replaced, still less than 50 percent of the value demonetised ${ }^{30}$. Pre-demonetisation levels of cash availability, as per the RBI data ${ }^{31}$, was around 17000 billion INR

[^9]at start of November 2017, which went down to about 7800 billion in December 2017. Even by end of March, cash availability was around 12000 billion INR, much lower than the pre-demonetisation levels. However, this lower cash availability may also be attributed to the reduced demand of cash for wealth accumulation.

Average currency requirement per transaction during the transient states can be calculated by extrapolation between the starting and the end states. The starting state here represents the situation just after demonetisation where certain demonetised denominations as well as new denominations had no availability. The new denominations are subsequently made available and the end state represents full availability of these denominations for making efficient transactions. During the transient phase, when the new denomination is not fully available, lesser number of optimal transactions are possible due to the additional constraints on availability. Scenario 5, as discussed earlier, considers 100 to be the only available denomination and therefore represents the starting state after demonetisation, as 100 was the highest available denomination just after demonetisation.

Table 8: Extrapolation of average requirement of notes for transient state calculation

|  | State | Uniform Distribution |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 100 | 500 | 1000 |
| Scenario 3 : 500 Introduced first | Start State (0\%) | 1.58 | 5.52 | 10.50 |
|  | End State (100\%) | 1.07 | 0.96 | 10.50 |
| Scenario 2: 2000 Introduced first | Start State (0\%) | 1.58 | 5.52 | 10.50 |
|  | End State (100\%) | 1.21 | 3.51 | 4.24 |
| Percent availability at which scenario 3 performs | 71.79 | 43.92 | 65.14 |  |
| equivalent to the end state of scenario 2 |  |  |  |  |

In Table 8, we extrapolate the average requirements of notes at transient states by using scenario 5 , which represents the starting state for prioritisation of 500 over 2000 (scenario 3) as well as prioritisation of 2000 over 500 (Scenario 2). Scenario 5 represents no availability ( 0 percent) of 2000 while Scenario 2 represents its full ( 100 percent) availability. Similarly, Scenario 5 also represents no ( 0 percent) availability of 500 while Scenario 3 represents its full ( 100 percent) availability. We extrapolate to evaluate percentage availability of 500 in Scenario 3 that has the same impact on the overall transaction efficiency as that of full availability of 2000 in Scenario 2. If we compare the transient states for Scenario 2 and 3, we find that for most of the distribution choices, even at partial availability of close to half the total requirement, introduction of 500 -rupee note has a similar impact on overall transaction efficiency as with the full availability of 2000 rupee note. The estimates of cash availability in the reports discussed suggests that more than six months would be required for complete replenishment of demonetised currency. Prioritisation of 500, thus would have provided similar transaction efficiencies atleast three months earlier resulting in better convenience for the end user.

## 8 Conclusion

This paper takes a supply chain orientation to study transitions in a currency denomination structure. We study a three stage supply chain comprising of the central bank, banks and ATMs and end users; and treating currency as a product with different denominations as substitutes. This supply chain framework allows us to explain change in demand of currency due to denomination structure change through substitute product demand. Substitutability is a function of step size of a denomination; smaller the step size, higher the substitutability. We also explain hoarding of currency using the well known effect in supply chain literature, the Bullwhip Effect.

We identify measures of operational performance for the remonetisation process based on the key tasks of printing, disbursal and transaction performed at different stages of the supply chain. Effective remonetisation focuses on the desired goal of achieving normalcy of economic transactions through availability of adequate value of currency to meet the transaction requirement. We define normalcy as achieving the pre-demonetisation levels of percentage of efficient transactions in the economy, where efficient transactions are the ones requiring minimum number of notes for the denomination structure. We identify the trade-offs involved in various stage-wise efficiency measures. Choosing currency notes of smaller size improved printing efficiency but was counterproductive for disbursal as recalibration of ATMs was needed for new notes. Similarly, choice of high value denomination increased printing efficiency but reduced disbursal efficiency and transaction efficiency thus resulting in an inefficient outcome for the end user.

Denomination structure choice impacts the overall currency requirement, as absence of certain denominations lead to inefficient transactions. The prioritisation of denominations during remonetisation process thus becomes an important policy decision for the central bank. We introduce the concept of overall transaction efficiency, compare alternate denomination structures and highlight adverse impact of prioritisation of 2000 over 500 . In doing so, we empirically check the distribution of transactions in the economy. Choice of 2000 as a denomination and its prioritisation allowed faster availability of the required value. However, it increased the average requirement of currency notes per transaction. We estimate the transition volatility experienced with the change in denomination structure through a sensitivity analysis. Transition volatility provides a lower bound to the total volatility, as additional volatility exists due to presence of hoarding.

Step size for the denominations changes during currency transition and thus impacts the demand for denominations not undergoing transition. Higher the step size, lower is the substitutability and robustness of a denomination to disruption becomes low. Our analysis estimates the increase in the demand of 100 , due to shortage of higher denominations and high step size. This increase of requirement results in a perceived shortage for 100 and further amplifies the requirement through a hoarding behaviour. Amplification of demand of certain denominations due to transition related hoarding as a reaction to a shortage situation is an important finding. Demand amplification by users and banks may lead to overproduction of denomination 100 by the central bank while printing of 500 would be more beneficial as it brings down requirement for 100 also.

Using the concept of Reverse Bullwhip Effect we explain why banks would experience maximum demand amplification. This is useful as the central bank need to ration supply of notes and plan production of denominations with limited capacities. We also evaluate alternate denomination structure of 100,200 and 500 and find it as having higher overall transaction efficiency than post-demonetisation case with 100 and 2000 as available denominations. It is however, less efficient as compared to the pre-demonetisation structure of 100,500 and 1000 . Thus, we recommend 200 as an additional denomination along with 1000 and not as its replacement.

Further extension of this work needs to evaluate the impact on average currency requirement for transaction during the transient states of availability as substantial time might be required to provide full replenishment of the value impacted in the currency transition. Currency transition is a gradual process and thus the impact on transaction efficiency during the transient state varies from the full availability scenario, which also needs to be factored in the planning process. A more detailed transient state analysis can also incorporate dynamic decisions like periodic prioritisation of denominations and allocation to various currency production facilities. Quantification of the impact on demand variability due to Bullwhip Effect also presents an interesting extension.

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[^0]:    The main objective of the Working Paper series of IIMA is to help faculty members, research staff, and doctoral students to speedily share their research findings with professional colleagues and to test out their research findings at the pre-publication stage.

[^1]:    ${ }^{1}$ https://www.treasury.gov/resource-center/faqs/Currency/Pages/denominations.aspx

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[^6]:    ${ }^{24}$ http://www.livemint.com/Industry/IbqZxSMbvhbOg5oAH3VnzJ/RBI-clears-proposal-to-introduce-Rs-200notes.html

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    ${ }^{26}$ http://www.censusindia.gov.in/2011census/hh-series/hh01.html
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[^8]:    ${ }^{28}$ http://economictimes.indiatimes.com/industry/services/retail/demonetisation-kishore-biyanis-future-group-seeks-discounts-more-credit-days-from-goods-firms/articleshow/55545773.cms

[^9]:    ${ }^{29}$ Refer footnote \# 7
    ${ }^{30}$ http://timesofindia.indiatimes.com/india/withdrawal-curbs-imposed-for-note-ban-will-end-on-holi/ articleshow/57050428.cms

    31 http://www.business-standard.com/article/economy-policy/modi-s-digital-india-dream-souring-value-of-digital-transactions-recedes-117032100113_1.html

