Benders decomposition for capacitated multi-period maximal covering location problem with server uncertainty

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W.P. No. 2019-03-03
March 2019

INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD – 380015
INDIA

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Abstract

Primary Health Centers (PHCs), which are single doctor clinics and vital to health care in rural areas of developing countries, often remain inoperative due to shortage of doctors. When doctors become available, the health administrator needs to decide that which of the inoperative PHCs should the doctor be assigned. For transparency and operational efficiency, the sequence in which the inoperative PHCs will be assigned doctors needs to be decided in advance at the start of the planning horizon. Further, the number of doctors that will become available in each period of the planning horizon is uncertain. Moreover, the health guidelines set the capacity target, i.e. the maximum population a PHC can provide service to.

We introduce and study the capacitated multi period maximal covering facility location problem under server uncertainty. We provide a formulation for the problem based on the minimax regret approach. Further, we solve the problem using CPLEX MIP solver and observe that it can only solve very small instances. Hence, we provide Benders decomposition based solution methods and refinements thereof, which is $100 - 5000$ times faster and could solve practical size instances in reasonable time.
Keywords: OR in health services, Primary health centers, Benders decomposition, Uncertainty

1 Introduction and literature review

The Alma-Ata declaration by the members of World Health Organization (WHO) in 1978 expressed serious concern at the existing gross inequality in the health status of the people, particularly between developed and developing countries. It further articulated primary health care as the key to the attainment of the goal of "Health for All" by the year 2000 (Lawn et al., 2008). However, "health for all" still remains an elusive dream for most of the developing countries (Walley et al., 2008; Rohde et al., 2008). In 2008, the Commission on Social Determinants of Health made a compelling call for close attention to health in all government policies, in all sectors, and further reiterated the importance of primary health care towards the attainment of "Health for All" (Lawn et al., 2008). Universal Health Coverage (UHC) is now a part of sustainable development goals, a global political commitment adopted by the United Nations General Assembly in 2015 (Buse & Hawkes, 2015; WHO, 2016). In accordance with the above declarations/commitments, many developing countries, including India, have opened Primary Health Centers (PHCs), which are clinics, often managed by a single doctor, to provide the basic health care services.

The number of PHCs in India has grown over the years, especially since the National Health Policy (1982) was sanctioned. However, despite the efforts for so many years, there is still a shortage of health care facilities in rural areas. The problem is quite severe in several states, like Bihar, Chhattisgarh, Gujarat, Madhya Pradesh, etc. To address this problem, the Government of India launched the National Rural Health Mission for improving the health care delivery across rural India, and increased the Health funding for the period 2005-2012. During this period, many PHCs were opened in these states. However, despite these initiatives, many of these states still fall far behind the target of a PHC per 30,000 population. Further, in many states where PHCs do exist, they remain inoperative due to shortage of doctors, as illustrated in Table 1 (National Rural Health Mission, 2011). Under such a circumstance, the problem facing a health

Table 1: Shortfall of doctors in PHCs

State	Required	Shortfall	% Shortfall
Bihar	1863	298	16.0%
Chhattisgarh	716	139	19.4%
Gujarat	1096	259	23.6%
Madhya Pradesh	1155	614	53.2%
Uttar Pradesh	3692	831	22.5%
All India	**23673**	**2433**	**10.3%**

administrator is to decide the sequence in which PHCs need to be assigned doctors as and when they become available over a planning horizon. However, the number of doctors that will become available in future periods is generally uncertain. Further, for transparency in policy making and operational effectiveness, it is desirable that such a sequence be decided and declared apriori.

Motivated by the above discussion, in this paper, we address the problem of assigning doctors (servers) to PHCs (facilities). A PHC becomes operational once a doctor is assigned to it. Hence, assigning a doctor (server) to a given PHC (facility) is tantamount to opening a facility in the lietrature facility location problem (FLP). However, the population (user demand) that needs to be served by the PHCs changes over time, and so does the number doctors that become available from one period to the next. In such a scenario, an optimal facility location (operational PHC) decision in one period may become sub-optimal in future periods, and thus may need to be revisited. Revisiting facility location decision may involve relocating/closing an open facility, which may be costly or prohibitive. So, it is desirable to plan ahead for the entire planning horizon. Further, as discussed above, transparency in policy making also demands that such a decision be taken and announced in advance. This results in a multi-period facility location problem (MFLP), wherein the problem parameters (like demand, number of facilities to open) vary with time.

MFLP has been fairly well studied in the literature (readers are referred to the book chapter by Nickel & da Gama (2015) for an extensive review). A subset of the literature on MFLP accounts for uncertainties with respect to demand, cost/distance, etc. (Averbakh & Berman, 1997, 2000; Chen & Lin, 1998; Berman & Wang, 2011). In our current study, demand corresponds to the population of villages, which can be estimated fairly accurately using the current populations and the growth rates, which in turn can be estimated fairly accurately. Hence, we do not account for demand uncertainty. However, uncertainties in MFLP may also arise from the supply side (e.g., related to coverage capability and the number of servers available). This is especially pertinent in the context of PHCs, wherein the availability of doctors (and hence the number of facilities to open) in each of the future periods in the planning horizon is uncertain. Hence, this problem is an MFLP with sever uncertainty (MFLPSU). Majority of the papers on MFLP under uncertainty account for the uncertainty in demand and cost/distance. Supply side uncertainties (related to coverage capability and the number of available servers), on the other hand, has not received as much attention. Table 2 classifies the MFLP literature based on different types of uncertainties. As evident from the Table, Marín et al. (2018); Zarrinpoor et al. (2018); Vatsa & Jayaswal (2016) are the only few studies on MFLP that consider supply side uncertainty. Marín et al. (2018) have studied a generalized version of multi-period covering problems (set covering and maximal covering) under demand and supply side uncertainties. Uncertainty in their work arises with respect to the demand and the capability of the service facilities to cover the demand nodes. More recently, Zarrinpoor et al. (2018) have studied a two-level, multi-flow, hierarchical location-allocation problem with service referral, arising in the context of health service network design. The uncertainty in their work is associated with demand, service capability and geographical accessibility.

Uncertainty in the number of facilities to be sited (which may be due to the uncertainty in the number of servers available) has been addressed by Current et al. (1998). However, their work pertains to a single period FLP. Specifically, they study a p-median FLP, wherein the final value of p is uncertain and they find the optimal set of locations to open in the initial siting decision.

Vatsa & Jayaswal (2016), to the best of our knowledge, is the only study in the MFLP literature to have accounted for the uncertainty in the available severs (and hence the number of facilities to open) in each period of the planning horizon, and hence closest to our work. However, Vatsa & Jayaswal (2016) study an uncapacitated problem, without any limit on the demand that can be served by an open facility. However, in the context of PHCs, there are clear guidelines on the maximum population to be catered by any PHC. Specifically, India has, in line with Alma-Ata recommendations, set a target of having an operational PHC for every 30,000 population (National Rural Health Mission, 2011). To the best of our knowledge, capacitated version of MFLP with supply side uncertainty, specifically uncertainty in the number of facilities to open (due to number of doctors becoming available) in each period, has not been studied. Hence, our work is the first to study capacitated version of MFLP with server uncertainty. We make the following contribution to the scarce literature on MFLP with uncertainty in server availability:

1. We present formulation for the capacitated maximal covering MFLP with an uncertainty in server availability and discuss a useful special case of the problem.
2. We present a Benders decomposition based exact solution method, and refinements thereof, to solve realistic problem instances.

The remainder of the paper is organized as follows. Section 2 describes the problem in detail, and provides the problem formulation. Section 3 presents a Benders decomposition based solution approach, followed by computational experiments in Section 4. The paper concludes with a summary and directions for future research in section 5.

2 Problem Description

The problem described in this paper pertains to assigning doctors, as they become available over time in a planning horizon, to PHCs in a given district. To describe the problem setting, we assume a planning horizon consisting of discrete time periods $t \in T = \{1, 2, ..., |T|\}$. Furthermore, we consider the population

Table 2: Uncertainty related to parameters in MFLP literature

Papers	Demand	Cost/ Distance	Coverage Capability	No. of facilities to open	Capacitated
Averbakh & Berman (1997)	✓				
Chen & Lin (1998)	✓	✓			
Vairaktarakis & Kouvelis (1999)	✓	✓			
Averbakh & Berman (2000)	✓				
Killmer et al. (2001)	✓	✓			✓
Burkard & Dollani (2002)	✓	✓			
Albareda-Sambola et al. (2011)	✓				✓
Berman & Wang (2011)	✓				
Baron et al. (2011)	✓				
Vatsa & Ghosh (2014)				✓	
Vatsa & Jayaswal (2016)				✓	
Marín et al. (2018)	✓		✓		
Zarrinpoor et al. (2018)	✓		✓		

requiring PHC services to be centered at each village of the district, which we refer to as a demand node $i \in I = \{1, 2, ..., m\}$, with a demand (population) d_{it} in period t. Let J^b be the set of PHCs already operational at the beginning of the planning horizon (i.e., at $t = 0$), while $J = \{1, 2, ..., n\}$ be the set of candidate facility locations to be opened (i.e., currently without any doctor assigned) in the planning horizon. Let δ_{ij} be the distance between a demand node $i \in I$ and a facility (either operational or a candidate for opening) $j \in J \cup J^b$. If the distance $\delta_{ij} \leq \delta_{min}$, then the demand node $i \in I$ can be fully covered by the facility $j \in J \cup J^b$. On the other hand, $\delta_{ij} \geq \delta_{max}$ implies the demand node i cannot be covered at all by the facility j. For intermediate values of the distance (i.e., $\delta_{min} < \delta_{ij} < \delta_{max}$), the coverage of node i by facility j can be partial. If $a_{ij} \in [0, 1]$ denotes the extent of coverage facility $j \in J \cup J^b$ can provide to node i, then a_{ij} can be described as:

$$a_{ij} = \begin{cases} 1 & \text{if } \delta_{ij} \leq \delta_{min}, \\ f(\delta_{ij}) & \text{if } \delta_{min} < \delta_{ij} < \delta_{max}, \text{ where } 0 < f(\delta_{ij}) < 1 \\ 0 & \text{if } \delta_{ij} \geq \delta_{max}, \end{cases}$$

where $f(\delta_{ij})$ is a linear, step or any other function of δ_{ij} (Karasakal & Karasakal, 2004; Berman et al., 2010; Vatsa & Ghosh, 2014; Vatsa & Jayaswal, 2016). Let $N_i := \{j \in J : \delta_{ij} < \delta_{max}\}$ and $N_i^b := \{j \in J^b : \delta_{ij} < \delta_{max}\}$. Further, each facility j has a capacity restriction of cap_j.

The number of doctors(servers) that will become available in each period $t \in T$ is uncertain. We represent this uncertainty using a set of server availability scenarios $s \in S$. The uncertainty is with respect to parameter p_{ts} which is the number of new servers that become available at time t under server availability scenario $s \in S$.

If the district administration would have known the exact number of doctors (servers) that will become available in each period of the planning horizon, they would have assigned the doctors to PHCs so as to maximize the total population that can be served in the complete planning horizon. Let ζ_s^* be the maximum coverage that could have been attained in scenario s. Then, regret from a proposed solution in any scenario is the difference between the maximum coverage that could have been attained and the coverage actually achieved using the proposed solution i.e. $\zeta_s^* - \zeta_s$.

In presence of uncertainty, minimizing the maximum regret from the proposed solution is one of the plausible objective (Kouvelis & Yu, 1996; Snyder, 2006). For PHCs without doctors, the district administration would like to find a sequence in which they should be assigned doctors so as to minimize the maximum regret associated with any such sequence across all scenarios. We refer to the resulting problem as Capacitated Multi-period Maximal Covering Location Problem under Server Uncertainty (CMMCLPSU). We summarize below the list of notations used to define the problem:

T: Set of time periods in the planning horizon, $t \in T$

S: Set of all possible server availability scenarios, $s \in S$

p_{ts}: Number of new servers that become available at time t under scenario s

I: Set of demand nodes, $i \in \{1, 2, ..., m\}$

d_{it}: Demand of demand node i in time period t

J: Set of candidate facility locations, $j \in \{1, 2, ..., n\}$

J^b: Set of initially open facilities

δ_{ij}: Distance between demand node i and candidate facility j

δ_{min}: Covering distance within which a candidate facility j can completely cover node i, i.e. $a_{ij} = 1$ if $\delta_{ij} \leq \delta_{min}$

δ_{max}: Covering distance outside which a candidate facility j cannot cover node i, i.e. $a_{ij} = 0$ if $\delta_{ij} \geq \delta_{max}$

N_i: Set of candidate facilities that are within the maximum covering distance δ_{max} from demand node i

N_i^b: Set of facilities open at the beginning of the planning horizon that lie within the maximum covering distance δ_{max} from demand node i

a_{ij}: Level of coverage provided by the facility at j to the demand node i

cap_j: Capacity limitation at candidate location j

ζ_s^*: Maximum demand that can be covered in scenario s over the complete planning horizon

To mathematically model the problem, we define the following decision variables:

z_{jk}: 1 if candidate facility j is one of the first k facilities in the the sequence, 0 otherwise

x_{ijts}: Demand at node i assigned to facility location j in period t and scenario s

Note that we could have used a binary decision variable y_{jts} to indicate if a candidate facility has been opened in time period t and scenario s. However, as shown by Vatsa & Jayaswal (2016) for the uncapacitated problem, the formulation using z_{jk} is tighter and smaller.

The facilities that have been opened (within planning horizon or prior to it) are assumed to remain open. Hence, to know if a facility is open in any period and scenario (i.e y_{jts} value) we only need to know how many candidate facilities have been opened till period t in scenario s (i.e. $k = \sum_{t' \leq t} p_{t's}$) and if the facility j is one of the first k facilities in the sequence.

Further, even though demand from far away nodes can be assigned to facility at j, not all of them necessarily take service. In other words there is demand leakage when the demand node is far from the facility it has been assigned to. The reason being, when a demand node is assigned to a far away facility (that provides partial coverage), some demand of that demand node will not prefer getting service from the assigned facility (will choose other alternatives). Therefore, all the assigned demand will not contribute to the workload of an open facility. We provide the following formulation for the Capacitated Multi-period Maximal Coverage Location Problem under Server Uncertainty with Partial coverage (CMMCLPSU-P) based on the stronger formulation given by Vatsa & Jayaswal (2016) for uncapacitated problem:

[CMMCLPSU-P:]

$$\text{Min} \quad \theta \tag{1}$$

$$s.t. \quad \theta \geq \zeta_s^* - \sum_{i \in I} \sum_{j \in N_i \cup N_i^b} \sum_{t \in T} a_{ij} x_{ijts} \qquad \forall s \in S \tag{2}$$

$$\sum_{i \in N_j} a_{ij} x_{ijts} \leq cap_j z_{jk} \qquad \forall j \in J \cup J^b, t \in T, s \in S : k = \sum_{t' \leq t} p_{t's} \tag{3}$$

$$\sum_{j \in N_i \cup N_i^b} x_{ijts} \leq d_{it} \qquad \forall i \in I, t \in T, s \in S \tag{4}$$

$$x_{ijts} \leq d_{it} z_{jk} \qquad \forall i \in I, j \in N_i, \forall t \in T, s \in S : k = \sum_{t' \leq t} p_{t's} \tag{5}$$

$$z_{jk} \geq z_{j(k-1)} \qquad \forall j \in J, k \in \{1, 2, ...n\} \tag{6}$$

$$\sum_{j \in J} z_{jk} = k \qquad \forall k \in \{0, 1, 2, ...n\} \tag{7}$$

$$x_{ijts} \geq 0 \qquad \forall i \in I, j \in N_i \cup N_i^b, t \in T, s \in S \tag{8}$$

$$\theta \geq 0 \tag{9}$$

$$z_{jk} \in \{0, 1\} \qquad \forall j \in J, k \in \{0, 1, 2, ...n\} \tag{10}$$

(1) and (2) help linearize the objective of minimizing the maximum regret. ζ_s^* is the maximum coverage possible in a given scenario $s \in S$ obtained by solving Capacitated Multi-period Maximal Covering Location Problem with Partial coverage(CMMCLP-P) as given in (11) through (18). Constraint set (3) specify the capacity restriction on the facilities. Notice that in (3), a fractional a_{ij} implies that not all demand at node i that has been assigned to facility at j, will contribute to the workload of the facility (some demand leakage will be there). Whereas, $a_{ij} = 1$, ensures that all the demand at node i that has been assigned to facility at j will contribute to its workload. Constraint set (4) is specify demand at each node while the constraint set (5) ensures that demand nodes are assigned only to open facilities. Notice that while the constraint set (5) is redundant, it makes the formulation stronger. Constraint set (6) ensure that if any facility is one of the $k-1$ open facilities, it is also one of the k open facilities. Constraint set (7) restricts total open facilities to k. Constraints (8), (9) and (10) are the non-negativity and binary constraints.

[CMMCLP-P:]

$$\text{Max} \quad \zeta_s = \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} a_{ij} x_{ijts} \tag{11}$$

$$
\begin{aligned}
s.t.\quad & x_{ijts} \le d_{it} y_{jts} && \forall i \in I, j \in N_i, \forall t \in T && (12)\\
& \sum_{i \in N_j} a_{ij} x_{ijts} \le cap_j z_{jk} && \forall j \in J \cup J^b, t \in T, s \in S : k = \sum_{t' \le t} p_{t's} && (13)\\
& \sum_{j \in N_i \cup N_i^b} x_{ijts} \le d_{it} && \forall i \in I, t \in T && (14)\\
& y_{jts} \ge y_{j(t-1)s} && \forall j \in J, t \in T \backslash \{1\} && (15)\\
& \sum_{j \in J} y_{jts} = \sum_{t' \le t} p_{t's} && \forall t \in T && (16)\\
& x_{ijts} \ge 0 && \forall i \in I, j \in N_i \cup N_i^b, t \in T && (17)\\
& y_{jts} \in \{0, 1\} && \forall j \in J, t \in T && (18)
\end{aligned}
$$

All the constraints of CMMCLP-P are also implied in CMMCLPSU-P. Here, constraint set (15) ensures that a facility once opened remains open throughout the planning horizon. Such a constraint is redundant in CMMCLPSU-P, as it is already implied by the use of sequence variable z_{jk}.

For any given scenario, with infinite capacity limit and binary a_{ij}, the above problems reduce to a Multi-period Maximal Covering Location Problem (MMCLP), which is the multi-period version of the MCLP. MMCLP is NP-hard since its single period version, MCLP is known to be NP-Hard (Drezner & Hamacher, 2001). We observed that CPLEX MIP solver fails to solve any practical size instances of this problem. Hence, we provide the Benders decomposition based solution method in the next section to solve the practical size problems.

3 Benders decomposition based solution method

Benders decomposition is a partition based solution technique where an original problem is partitioned into a sub problem and a master problem (Benders, 1962). The complicating variables from the original problem is identified and the master problem typically contains these variables and their associated constraints. In a MIP problem, integer variables are typically the complicating variable. Further, in each iteration a relaxed master problem is solved and we obtain a lower bound (for minimization problem) of the objective function value. This master problem solution is used to solve the sub problem, which provides the upper bound. Further, the dual solution to the sub problem also provides a Benders cut which is added back to the master problem. This iterative process continues till the upper bound and lower bound converge.

The Benders decomposition based solution technique has been widely applied in facility location (Wentges, 1996), hub location (de Camargo et al., 2009, 2011; Contreras et al., 2011) etc. Costa (2005) provides a detailed review of application of Benders decomposition to solve some of the above problems.

In the CMMCLPSU-P, by fixing the binary variables z_{jk} as $\bar{z}_{jk}$ we obtain the following primal sub-problem :

[CMMCLPSU-P-PSP:]

$$
\text{Min} \quad \theta \qquad (19)
$$

$$s.t. \quad \theta \geq \zeta_s^* - \sum_{i\in I} \sum_{j\in N_i \cup N_i^b} \sum_{t\in T} a_{ij} x_{ijts} \qquad \forall s \in S \tag{20}$$

$$\sum_{i\in N_j} a_{ij} x_{ijts} \leq cap_j \bar{z}_{jk} \qquad \forall j \in J \cup J^b, t \in T, s \in S : k = \sum_{t'\leq t} p_{t's} \tag{21}$$

$$\sum_{j\in N_i \cup N_i^b} x_{ijts} \leq d_{it} \qquad \forall i \in I, t \in T, s \in S \tag{22}$$

$$x_{ijts} \leq d_{it} \bar{z}_{jk} \qquad \forall i \in I, j \in N_i, \forall t \in T, s \in S : k = \sum_{t'\leq t} p_{t's} \tag{23}$$

$$x_{ijts} \geq 0 \qquad \forall i \in I, j \in N_i \cup N_i^b, t \in T, s \in S \tag{24}$$

$$\theta \geq 0 \tag{25}$$

Note that the constraint sets (21) and (22) make constraint set (23) redundant in CMMCLPSU-P-PSP. Further, this primal problem is an LP problem, and including (23) does not make the formulation any stronger. However, Van Roy (1986); Wentges (1996) have shown that for the capacitated facility location problelm (CFLP), this constraint set helps to generate stronger Benders cuts. Let $\alpha_s, \beta_{jts}, \gamma_{its}$ and ρ_{ijts} be the dual variables associated with the constraint sets (20), (21), (22), and (23) respectively. The dual of CMMCLPSU-P-PSP is formulated as follows:

[CMMCLPSU-P-DSP:]

$$\begin{aligned} \text{Max} \quad & \sum_{s\in S} \zeta_s^* \alpha_s - \sum_{i\in I} \sum_{t\in T} \sum_{s\in S} d_{it} \gamma_{its} - \sum_{j\in J} \sum_{t\in T} \sum_{s\in S} cap_j \beta_{jts} \bar{z}_{jk:\ k=\sum_{t'\leq t} p_{t's}} \\ & - \sum_{i\in I} \sum_{j\in N_i} \sum_{t\in T} \sum_{s\in S} d_{it} \rho_{ijts} \bar{z}_{jk:\ k=\sum_{t'\leq t} p_{t's}} \end{aligned} \tag{26}$$

$$s.t. \quad a_{ij}\alpha_s - a_{ij}\beta_{jts} - \gamma_{its} - \rho_{ijts} \leq 0 \qquad \forall i \in I, j \in N_i \cup N_i^b, t \in T, s \in S \tag{27}$$

$$\sum_{s\in S} \alpha_s \leq 1 \tag{28}$$

$$\alpha_s, \beta_{jts}, \gamma_{its}, \rho_{ijts} \geq 0 \qquad \forall i \in I, j \in J \cup J^b, t \in T, s \in S \tag{29}$$

Let H denote the set of all extreme points of CMMCLPSU-P-DSP. For each extreme point $h \in H$, we denote the corresponding values of the dual variables as $\alpha_s^h, \beta_{jts}^h, \gamma_{its}^h$, ρ_{ijts}^h, and the corresponding values of the primal variables as x_{ijts}^h, θ^h. Let N_j be the set of demand nodes that can be covered completely or partially by any candidate facility j, i.e., $N_j = \{i \in I : a_{ij} > 0\}$. Then, the master problem can be formulated as follows:

[CMMCLPSU-P-MP:]

$$\text{Min} \quad \eta \tag{30}$$

$$s.t. \quad z_{jk} \geq z_{j(k-1)} \qquad \forall j \in J, k \geq 1 \tag{31}$$

$$\sum_{j\in J} z_{jk} = k \qquad \forall k \in \{0, 1, ..., n\} \tag{32}$$

$$\begin{aligned} \eta \geq & \sum_{s\in S} \zeta_s^* \alpha_s^h - \sum_{i\in I} \sum_{t\in T} \sum_{s\in S} d_{it} \gamma_{its}^h \\ & - \sum_{j\in J} \sum_{t\in T} \sum_{s\in S} cap_j \beta_{jts}^h \bar{z}_{jk} - \sum_{j\in J} \sum_{t\in T} \sum_{s\in S} \left(\sum_{i\in N_j} d_{it} \rho_{ijts}^h \right) \bar{z}_{jk} \qquad \forall h \in H : k = \sum_{t'\leq t} p_{t's} \end{aligned} \tag{33}$$

$$z_{jk} \in \{0,1\} \qquad \forall j \in J, k \in \{0,1,..,n\} \quad (34)$$

Proposition 1. *The primal sub-problem CMMCLPSU-P-PSP is always feasible and bounded for any* $\bar{z}_{jk}$ *feasible to CMMCLPSU-P-MP.*

Proof. A solution $\bar{z}_{jk}$ of CMMCLPSU-P-MP is essentially a facility opening sequence. Once this sequence is known, we can always determine which all facilities are open in any given period t and scenario s. Consequently, we can always find the overall coverage and regret associated with each scenario s. Hence, maximum regret (which is bounded), associated with any master problem solution $\bar{z}_{jk}$ can always be obtained. □

The efficiency of the Benders decomposition solution method depends to a large extent on how efficiently the sub-problem can be solved. Hence, for a given $\bar{z}_{jk}$, we decompose CMMCLPSU-P-PSP, for each time period $t \in T$ and scenario $s \in S$. The decomposed problem for each $t \in T$ and $s \in S$ is stated as:

$$\text{Max} \quad \zeta'_{ts} = \sum_{i \in I} \sum_{j \in N_i \cup N_i^b} a_{ij} x_{ijts} \tag{35}$$

$$s.t. \quad \sum_{i \in N_j} a_{ij} x_{ijts} \leq cap_j \bar{z}_{jk} \qquad \forall j \in J \cup J_b, k = \sum_{t' \leq t} p_{t's} \tag{36}$$

$$\sum_{j \in N_i \cup N_i^b} x_{ijts} \leq d_{it} \qquad \forall i \in I \tag{37}$$

$$x_{ijts} \geq 0 \qquad \forall i \in I, j \in N_i \cup N_i^b \tag{38}$$

Regret associated with scenario s is $\zeta^*_s - \sum_{t \in T} \zeta'_{ts}$ and the maximum regret across scenarios (θ), is given by $\theta = \max_{s \in S} \left(\zeta^*_s - \sum_{t \in T} \zeta'_{ts} \right)$. The above problem (35)-(38) is a LP problem which can be solved efficiently. For the special case $a_{ij} \in \{0,1\}$, i.e. for the problem with complete coverage, the above problem (35)-(38) takes a network flow problem structure. Such a problem can be solved much more efficiently compared to a general LP problem using a network flow algorithm.

We now provide algorithms to solve the sub problem. For a given master problem solution $\bar{z}_{jk}$, let $j \in OPEN_{ts}$ and $j \in CLOSE_{ts}$ represent the set of open and closed facilities respectively in period t and scenario s, i.e., $OPEN_{ts} = \{j \in J \cup J^b : \bar{z}_{jk} = 1, k = \sum_{t' \leq t} p_{t's}\}$ and $CLOSE_{ts} = \{j \in J : \bar{z}_{jk} = 0, k = \sum_{t' \leq t} p_{t's}\}$. We will solve the decomposed problem (35)-(38) and use the results to solve CMMCLPSU-P-DSP to optimality. Algorithm 1 provides one of the alternate solutions for CMMCLPSU-P-DSP without considering redundant constraint (23), i.e with ρ_{ijts} values set as 0. The solution algorithm is given as follows:

Proposition 2. *For a given solution* $\bar{z}_{jk}$ *to CMMCLPSU-MP, Algorithm 1 gives an optimal solution to CMMCLPSU-DSP.*

Proof. First, we prove that Algorithm 1 gives a feasible solution to CMMCLPSU-P-DSP. Clearly, steps 1 and 2 give an optimal solution to CMMCLPSU-P-PSP. The solution to CMMCLPSU-P-DSP is obtained in steps 3 to 5 using complementary slackness conditions between CMMCLPSU-P-PSP and CMMCLPSU-P-DSP.

Algorithm 1 Solution algorithm for CMMCLPSU-P-DSP without redundant constraint (23)

1: for all $t \in T$ and $s \in S$ solve (35)-(38) to obtain x_{ijts}, ζ'_{ts}, and dual solution $\gamma'_{its} \quad \forall i \in I$ and $\beta'_{jts} \quad \forall j \in OPEN_{ts}$;
2: $\theta \leftarrow \max\limits_{s\in S}\left(\zeta^*_s - \sum\limits_{t\in T}\zeta'_{ts}\right)$, $\xi \leftarrow \operatorname{argmax}\limits_{s\in S}\left(\zeta^*_s - \sum\limits_{t\in T}\zeta'_{ts}\right)$, ties are broken arbitrarily; ;
3: set $\alpha_\xi \leftarrow 1, \alpha_s \leftarrow 0 \quad \forall s \in S\backslash\xi$;
4: set $\gamma_{it\xi} \leftarrow \gamma'_{it\xi} \quad \forall i \in I, t \in T$, and $\beta_{jt\xi} \leftarrow \beta'_{jt\xi} \quad t \in T, j \in OPEN_{t\xi}$, set $\gamma_{its} \leftarrow 0, \beta_{jts} \leftarrow 0 \quad \forall s \in S\backslash\xi$;
5: for all $t \in T$, $s \in S$ and $j \in CLOSE_{ts}$ set $\beta_{jts} \leftarrow \max\{0, \max\limits_{i\in N_j}(\alpha_s - \gamma_{its}/a_{ij})\}$ when $a_{\bar{i}j} > 0, \beta_{jt\xi} \leftarrow 0$ when $a_{\bar{i}j} = 0$;
6: output $\alpha_s, \beta_{jts}, \gamma_{its} \quad \forall i \in I, j \in N_i, t \in T, s \in S$.

In step 1, we obtain γ'_{its} from the dual solution of (35)-(38). Applying complementary slackness condition to (20) gives: $(\theta + \sum\limits_{i\in I}\sum\limits_{j\in N_i\cup N^b_I}\sum\limits_{t\in T} d_{it}x_{ijts} - \zeta^*_s)\alpha_s = 0 \quad \forall s \in S$. This, together with (28) gives as feasible solution $\alpha_\xi = 1$, where $\xi = \operatorname{argmax}\limits_{s\in S}(\zeta^*_s - \sum\limits_{i\in I}\sum\limits_{j\in N_i\cup N^b_i}\sum\limits_{t\in T} a_{ij}x_{ijts})$ and $\alpha_s = 0 \quad \forall s \in S\backslash\xi$ in step 3. $\gamma_{it\xi}$ is obtained in step 4 from $\gamma'_{it\xi}$ values, $\gamma_{its} = 0 \quad \forall s \in S\backslash\xi$. Similarly, $\beta_{jt\xi}$ is obtained in step 4 from $\beta'_{jt\xi}$ values, $\beta_{jts} = 0 \quad \forall s \in S\backslash\xi$. Furthermore, β_{jts} for closed facilities $j \in CLOSE_{ts}$ is obtained in step 5 using the values of α_s and γ_{its} in (27).

We now show that this solution is an optimal solution to CMMCLPSU-P-DSP. With this solution obtained using Algorithm 1, CMMCLPSU-P-DSP objective function (26) is expressed as:

$$\zeta^*_\xi - \sum_{i\in I}\sum_{t\in T} d_{it}\gamma_{it\xi} - \sum_{j\in J}\sum_{t\in T} cap_j\beta_{jt\xi}\bar{z}_{jk:\ k=\sum\limits_{t'\leq t} p_{t'\xi}} \tag{39}$$

For a given scenario ξ and time t, the dual objective function of (35)-(38) can be expressed as $\sum\limits_{i\in I} d_{it}\gamma_{it\xi} - \sum\limits_{j\in J} cap_j\beta_{jt\xi}\bar{z}_{jk:\ k=\sum\limits_{t'\leq t} p_{t'\xi}}$. Hence, the last two terms of (39) evaluate to the total demand covered with scenario ξ. Consequently, CMMCLPSU-P-DSP and CMMCLPSU-P-PSP objective function value are same (equal to θ) and hence the Algorithm 1 solves the CULLPSU-P-DSP to optimality. □

Van Roy (1986); Wentges (1996) discuss that the convergence of Benders decomposition for Capacitated Facility Location Problem (CFLP) is poor when the sub-problem solution found by Algorithm 1 is used for the Benders cut. The constraint set (23) in CMMCLPSU-P-PSP can be used to strengthen the Benders cut. When $\bar{z}_{jk} = 0$ for any scenario s and time t, i.e. when the facility j is closed, changing the β_{jts} and ρ_{ijts} for $j \in CLOSE_{ts}$ does not change the objective function value of CMMCLPSU-P-DSP. However, while changing their values, we need to ensure that β_{jts} and ρ_{ijts} satisfy constraint set (27). We present the algorithm based on Van Roy (1986). For all $j \in CLOSE_{t\xi}$, in order to get stronger Benders cut, $\beta_{jt\xi}$, $\rho_{ijt\xi}$ values is obtained from the following linear program:

$$\text{Min} \quad cap_j\beta_{jt\xi} + \sum_{i\in N_j} d_{it}\rho_{ijt\xi} \tag{40}$$

$$s.t. \quad a_{ij} - a_{ij}\beta_{jt\xi} - \gamma_{it\xi} - \rho_{ijt\xi} \leq 0 \qquad \forall i \in I \tag{41}$$

$$\beta_{jt\xi}, \rho_{ijt\xi} \geq 0 \qquad \forall i \in I \tag{42}$$

The above formulation can be expressed as follows:

$$\text{Min} \quad cap_j\beta_{jt\xi} + \sum_{i\in N_j} d_{it}\max\{0, a_{ij} - a_{ij}\beta_{jt\xi} - \gamma_{it\xi}\} \tag{43}$$

$$\beta_{jt\xi} \geq 0 \tag{44}$$

Proposition 3. (43)-(44) *can be solved to optimality using Algorithm 2.*

Algorithm 2 Benders cut strengthening based on Van Roy (1986)

1: sort $I = \{1, 2, .., m\}$ in descending order of the values $a_{ij} - \gamma_{it\xi}$, set $I^* \leftarrow \phi$;
2: find the demand node $\bar{i} \in I$ with the highest value of $a_{ij} - \gamma_{it\xi}, i \in I$;
3: **if** $a_{\bar{i}j} - \gamma_{\bar{i}t\xi} \leq 0$ or $\sum_{i\in I^*\cup\{\bar{i}\}} a_{ij}d_{it} \geq cap_j$ **then** go to step 6;
4: **else** set $I^* \leftarrow I^* \cup \{\bar{i}\}, I \leftarrow I\backslash\{\bar{i}\}$ and go to step 2;
5: **end if**
6: **if** $I = \phi$ **then** set $\beta_{jt\xi} \leftarrow 0$;
7: **else** set $\beta_{jt\xi} \leftarrow \max\{0, 1 - \gamma_{\bar{i}t\xi}/a_{\bar{i}j}\}$ when $a_{\bar{i}j} > 0, \beta_{jt\xi} \leftarrow 0$ when $a_{\bar{i}j} = 0$;
8: **end if**
9: set $\rho_{ijt\xi} \leftarrow \max\{0, a_{ij} - a_{ij}\beta_{jt\xi} - \gamma_{it\xi}\}$;
10: output $\beta_{jt\xi}, \rho_{ijt\xi} \quad \forall i \in I$.

Proof. Van Roy (1986); Wentges (1996) discuss a similar algorithm for CFLP and we extend that algorithm for CMMCLPSU-P-DSP. The objective function (43) can be expressed as:

$$[\beta_{jt\xi}(cap_j - a_{1j}d_{1t} - a_{2j}d_{2t} - .... - a_{i'j}d_{i't})] + [d_{1t}(a_{1j} - \gamma_{1t\xi}) + d_{2t}(a_{2j} - \gamma_{2t\xi}) + ... \\ + ...d_{i't}(a_{i'j} - \gamma_{i't\xi})] \tag{45}$$

where $i' \in I$ is such that $a_{i'j} - a_{i'j}\beta_{jt\xi} - \gamma_{i't\xi} > 0$. Clearly, for minimum value of the above expression , $\beta_{jt\xi}$ should be given a value 0 if $(cap_j - a_{1j}d_{1t} - a_{2j}d_{2t} - .... - a_{i'j}d_{i't})$ is positive (i.e. in step 6 or when $a_{\bar{i}j} - \gamma_{\bar{i}t\xi} \leq 0$ in step 3). $\beta_{jt\xi}$ should be given a maximum possible value as indicated in step 7, if $(cap_j - a_{1j}d_{1t} - a_{2j}d_{2t} - .... - a_{i'j}d_{i't})$ is negative. □

Proposition 4. *The Benders cut* (33) *can be expressed as:*

$$\eta \geq \zeta^*_{\xi^h} - \sum_{i\in I}\sum_{t\in T} d_{it}\gamma^h_{it\xi^h} - \sum_{j\in J}\sum_{t\in T} cap_j\beta^h_{jt\xi^h}\bar{z}_{jk} - \sum_{j\in J}\sum_{t\in T}\left(\sum_{i\in N_j} d_{it}\rho^h_{ijt\xi^h}\right)\bar{z}_{jk} \\ \forall h \in H : k = \sum_{t'\leq t} p_{t'\xi^h} \tag{46}$$

where, ξ^h associated with the extreme point $h \in H$, is found in step 2 of Algorithm 1.

Proof. This follows directly from substituting the values of dual variables in (33), using z_{jk} as a variable, and rearranging the terms. □

Wentges (1996) further argue that in such cases where a demand node is serviced most efficiently only by one unique facility, Benders cut will be stronger if it captures this information. Let $j(i) \in OPEN_{t\xi}$ represent the facility, demand node i is assigned to (after solving (35)-(38)). If there is no other facility that gives such good coverage to node i as given by facility $j(i)$, $\gamma_{it\xi}$ can be decreased by the extra coverage that node i gets from $j(i)$, and $\rho_{ij(i)t\xi}$ can be raised by a similar amount. Clearly, these changes do not alter the dual objective function value given by expression (26). Wentges (1996) show the pareto optimality of the modified Benders cut for CFLP. Clearly, even for CMMCLPSU-P, it is evident from (46) that such modifications will give a Benders cut that is no weaker than Benders cut (46). We incorporate these improvements suggested by Wentges (1996) for the CFLP in our capacitated problem CMMCLPSU-P:

Algorithm 3 Benders cut strengthening based on Wentges (1996)

```
perform Algorithm 2 to find the dual variables β_{jtξ}, ρ_{ijtξ} for j ∈ CLOSE_{tξ};
for all i ∈ I do
    find the highest(H_i^1) and second highest(H_i^2) value of the set {a_{ij} − a_{ij}β_{jtξ} : j ∈ OPEN_{tξ}}. It follows
  H_i^1 = a_{ij(i)} − a_{ij(i)}β_{j(i)tξ} = γ_{itξ} for some j(i) ∈ OPEN_{tξ}.
    find the highest element H_c of the set {a_{ij} − a_{ij}β_{jtξ} : j ∈ CLOSE_{tξ}}
    set Δ_i ← max{0, min{H_i^1 − H_i^2, H_i^1 − H_c}}, set ρ_{ijtξ} ← 0  ∀j ∈ OPEN_{tξ}, j ≠ j(i);
    if Δ_i > 0 then set γ_{itξ} ← γ_{itξ} − Δ_i, ρ_{ij(i)tξ} ← Δ_i;
    else γ_{itξ} ← γ_{itξ}, ρ_{ij(i)tξ} ← 0;
    end if
end for
once again apply Algorithm 2 to find the final values of dual variables β_{jtξ}, ρ_{ijtξ} for j ∈ CLOSE_{tξ};
output β_{jtξ}, ρ_{ijtξ}   ∀i ∈ I.
```

4 Computational study

In this section we discuss the data generation scheme that was used for our computational experiments. Further, we also report and discuss the results obtained in our experiments. We report our results for the CMMCLPSU-P and the special case of complete coverage wherein $a_{ij} \in \{0, 1\}$. We refer to this special case as CMMCLPSU.

4.1 Data generation

In our study, we have conducted experiments with different problem sizes. The number of demand nodes that have been considered in our experiments are $m \in \{100, 200, 300, 400, 500\}$. We have used X_i and Y_i as the coordinates of demand nodes $i \in I$. Further we generated these coordinates as $X_i \sim U[0, 100]$ and $Y_i \sim U[0, 100]$.

The number of candidate facilities $n \in \{5, 10, 15\}$ for CMMCLPSU and $n \in \{5, 8, 10, 15\}$ for CMMCLPSU-P. We have randomly selected this list of n candidate facilities from the set I of demand nodes. In our experiments, we have taken set $J^b = \phi$, i.e. none of the facilities were open at the start of the planning horizon. Further, we have used the X and Y coordinates to generate the Euclidean distance δ_{ij} between demand node i and candidate facility location j, i.e. $\delta_{ij} = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}$. We have considered the covering distance in CMMCLPSU as $\delta_0 = 20$. Maximum and minimum covering distances in CMMCLPSU-P are fixed as $\delta_{max} = 30$ and $\delta_{min} = 20$. Further the coverage function in CMMCLPSU-P assumed to be linearly decreasing between δ_{min} and δ_{max} as indicated below:

$$a_{ij} = \begin{cases} 1 & \text{if } \delta_{ij} \leq \delta_{min}, \\ 1 - \frac{\delta_{ij}-\delta_{min}}{\delta_{max}-\delta_{min}} = \frac{\delta_{max}-\delta_{ij}}{\delta_{max}-\delta_{min}} & \text{if } \delta_{min} < \delta_{ij} \leq \delta_{max}, \\ 0 & \text{if } \delta_{ij} > \delta_{max}. \end{cases}$$

The demand at a demand node i in all periods has been generated using the first period demand $d_{i1} \sim U[50, 1500]$ and a constant (over all periods) demand growth rate $g_i \sim U[-0.04, 0.10]$ i.e., $d_{it} = d_{i(t-1)}(1 + g_i) \;\; \forall t \in \{2, 3, .., |T|\}$. This data generation scheme has been motivated from the population and growth rates of villages in Dang district of Gujarat, India. Further, length of the planning horizon in all experiments is assumed to be 4 periods, i.e. $|T| = 4$. Server availability scenarios are generated using the same scheme as Vatsa & Jayaswal (2016). Further, we use the capacity limit of 30,000 for all candidate facilities. This capacity limit is motivated from the health guideline that recommends a PHC for every 30,000 population (National Rural Health Mission, 2011).

4.2 Computational results

We perform the computational study on the data generated with the above mentioned scheme. All the experiments are run on a personal computer with Intel Core i5 (3.30 GHz) processor; 4 GB RAM; and windows 64-bit operating system. Solution algorithms are coded in C++ (Visual Studio 2010) and we use IBM ILOG CPLEX as the MIP solver. Further, in all our experiments, the maximal coverage ζ_s^* for each scenario $s \in S$ is obtained by solving CMMCLP for complete coverage and CMMCLP-P for gradual coverage using CPLEX MIP solver. Tables 3 and 4 report the total time taken to obtain ζ_s^* for each scenario by solving CMMCLP and CMMCLP-P respectively. These times are much smaller than the CPU time taken by CPLEX MIP solver to solve CMMCLPSU and CMMCLPSU-P, respectively. Hence, we do not include these times in the total CPU times reported in all our further experiments. Further, after finding ζ_s^* for all scenarios, we check which scenarios can be ignored for any further consideration. All those scenarios that are dominated (by some other scenario), will not influence the optimal solution and can be removed from the problem. This step leads to a huge reduction in the problem size and consequently in the solution times. The scenario dominance rules used by us are motivated from Vatsa & Ghosh (2014) who use similar scenario dominance rules for the uncapacitated problem.

Table 3: Total CPU time to solve CMMCLP for each scenario

Ins.	CPU(s)														
	$n=5$					$n=10$					$n=15$				
Ins.\\m	100	200	300	400	500	100	200	300	400	500	100	200	300	400	500
1	0.8	1.6	2.1	2.6	4.1	11.8	28.6	43.0	110.8	110.2	79.7	351.5	453.2	1131.7	2130.2
2	0.7	1.3	1.9	2.3	4.1	11.8	26.5	63.9	52.7	172.7	90.0	251.6	582.0	1260.8	1797.8
3	0.8	1.1	1.8	2.7	2.9	12.6	22.5	82.2	83.5	107.3	131.1	181.7	725.5	1101.0	781.3
4	0.7	1.1	2.7	3.9	3.0	12.7	19.9	55.6	91.5	68.9	73.8	169.3	354.8	953.1	568.6
5	0.8	1.2	2.0	3.4	5.0	13.4	24.2	46.2	140.1	124.9	116.0	260.1	408.2	2217.7	2083.6
6	0.7	1.3	3.3	3.0	3.8	13.6	20.3	92.1	88.6	92.3	116.4	159.9	1410.7	1436.5	1945.9
7	0.8	1.2	2.0	3.6	4.6	13.1	21.4	122.4	119.5	143.7	184.1	158.5	1125.3	2418.3	1142.3
8	0.9	1.2	1.9	3.9	4.2	14.2	26.7	33.3	95.3	104.2	139.7	372.7	724.3	790.2	1180.0
9	0.7	1.4	2.1	3.7	3.6	12.1	31.6	70.4	194.9	128.0	103.6	212.2	627.1	1885.6	2016.0
10	0.8	1.2	2.0	2.8	3.3	12.3	20.4	90.7	90.4	120.5	87.5	136.3	893.2	898.6	1575.6
Avg.	0.8	1.3	2.2	3.2	3.8	12.8	24.2	70.0	106.7	117.3	112.2	225.4	730.4	1409.4	1522.1
Max.	0.9	1.6	3.3	3.9	5.0	14.2	31.6	122.4	194.9	172.7	184.1	372.7	1410.7	2418.3	2130.2

Table 4: Total CPU time to solve CMMCLP-P for each scenario

Ins.	CPU (s)							
	$n=5$				$n=8$			
Ins.\\m	100	200	300	400	100	200	300	400
1	1.4	4.4	5.0	4.7	7.5	43.2	63.7	42.3
2	1.3	3.1	5.9	4.9	6.5	44.5	55.7	37.8
3	1.3	2.1	4.6	6.1	9.1	15.1	63.3	165.5
4	1.2	1.6	6.4	5.9	7.7	14.9	55.4	99.4
5	1.5	2.3	3.9	8.0	8.4	14.5	43.2	68.4
6	1.3	2.5	6.0	6.6	9.8	31.8	91.5	55.0
7	1.3	2.7	4.4	8.4	8.1	13.7	42.3	62.8
8	1.4	2.2	3.7	7.8	9.3	26.0	65.5	57.5
9	1.2	3.3	4.9	7.7	7.4	23.6	64.0	54.8
10	1.6	2.5	4.6	6.6	11.0	21.4	91.3	56.8
Avg.	1.3	2.7	4.9	6.7	8.5	24.9	63.6	70.0
Max.	1.6	4.4	6.4	8.4	11.0	44.5	91.5	165.5
	$n=10$				$n=15$			
Ins.\\m	100	200	300	400	100	200	300	400
1	21.5	113.9	228.1	128.2	121.4	866.3	1324.1	1827.3
2	18.9	126.3	239.4	146.6	113.6	809.0	2355.6	3755.8
3	21.9	60.8	267.2	502.2	130.5	397.4	1665.2	6113.3
4	22.3	58.5	177.0	347.0	115.3	457.3	1620.3	4554.1
5	24.5	85.8	165.5	233.3	126.2	452.3	2555.8	3621.6
6	26.1	74.7	369.9	203.6	158.4	526.2	3261.2	4325.6
7	24.6	37.8	235.0	235.1	166.9	416.7	2547.2	4549.9
8	29.7	80.6	120.4	189.8	163.8	799.9	2584.8	2212.0
9	20.0	62.3	268.3	351.5	104.7	577.6	3914.4	3263.0
10	32.4	61.5	367.0	264.3	137.2	373.9	3754.1	2563.6
Avg.	24.2	76.2	243.8	260.2	133.8	567.6	2558.2	3678.6
Max.	32.4	126.3	369.9	502.2	166.9	866.3	3914.4	6113.3

We conduct our computational experiments with 10 instances for each of the problem sizes described in section 4.1. For CMMCLPSU, Table 5 reports the objective function value (Obj), time taken by CPLEX MIP solver (CPLEX CPU(s)), time and number of cuts required by the callback versions of Benders decomposition method while incorporating Algorithm 2 (BD-VR) and Algorithm 3 (BD-Wen). Clearly, BD-VR and BD-Wen out-perform the CPLEX MIP solver. For example, the computation time taken by the CPLEX MIP solver for the problem size of $n = 10, m = 300$ is on average more than 250 times the time taken by BD-VR and BD-Wen. Further, CPLEX solver could not solve CMMCLPSU instances beyond problem size $n = 10, m = 300$ within the time limit of 20 hours. Benders-VR and BD-Wen, on the other hand, could solve most of the problem instances till $n = 15, m = 500$ in less than 1 hour on average. At the same time, we notice that the number of cuts, and hence the CPU time, required by BD-VR and BD-Wen increases with the problem size. We notice that even though BD-Wen takes fewer number of cuts on average compared to BD-VR, the difference is not that significant.

Table 6 provides a comparison between CPLEX MIP solver, BD-VR and BD-Wen for CMMCLPSU-P. We report results with $n \in \{8, 10, 15\}$ and $m \in \{100, 200, 300, 400\}$ as we find that with $n = 5$ or with $m = 500$ most of the instances give zero regret solution with the data used in our experiments. With a large number of demand nodes compared to number of facilities, and allowing for higher covering distance, the full capacity of all the open facilities will be used. Hence, the regret with any of the facility opening sequence will be zero. Further, from Table 6, it is evident that BD-VR and BD-Wen solve much larger instances of CMMCLPSU-P compared to CPLEX MIP solver within the time limit of 20 hours. For example, CPLEX MIP solver could not solve instances of problem size $n = 8, m = 300$ and $n = 10, m = 200$ within the 20 hour limit, while BD-VR could solve all instances even of size $n = 15, m = 300$ in close to 2 hours on average. Further, for the larger instances that CPLEX MIP solver could solve within the time limit, BD-VR is of the order of 100 times faster. Moreover, comparing the results in Table 5 and 6, we notice that the BD-VR is much faster for CMMCLPSU, compared to CMMCLPSU-P. This is due to the resulting network flow structure in the decomposed sub-problems of CMMCLPSU, which we are able to exploit.

We observe that the number of instances with zero regret solution is higher for the capacitated problems compared to the uncapacitated problems reported by Vatsa & Jayaswal (2016). Moreover, we also see that objective function value is zero for most of the instances with $m = 500$. The reason is, with a large number of demand nodes (and demand values) compared to number of facilities the full capacity of all the open facilities will be used. Hence, the regret with any of the facility opening sequence will be zero. This also suggests that the decrease in capacity limit will result in higher number of instances with zero regret solution.

5 Conclusion

In this paper, we identified a gap in the capacitated facility location literature with an uncertainty related to server availability. This problem is motivated from the real world problem of assigning doctors to Primary Health Centres (PHCs), which are single doctor clinics and have capacity limits. In developing countries there is a shortage of doctors and many of the PHCs (facility) remain inoperative. The district administration wants to find a sequence of opening these PHCs so that doctors can be assigned to them as an when they become available over the planning horizon. Since, the number of doctors becoming available in each period of the planning horizon is uncertain, the district administration would like to consider such a sequence that minimizes the maximum regret across all scenarios of doctor availability.

We provided a formulation for this problem. However, we observed that CPLEX MIP solver was unable to solve even the medium size instances within time limit of 20 hours. Consequently, we provided Benders based solution method which enables us to solve larger size instances in reasonable time. We further found that one of the special case of the problem, where any demand node is either fully covered or not covered (complete coverage), can be solved much faster than the general case. Moreover, for the instances that CPLEX MIP solver could solve within a time limit of 20 hours, our proposed solution method turned out to be of the order of $100 - 5000$ times faster for the problems with complete coverage, and around $100 - 500$ times faster for gradual coverage.

Future research may consider uncertainty in demand along with the uncertainty in the number of servers. Furthermore, instead of minimax regret as used in our work, other regret criterion can be used.

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Table 5: Computational results with CMMCLPSU

Ins.	Obj	CPLEX	BD-VR		BD-Wen		Obj	CPLEX	BD-VR		BD-Wen	
		CPU(s)	CPU(s)	Cuts	CPU(s)	Cuts		CPU(s)	CPU(s)	Cuts	CPU(s)	Cuts
			$n = 5, m = 100$						$n = 5, m = 200$			
1	0.0	2.3	0.2	4	0.2	4	1004.6	12.7	0.8	9	0.8	9
2	0.0	2.0	0.2	6	0.2	5	0.0	5.8	0.3	6	0.3	7
3	0.0	2.8	0.3	8	0.2	5	0.0	4.2	0.2	1	0.2	1
4	0.0	2.9	0.2	6	0.2	4	115.0	3.4	0.9	13	0.9	12
5	0.0	2.7	0.2	7	0.2	6	0.0	5.4	0.7	9	0.6	6
6	0.0	1.7	0.4	12	0.4	12	159.0	3.7	0.8	15	0.8	15
7	0.0	2.7	0.3	7	0.3	6	0.0	4.3	0.3	3	0.4	4
8	16.6	3.0	0.3	8	0.3	8	0.0	5.4	0.8	12	0.8	12
9	0.0	2.3	0.3	9	0.3	8	6772.5	5.6	0.6	9	0.5	9
10	0.0	3.1	0.5	17	0.3	6	0.0	4.8	0.4	7	0.3	5
Avg.		2.5	0.3	8.4	0.3	6.4		5.5	0.6	8.4	0.6	8
Max.		3.1	0.5	17	0.4	12		12.7	0.9	15	0.9	15
			$n = 5, m = 300$						$n = 5, m = 400$			
1	652.8	17.0	1.3	12	1.4	13	0.0	8.3	0.3	2	0.3	2
2	0.0	7.0	0.1	0	0.1	0	0.0	6.9	0.5	6	0.4	5
3	0.0	6.8	0.6	6	0.5	6	0.0	6.9	0.6	6	0.7	8
4	174.0	9.9	2.4	16	1.7	11	850.0	64.7	1.0	7	1.0	7
5	0.0	7.5	0.4	4	0.4	4	0.0	11.1	0.3	2	0.3	2
6	0.0	30.7	0.2	2	0.2	2	0.0	9.5	0.5	5	0.4	4
7	0.0	5.6	1.0	7	1.0	8	0.0	12.3	0.2	2	0.2	2
8	0.0	11.2	0.5	6	0.5	6	0.0	14.6	0.4	4	0.4	4
9	0.0	6.8	0.8	10	0.6	7	0.0	96.4	0.2	0	0.2	0
10	434.0	9.7	0.8	10	0.8	10	318.0	7.6	0.4	4	0.4	4
Avg.		11.2	0.8	7.3	0.7	6.7		23.8	0.4	3.8	0.4	3.8
Max.		30.7	2.4	16	1.7	13		96.4	1.0	7	1.0	8
			$n = 10, m = 100$						$n = 10, m = 200$			
1	159.6	138.8	14.4	33	11.5	32	6176.4	24487.1	71.1	108	74.2	87
2	0.0	444.1	13.2	47	17.1	43	7509.0	7894.8	24.7	52	28.0	47
3	1532.4	1045.4	7.3	28	6.0	21	19828.6	2880.7	49.6	54	64.6	53
4	3822.3	1257.9	7.4	32	8.8	32	115.7	827.9	44.1	50	42.1	39
5	3643.9	3022.6	9.3	33	9.7	44	0.0	1851.9	39.0	39	32.1	33
6	6197.2	5240.4	38.9	85	26.4	74	8758.8	1848.8	52.7	76	70.7	63
7	2507.9	1449.2	15.0	46	13.1	38	0.0	1170.8	13.1	20	13.6	19
8	2911.0	1954.1	23.1	55	22.0	49	0.0	14223.0	38.0	51	47.1	50
9	2442.2	610.7	31.9	78	35.7	72	13179.4	11653.6	13.5	41	18.6	33
10	249.7	754.1	8.6	34	10.5	38	0.0	1368.7	14.7	16	33.2	36
Avg.		1591.7	16.9	47.1	16.1	44.3		6820.7	36.0	50.7	42.4	46
Max.		5240.4	38.9	85	35.7	74		24487.1	71.1	108	74.2	87
			$n = 10, m = 300$						$n = 10, m = 400$			
1	8557.7	8266.2	179.5	170	188.8	127	2632.9	*	49.9	46	55.5	43
2	336.4	80763.9	23.9	60	34.6	49	1246.0	*	23.1	23	38.6	21
3	245.0	29232.4	105.4	97	94.1	98	395.0	*	50.9	65	117.5	78
4	2395.9	2922.4	65.1	66	83.9	53	1763.1	*	60.2	36	59.2	33
5	4951.6	17290.9	42.2	32	57.4	37	0.0	*	29.6	32	36.1	23
6	211.4	3153.9	31.4	58	45.2	68	0.0	*	41.3	44	26.6	14
7	7830.8	37922.4	170.6	125	216.7	129	1634.0	*	189.3	109	184.5	123
8	0.0	2853.6	34.1	46	77.5	77	12091.9	*	27.4	25	36.7	26
9	347.8	63384.8	52.9	60	52.6	48	10502.1	*	77.5	76	150.5	83
10	1518.5	5378.5	51.5	62	107.1	51	4009.8	*	30.8	56	33.6	37
Avg.		25116.9	75.7	77.6	95.8	73.7			58.0	51.2	73.9	48.1
Max.		80763.9	179.5	170	216.7	129			189.3	109	184.5	123

Table 5 (continued)

Ins.	Obj	CPLEX	BD-VR		BD-Wen		Obj	CPLEX	BD-VR		BD-Wen	
		CPU(s)	CPU(s)	Cuts	CPU(s)	Cuts		CPU(s)	CPU(s)	Cuts	CPU(s)	Cuts
			$n = 10, m = 500$						$n = 15, m = 100$			
1	7325.2	*	498.9	149	245.0	149	159.6	*	109.8	69	97.1	59
2	172.2	*	410.8	314	724.1	317	73.7	*	206.5	117	158.7	100
3	3599.7	*	146.3	144	203.6	84	6075.3	*	143.7	97	126.4	71
4	0.0	*	20.3	10	25.6	10	18.8	*	64.4	52	80.6	50
5	0.0	*	59.2	30	48.5	27	3221.5	*	63.6	38	55.4	34
6	0.0	*	9.2	3	134.4	3	5471.8	*	351.9	257	272.0	177
7	466.3	*	109.4	99	178.2	73	7631.9	*	377.6	211	354.1	182
8	0.1	*	70.3	29	86.0	31	4106.3	*	139.2	100	162.5	109
9	4081.7	*	357.6	65	414.8	57	3293.7	*	194.7	130	239.3	134
10	10637.1	*	85.4	41	86.1	32	249.7	*	90.0	49	82.5	47
Avg.			176.8	88.4	214.6	78.3			174.1	112	162.9	96.3
Max.			498.9	314	724.1	317			377.6	257	354.1	182
			$n = 15, m = 200$						$n = 15, m = 300$			
1	13335.4	*	1602.5	581	2016.9	558	1757.4	*	422.9	90	710.1	81
2	7509.0	*	475.4	140	609.0	166	6734.8	*	747.6	268	663.0	202
3	4833.0	*	594.4	176	1018.2	162	10131.0	*	2061.9	369	1632.0	236
4	9274.4	*	535.6	226	668.1	200	2526.5	*	819.7	278	598.3	135
5	3608.8	*	443.1	122	426.5	119	6623.3	*	613.0	138	516.6	95
6	904.0	*	991.9	225	1051.3	229	5176.2	*	541.4	275	511.6	164
7	3215.5	*	498.0	160	499.4	146	6379.9	*	2526.9	413	2494.8	367
8	6263.6	*	2663.7	460	2824.8	460	10606.9	*	778.6	160	619.2	150
9	16059.5	*	570.8	112	406.9	106	12108.7	*	646.6	125	671.4	137
10	0.0	*	236.3	72	186.7	49	3155.2	*	1122.4	262	1160.2	281
Avg.			861.2	227.4	970.8	219.5			1028.1	237.8	957.7	184.8
Max.			2663.7	581	2824.8	558			2526.9	413	2494.8	367
			$n = 15, m = 400$						$n = 15, m = 500$			
1	8141.8	*	1145.1	158	1209.7	145	6318.6	*	6309.0	592	5052.6	521
2	829.2	*	1550.9	346	469.2	90	2482.1	*	4224.6	487	4649.6	536
3	4182.5	*	1637.4	265	2081.8	322	0.0	*	330.3	27	325.4	28
4	12039.8	*	1739.0	210	1254.7	145	8844.9	*	593.8	52	514.5	53
5	10404.5	*	4705.2	747	3553.8	620	2876.8	*	2652.6	108	2346.9	105
6	1494.8	*	1552.5	339	1299.8	245	0.0	*	3323.4	121	6474.1	155
7	8624.1	*	1824.4	360	2325.8	422	1765.0	*	1968.4	89	1291.4	50
8	4567.3	*	167.5	30	250.6	43	21.6	*	4201.7	206	4059.3	214
9	11109.8	*	7632.6	488	6788.1	434	3684.3	*	3299.3	106	2476.4	78
10	4009.7	*	438.2	78	1964.3	86	12214.2	*	6928.5	257	4887.2	206
Avg.			2239.3	302.1	2119.8	255.2			3383.2	204.5	3207.7	194.6
Max.			7632.6	747	6788.1	620			6928.5	592	6474.1	536

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Table 6: Computational results with CMMCLPSU-P

Ins.	Obj	CPLEX	BD-VR		BD-Wen		Obj	CPLEX	BD-VR		BD-Wen	
		CPU(s)	CPU(s)	Cuts	CPU(s)	Cuts		CPU(s)	CPU(s)	Cuts	CPU(s)	Cuts
			n=8, m=100						n=8, m=200			
1	4052.4	148.5	32.9	34	29.6	39	12279.6	18512.5	83.1	42	54.9	31
2	0.0	224.6	23.5	20	14.0	11	1348.8	7547.5	62.5	38	39.6	30
3	3115.1	121.8	26.3	15	22.7	15	79.1	1887.5	79.5	48	50.2	37
4	3170.0	1410.5	20.3	17	25.2	16	457.9	1522.8	41.7	21	31.8	19
5	29.2	219.8	24.2	21	18.6	17	0.0	648.7	46.9	21	37.6	18
6	6144.5	821.0	69.5	49	44.7	36	9979.7	7038.5	77.3	33	66.2	40
7	38.8	93.1	25.2	25	27.6	34	3079.5	1612.3	61.3	31	53.6	34
8	0.0	2417.3	32.7	33	31.7	38	1127.2	2841.0	68.3	31	54.8	32
9	0.0	246.4	19.7	14	16.7	13	2667.2	746.8	41.0	22	33.8	25
10	2419.9	532.7	57.0	43	48.0	41	927.8	5183.0	64.3	33	59.8	34
Avg.		623.6	33.1	27.1	27.9	26		4754.1	62.6	32	48.2	30
Max.		2417.3	69.5	49	48.0	41		18512.5	83.1	48	66.2	40
			n=8, m=300						n=8, m=400			
1	108.8	*	46.6	21	37.4	17	0.0	*	46.8	6	38.2	5
2	0.0	*	25.9	1	24.9	1	0.0	*	39.2	6	40.2	11
3	0.0	*	33.2	10	32.9	12	1614.4	*	234.9	74	203.9	71
4	350.5	*	35.7	13	33.7	16	813.6	*	183.4	96	153.4	90
5	0.0	*	24.7	6	25.7	9	0.0	*	38.1	1	33.5	1
6	0.0	*	47.3	15	45.6	18	2461.3	*	201.6	82	194.8	93
7	0.0	*	36.0	10	29.7	5	0.0	*	39.0	2	34.3	2
8	820.8	*	73.4	47	56.8	36	0.0	*	48.1	8	42.0	9
9	0.0	*	37.5	12	31.0	8	0.0	*	38.7	2	33.1	2
10	1141.6	*	72.3	22	73.9	27	0.0	*	38.7	2	32.9	2
Avg.			43.3	15.7	39.1	14.9			90.9	27.9	80.6	28.6
Max.			73.4	47	73.9	36			234.9	96	203.9	93
			n=10, m=100						n=10, m=200			
1	280.6	426.9	103.7	50	100.6	48	12279.6	*	206.1	74	205.0	71
2	7.3	1414.9	90.9	40	94.4	40	2107.6	*	154.1	45	142.3	43
3	3908.7	412.1	53.1	24	53.1	23	10549.3	*	283.2	54	396.0	38
4	3262.5	2046.2	64.1	35	73.4	41	1913.1	*	161.4	42	179.9	44
5	1920.6	14923.5	124.9	44	85.3	33	637.6	*	93.2	22	136.8	39
6	10066.3	11460.0	180.4	70	178.0	62	4497.8	*	262.3	58	284.5	67
7	8496.6	3407.5	84.2	32	105.5	40	3079.5	*	137.2	45	106.8	33
8	5663.0	5899.7	110.6	50	91.1	40	2901.2	*	276.2	53	263.5	48
9	10.0	1042.9	89.7	38	99.2	47	4972.4	*	75.9	25	72.8	24
10	5097.5	9381.6	126.0	55	133.5	59	3390.4	*	186.4	46	170.5	41
Avg.		5041.5	102.8	43.8	101.4	43.3			183.6	46.4	195.8	44.8
Max.		14923.5	180.4	70	178.0	62			283.2	74	396.0	71
		n=10, m=300						n=10, m=400				
1	0.0	*	155.1	17	140.5	15	1198.1	*	167.6	9	155.5	9
2	0.0	*	223.9	29	201.7	25	0.0	*	116.4	5	107.8	6
3	0.0	*	181.6	27	183.5	26	0.0	*	519.4	68	430.3	54
4	7217.6	*	205.4	31	200.4	31	813.6	*	1349.9	220	1210.9	216
5	0.0	*	178.4	33	172.5	31	0.0	*	162.0	7	120.0	4
6	1614.3	*	411.8	73	453.8	85	2461.3	*	267.7	24	383.4	48
7	888.7	*	510.3	93	379.3	70	1412.6	*	403.2	35	408.6	45
8	0.0	*	177.8	49	117.7	24	1740.3	*	336.7	31	275.5	28
9	0.0	*	232.6	32	161.3	18	186.0	*	784.7	108	718.4	114
10	6266.3	*	860.7	120	773.9	109	2.3	*	336.4	24	280.9	21
Avg.			313.8	50.4	278.5	43.4			444.4	53.1	409.1	54.5
Max.			860.7	120	773.9	109			1349.9	220	1210.9	216

Table 6 (continued)

Ins.	Obj	CPLEX	BD-VR		BD-Wen		Obj	CPLEX	BD-VR		BD-Wen	
		CPU(s)	CPU(s)	Cuts	CPU(s)	Cuts		CPU(s)	CPU(s)	Cuts	CPU(s)	Cuts
			n=8, m=100						n=8, m=200			
1	5162.9	*	694.6	60	864.3	81	18970.4	*	4480.8	302	3392.6	226
2	4025.5	*	751.9	86	789.3	93	3989.2	*	1786.9	126	1815.3	127
3	4336.9	*	564.4	59	570.9	57	5026.7	*	4247.0	121	6501.7	191
4	1478.7	*	564.4	65	510.0	59	5997.5	*	3571.0	136	3538.0	127
5	10212.4	*	475.9	57	344.8	39	12606.1	*	1309.0	70	1498.0	83
6	10131.6	*	1504.9	197	1198.6	159	4887.0	*	2739.6	119	3100.0	137
7	11693.0	*	1069.2	114	735.5	80	3686.8	*	2851.2	132	2641.4	120
8	3226.9	*	587.4	74	586.6	78	5826.2	*	5506.4	122	7703.1	174
9	0.0	*	392.6	47	189.6	16	7687.6	*	1252.3	62	1138.0	51
10	9443.1	*	764.6	92	670.8	79	3907.2	*	3741.5	162	3807.9	167
Avg.			737.0	85.1	646.0	74.1			3148.6	135.2	3513.6	140.3
Max.			1504.9	197	1198.6	159			5506.4	302	7703.1	226
			n=15, m=300						n=15, m=400			
1	6063.4		2756.0	100	2924.6	113	5740.2	*	4628.5	77	4830.0	87
2	5546.6		14229.1	440	16792.9	522	7146.1	*	12596.0	262	15067.6	321
3	279.7		6651.6	141	7977.7	165	*	*	*	*	*	*
4	2575.9		5710.0	119	5465.8	105	2539.2	*	48221.7	686	46278.1	665
5	10088.5		4009.9	185	6667.8	317	568.0	*	9959.9	213	9200.3	195
6	869.4		6121.6	260	5296.5	215	5518.4	*	88650.5	1486	87067.1	1458
7	4541.9		15304.2	345	10207.8	227	2426.4	*	5953.5	154	5423.6	141
8	7841.6		9212.1	360	6207.4	241	4668.2	*	4727.7	122	4609.2	118
9	401.5		3327.5	119	2673.7	92	598.4	*	12233.1	256	11560.3	240
10	7181.4		6465.0	263	5160.1	207	558.8	*	4138.7	68	3957.5	64
Avg.			7378.7	233.2	6937.4	220.4			21234.4	369.3	20888.2	365.4
Max.			15304.2	440	16792.9	522			88650.5	1486	87067.1	1458

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