A computational algorithm to analyze unobserved sequential reactions of the central banks: Inference on complex lead-lag relationship in evolution of policy stances

Anindya S. Chakrabarti[a]
Sudarshan Kumar

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[a](corresponding author) anindyac@iima.ac.in. Economics area, Indian Institute of Management, Vastrapur, Ahmedabad, Gujarat-380015, India.

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INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD – 380015
INDIA

A computational algorithm to analyze unobserved sequential reactions of the central banks: Inference on complex lead-lag relationship in evolution of policy stances[1]

Anindya S. Chakrabarti[2]
Sudarshan Kumar[3]

Abstract

Central banks of different countries are some of the largest economic players at the global scale and they are not static in their monetary policy stances. They change their policies substantially over time in response to idiosyncratic or global factors affecting the economies. A very prominent and empirically documented feature arising out of central banks' actions, is that the relative importance assigned to inflation vis-a-vis output fluctuations evolve substantially over time. We analyze the leading and lagging behavior of central banks of various countries in terms of adopting low inflationary environment vis-a-vis high weight assigned to counteract output fluctuations, in a completely data-driven way. To this end, we propose a new methodology by combining *complex Hilbert principle component analysis* with state-space models in the form of Kalman filter. The CHPCA mechanism is non-parametric and provides a clean identification of leading and lagging behavior in terms of phase differences of time series in the complex plane. We show that the methodology is useful to characterize the extent of coordination (or lack thereof), of monetary policy stances taken by central banks in a cross-section of developed and developing countries. In particular, the analysis suggests that US Fed led other countries central banks in the pre-crisis period in terms of pursuing low-inflationary regimes.

Keywords: Monetary policy, Taylor rule, time-varying coefficients, Hilbert transformation, principle component.

JEL Codes: E37, E43, E47, E58, C32

1 Introduction

Global economy is a large complex system comprising strategic agents interacting in micro as well as macro-scales (Schweitzer et al. (2009)). As empirical observers, we observe the outcomes of the strategies that the agents pursue, for example in the form of price or trade or production data.

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[2](corresponding author) anindyac@iima.ac.in. Economics area, Indian Institute of Management, Vastrapur, Ahmedabad, Gujarat-380015, India.

[3]Email: sudarshank@iima.ac.in. Finance & accounting area, Indian Institute of Management, Vastrapur, Ahmedabad, Gujarat-380015, India.

However, frequently the underlying strategic aspect is not directly observable. One of the prime examples being how the central banks react to local or global factors. We see the movements in their policy instruments. But generally it is difficult to analyze whether the change in the policy instrument is reflective of deep changes in policy parameters or merely temporary adjustments[4]. Therefore, it is difficult to predict the consequences of such changes. Yet they are the biggest global players with their policies impacting the whole global economy. It is even more problematic for smaller central banks as they are often pushed by implicit policy changes of larger and more dominant central banks like US Fed who do not take into account the repercussions on rest of the world, indicating a coordination failure at a global scale (Mishra and Rajan (2016)). Therefore, it is an important question to figure out how do these global economic players change their policy stances and in particular, who follows whom chronologically? The difficulty in answering this question is that we can only observe aggregate data and most of the time, policy parameters are not announced explicitly. In this paper, we propose a novel methodology by combining Kalman filter to filter unobserved behavior of the central banks from aggregate economic data and complex Hilbert principle component analysis to extract the sequence of reactions of the central banks, by explicitly quantifying the concurrent evolution of policy regimes across countries.

The nominal interest rate set by the US Fed is generally perceived as a good indicator of its monetary policy stance (Bernanke and Blinder (1990)) with respect to movements of economic variables. Taylor rule is a simple mathematical expression summarizing dynamics of nominal interest rate vis-a-vis fluctuations in output and inflation. Ever since the demonstration of the fact that this equation actually captures the behavior of the US Federal Reserve reasonably well (Taylor (1993)), a large literature has been developed that analyzed this equation in multiple forms across different countries. The basic Taylor rule suggests that the central banks set the nominal interest rate as positively related to both output gap and inflationary gap in a linear fashion, with fixed weights attached to both. However, any given central bank's behavior typically changes over time along with its policy orientation and hence, it is highly unlikely that the relative importance of inflation vis-a-vis output fluctuations will remain fixed throughout a long period.

In this paper, we build on the idea that central banks across different countries will have concurrent but dissimilar evolution of the weights attached to output and inflationary fluctuations, reflecting policy changes of the central banks due to idiosyncratic as well as common factors that affect monetary policies. We analyze dynamics of time-varying policy changes to find the ordering of countries adopting a particular policy stances. One possibility is to simply look at the formal announcements made by the central banks to determine changes in the corresponding policy regimes. However, that has the obvious drawback that many a times implicit policy changes are not announced and frequently, a stated policy is not necessarily reflected immediately in action. Therefore, the non-trivial task is to infer the lags in central banks behavior purely inferred from data. In order to do that, we borrow a tool from geophysics literature in the form of complex Hilbert principle component analysis, which turns out to be quite useful for ranking countries in terms of their relative policy stances regarding responsiveness toward output and inflationary fluctuations.

The literature on time-varying behavior of the central bank is quite large. Romer and Romer (1989) described shifts in monetary policies, which were often deliberate and had real effects. Orphanides (2004) noted that in the relative importance of inflation and output to the central banks have changed dramatically over last five decades. In the 1970s, the US central bank was more concerned about output whereas slowly the focus shifted towards attaining low inflation. Another landmark event was the operational independence of the Bank of England to conduct monetary policy in the United Kingdom in 1997, with the stated objective of attaining a low inflationary environment. Similarly, Canada and New Zealand adopted explicit inflation targeting in the 1990s. However, a bigger issue for many other countries is that often they are affected by the policy stances of their peers. In other words, their own central banks adopt policy stances taken by the peer group, that is not necessarily a response to domestic economic development. Kannan (1999) for example,

[4]Unless in the time of extreme measures like the financial crisis where the policies were deliberately implemented to fight the recession.

discussed relative merits and de-merits of adoption of a similar policy stance by the Indian central bank, Reserve Bank of India. Eventually, in 2016 the Indian central bank adopted an explicit inflationary target. Thus there seems to be two features of the evolution of the central banks' policy stances. One, each country's central bank has their own trajectory of policy stances. Two, for various reasons, some countries were early adopters of specific agendas and some were late, creating a lead-lag relationship between countries. The main contribution of this paper is to provide a method to statistically infer this leading and lagging relationships.

Empirical literature on time-varying coefficients of the Taylor rule has focused on various modeling specifications. A large part of the literature focuses on regime-switching behavior, see e.g. Debortoli and Nunes (2014) and Assenmacher-Wesche (2006). Chen et al. (2017b) showed in a DSGE set-up that the conservatism regarding inflation targeting by the US Fed decreased substantially during times of financial turbulence. Givens and Salemi (2015) used a similar DSGE set up to show that the central bank's uncertainty regarding state of the economy in the real time, plays an important role under discretionary policy regime. In particular, the inferred policy stances evolve substantially over time. Dennis (2004) provided a complementary analysis by modeling the central bank's objective function in a NK-DSGE set up and showed that inflation targeting behavior has changed drastically over time. Boivin (2006) and Boivin and Giannoni (2006) described gradual changes in the US monetary policy stances along with their effectiveness (see also Orphanides and Williams (2005), Kim and Nelson (2006), Favero and Rovelli (2003)).

While a large chunk of the literature attempted to model and analyze US data, similar studies have been conducted for other countries central banks as well. For example, Kuzin (2006) documents sudden large shifts in the weights assigned to inflation aversion by the Bundesbank. Chen et al. (2017a) analyzed monetary policy of the Euro zone in a DGSE framework and showed that ECB maintained conservatism in line with the earlier policy framework of the Bundesbank (see also Rivolta (2018)). Literature on cross-country perspective of Taylor rule with time-varying coefficients is scarce. Tachibana (2004) provided a comparative picture of the preferences of central banks of Japan, US and UK and show that UK had the highest aversion towards output fluctuations while all three countries generally disliked fluctuations in inflation. Our paper falls into this category with a much more comprehensive coverage of countries.

We have adopted a simple state-space model of backward-looking Taylor rule with time-varying coefficients to model the central banks' preferences. Since we want to compare among a large set of countries, the data availability restricts the form of Taylor rule that we can estimate. For estimating more complicated versions of the rule, the set of countries will need to be reduced substantially. Once we extract the time-varying coefficients through a Kalman filter (see Harvey (1990) and Hamilton (1994) for textbook treatments), we utilize complex Hilbert principle component analysis to find the aggregate phase difference of the countries' coefficients in a complex domain.

The CHPCA technique is non-parametric in nature and provides a very clean picture of relative dynamics of multiple time series. First, we complexify the time series with a Hilbert transform, which allows us to map the evolution of the time series in the complex plane. Then by conducting eigendecomposition of contemporaneous correlation matrix of multiple time series, we can extract the phase differences As far as we know, this methodology was originally developed in geophysics literature (see Rasmusson et al. (1981), Horel (1984) and Barnett (1983)). Vodenska et al. (2016) applied this method on financial data by adopting it from the geophysics and signal processing literature. In their research monograph, Aoyama et al. (2017) described the technique in the context of Fourier transform in an elaborate manner and showed applications to sector-level production data as well as financial index data. Our discussion of this method is based on Aoyama et al. (2017) as they provided an intuitive description based on Fourier transform. To complement the discussion, we review the interpretation of the Hilbert transform in the context of Cauchy integral in a complex domain, in App. 4.1.

The rest of the paper is structured as follows. In Sec. 2, we describe the data (Sec. 2.1) and the methodology. In particular, Sec. 2.2 describes the state-space estimation of the time varying coefficients attached to inflationary and output fluctuations and Sec. 2.3 describes the application of the complex Hilbert PCA to multivariate data. After developing the tools and techniques, we apply them to a set of countries comprising developing as well as developed ones (Sec. 3). We describe the relative leading and lagging behavior of a mix of countries between 1980-2007 and only OECD countries (both in Sec. 3.1). Then we consider a set of countries in the post-crisis period which did not face the zero lower bound (Sec. 3.2). Sec. 4 summarizes and concludes the paper.

2 Empirical Data and Methodology

In this section, we first describe the data we have analyzed and then we describe the methodology. In particular, we elaborate the discussion on complex Hilbert principle component analysis in Sec. 2.3. Further discussion on the definition and existence of the integral can be found in App. 4.1.

2.1 Data Description

For the empirical analysis of the proposed framework, we need country-wise time series data of the central bank policy rate, inflation and GDP growth rate. Since not all countries provide historical data, we have chosen a set of countries by making sure that the set of countries cover both developed and developing ones along with having full set of required data. In table 1, we provide a list of all countries considered and their corresponding short forms that were used in diagrammatic representations. We use Datastream database (Source: Oxford economics) to get these variables. Time frame of our first analysis is from 1981 to 2007[5]. Central bank policy rate used in this study is the policy interest rate set by the respective central banks. For inflation, we use GDP price deflator (quarter on quarter). We obtained quarterly constant price seasonally adjusted GDP data from the Datastream and calculate GDP growth rate as annual percentage change in the quarterly data. Table 2 summarizes details of these variables and list of the countries used in the first analysis. For robustness, We use slightly different data set of the OECD countries for the period 1981 to 2007. We rely on OECD website for the inflation and GDP growth data. For the inflation, we use CPI inflation measured as the annual growth rate of the CPI index.[6] For the GDP growth rate we use sum of the quarterly GDP growth rate[7] of the last four quarters. For central bank policy rate, we primarily rely on the BIS (Bank of international settlements).[8]. For the countries, for which BIS does not provide the policy rate rate data for the above-mentioned period, we use short term interest rates data provided by OECD[9] Table 4 provides details of the data used in this analysis. Finally, we also use the same framework for post crisis period (2010-2018) for a new sample of countries, which did not have zero or close to zero interest rates in that period. Table 4 summarizes the details of the data used in this analysis.

2.2 Time-Varying Coefficients of Taylor Rule

First, let us describe the Taylor rule. The essential idea is that the central banks can use the policy rate i.e. the nominal interest rate, as a response to inflationary gap and output gap in the economy:

$$i_t = \Pi_t + r_t^* + \beta_\pi(\Pi_t - \Pi^*) + \beta_g(y_t - y_t^*) \quad (1)$$

[5]We restrict our analysis till 2007. Post 2007, interest rate in the USA and most of the European countries touched zero lower bound or went marginally negative. Hence, it was not possible for the central bank to implement monetary policy response function mandated by the Taylor rule.

[6]Source: https://data.oecd.org/price/inflation-cpi.htm

[7]Source: https://data.oecd.org/gdp/quarterly-gdp.htm

[8]https://www.bis.org/statistics/cbpol.htm

[9]https://data.oecd.org/interest/short-term-interest-rates.htm

where i_t denotes policy rate, Π denotes inflation, r_t^* denotes assumed long-run interest rate and y_t denotes income. In order to model the time-varying coefficients, we have used the following backward-looking variant of Taylor rule augmented with an error term:

$$i_{t+1} = \alpha + \beta_{\pi t}\Pi_t + \beta_{gt}g_t + \epsilon_{t+1}, \tag{2}$$

where α is a constant and g_t denotes the growth rate in income. In principle, one can use more complicated variants, but this specification suffices for our purpose.

2.2.1 State-Space Modeling of Hawkish Stance of the Central Banks

We utilize a standard Kalman filter to extract the time series of $\beta_{\pi t}$ and β_{gt}. For the sake of introducing the notation, let us denote the state variables by x_t and the observed variables by y_t. Then a general state space formulation can be written as

$$\begin{aligned} x_t &= \mu + \Gamma x_{t-1} + \phi_t \\ y_t &= \alpha + \gamma z_t + \Theta_t x_t + \chi_t \end{aligned} \tag{3}$$

where the error terms are independent and identically distributed as follows

$$\begin{bmatrix} \phi_t \\ \chi_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_\phi & 0 \\ 0 & \Sigma_\chi \end{bmatrix}\right)$$

and z_t are exogenous factors. There is a slight modification of the standard Kalman filter. Note that the coefficient matrix Θ_t here is time-dependent. We will discuss below why it is important. Essentially, this is the form that allows us to estimate a state-space model with stochastically varying coefficients.

In order to model the evolution of bank's hawkishness, we define the state variables as

$$x_t = \begin{bmatrix} \beta_{\pi t} \\ \beta_{gt} \end{bmatrix}$$

and the observed variable is

$$y_t = \begin{bmatrix} i_t \end{bmatrix}.$$

We restate the above state space model in the following way (we set $\gamma = 0$ in Eqn. 3):

$$\begin{bmatrix} \beta_{\pi t} \\ \beta_{gt} \end{bmatrix} = \mu + \Gamma \begin{bmatrix} \beta_{\pi(t-1)} \\ \beta_{g(t-1)} \end{bmatrix} + \phi_t$$

and

$$i_{t+1} = \alpha + [\Pi_t \ g_t] \begin{bmatrix} \beta_{\pi t} \\ \beta_{gt} \end{bmatrix} + \chi_t.$$

This model can be estimated using the standard procedure of Kalman filter. Textbook treatments can be found in Hamilton (1994). By estimating the model on interest rate, inflation and GDP growth rate data for all countries, we obtain a pair of estimated time-series for the time-varying coefficients reflecting the monetary policy stances, viz. $\{\hat{\beta}_{\pi it}, \ \hat{\beta}_{git}\}$ for all $i \in N$. In the next section, we need to introduce more notations to deal with the Hilbert transform of these estimated time-series pairs. To simplify the exposition, we refer to the estimated time series without the hat notation on β.

2.3 Complex Hilbert Principle Component Analysis and Phase Differences

From the Kalman filter, we have extracted a sequence of $\{\beta_{\pi it}, \beta_{git}\}_{t \in T}$ for each country ($i \in N$). To fix the notation, let us denote the time series pair for the i-th country as $\{\beta_{\pi it}, \beta_{git}\}_{i \in N, t \in T}$. Now we separately apply Hilbert transform to analyze the leading and lagging behavior of the two time series.

For the description of the methodology, let us consider the time series of measure of hawkishness $\{\beta_{\pi it}\}$ of the i-th country. First, we construct the Hilbert transformation of the time series:

$$\mathscr{H}(\beta_{\pi it}) = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{\beta_{\pi iu}}{t-u} du, \tag{4}$$

where PV denotes the Cauchy principle value of the integration. We can rewrite the above definition as[10]

$$\mathscr{H}(\beta_{\pi it}) = \frac{1}{\pi} \lim_{\epsilon \to 0^+} \left(\int_{t-1/\epsilon}^{t-\epsilon} \frac{\beta_{\pi iu}}{t-u} du + \int_{t+1/\epsilon}^{t+\epsilon} \frac{\beta_{\pi iu}}{t-u} du \right). \tag{5}$$

However, this is an integration over continuous domain. For numerical calculations, the approach proposed by Aoyama et al. (2017) is to consider the discrete counterpart of Fourier transform following Barnett (1983). A discretized Fourier transform would be given as

$$\mathscr{F}(\beta_{\pi it}) = \sum_{t=0}^{T-1} \beta_{\pi it} exp(-2i\pi kt/T). \tag{6}$$

Note that we can easily recover the original time series by simply using the inverse Fourier transform:

$$\beta_{\pi it} = \frac{1}{T} \sum_{t=0}^{T-1} \mathscr{F}(\beta_{\pi it}) exp(2i\pi kt/T). \tag{7}$$

Hilbert transform shifts the phase by $\pi/2$ in the complex plane and can be expressed as

$$\mathscr{H}(\beta_{\pi it}) = \frac{1}{T} \sum_{t=0}^{T-1} \mathscr{F}(\beta_{\pi it}) exp(2i\pi kt/T - i\pi/2) sgn(k - \frac{T}{2}). \tag{8}$$

Here the sign function is defined as

$$sgn(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0. \end{cases}$$

Now, we create a complexified time series of the estimated $\beta_{\pi it}$ for each country:

$$\gamma_{it}^{\pi} = \beta_{\pi it} + i\mathscr{H}(\beta_{\pi it}) \tag{9}$$

and similarly for estimated β_{git}:

$$\gamma_{it}^{g} = \beta_{git} + i\mathscr{H}(\beta_{git}). \tag{10}$$

In order to visualize the dynamics of the Hilbert transformed series in the complex space, we need to compute the Z-score for each of the series. Therefore, given each series $\{x_t\}$, we have to compute $\{y_t\}$ such that $y_t = \frac{x_t - \bar{x}}{\sigma_x}$ where $\bar{x}$ denotes the sample mean and σ_x denotes the sample standard deviation. From the resulting time series, we can now construct the complex correlation matrices:

$$\mathscr{C}^{\pi} = \frac{1}{T} \Gamma^{\pi} \Gamma^{\pi\prime} \quad \text{and} \quad \mathscr{C}^{g} = \frac{1}{T} \Gamma^{g} \Gamma^{g\prime} \tag{11}$$

where x' denotes the complex conjugate of matrix x. Here, we note that both $\mathscr{C}^{\pi}$ and $\mathscr{C}^{g}$ are Hermitian[11], i.e. self adjoint matrices. Hermitian matrices have a nice property that makes them amenable to principle component analysis quite easily. We utilize the following standard result from linear algebra.

[10] See App. 4.1 for a brief description based on Cauchy integration.

[11] A matrix is *Hermitian* if it is a complex square matrix such that it is equal to its own conjugate transpose.

Result 1 *For every Hermitian matrix in $\mathcal{C}^n$, there exists an orthonormal basis consisting of eigenvectors of the Hermitian matrix and all eigenvalues are real.*

Therefore, we can carry out the following eigendecompositions:

$$\mathscr{C}^{\pi} = \sum_{i=1}^{N} \lambda_i^{\pi} e_i^{\pi} e_i^{\pi\prime} \quad \text{and} \quad \mathscr{C}^{g} = \sum_{i=1}^{N} \lambda_i^{g} e_i^{g} e_i^{g\prime} \tag{12}$$

where $\lambda_i^{\pi} \in \mathbb{R}$ and $\lambda_i^{g} \in \mathbb{R}$ for all $i \in N$. We rank order all the eigenvalues and extract the dominant ones. Without loss of generalization, let us denote them by λ_1^{π} and λ_1^{g}; also, let us denote the corresponding eigenvector by e_1^{π} and e_1^{g} respectively.

As we will describe below, the dominant eigenvectors e_1^{π} and e_1^{g} contain information about the lead-lag relationship. However, before that we need to ensure that the result is not outcome of random noise. For that purpose, we consider the spectral radius given by λ_1^{π} and λ_1^{g} and compare them with those obtained from *i.i.d.* noise. We utilize the following result from random matrix theory (see Mehta (2004) and Aoyama et al. (2017)).

Result 2 (Spectral radius for complexified time series) *Let $N \to \infty$ and $T \to \infty$ such that $T > N$ for a matrix D of size $N \times T$ such that $D \sim N(0, I)$. Let us use κ to denote the ratio $2N/T$. Let W be the covariance matrix of D, i.e. $W = DD'$. Then the spectral radius $\lambda_+ = (1 + \sqrt{\kappa})^2$.*

Note that for correlation matrix constructed from real time series, the corresponding κ would be only N/T which is the standard Marčenko-Pastur bound (see Marčenko and Pastur (1967), Sengupta and Mitra (1999) and Mehta (2004)). We confirmed that in all cases, the dominant eigenvalues were larger than the theoretical bounds for random matrices.

2.3.1 Phase Differences and Leading Relations

Consider one leading eigenvector e_1^{π}. Clearly it has N number of complex elements of the form

$$e_1^{\pi} = \begin{bmatrix} x_1^{\pi} + iy_1^{\pi} \\ x_2^{\pi} + iy_2^{\pi} \\ \vdots \\ x_N^{\pi} + iy_N^{\pi} \end{bmatrix}.$$

Then on the complex plane, we plot the sequence of points as $\{x_1^{\pi},\ y_1^{\pi}\}$, $\{x_2^{\pi},\ y_2^{\pi}\}$, ... , $\{x_N^{\pi},\ y_N^{\pi}\}$. Comparison of the respective angles ($arctan(y_k^{\pi}/x_k^{\pi})$) determine the phase for the k-th country. The same mechanism works for e_1^{g} as well. The pairwise phase difference can be easily quantified by the difference between phases of a pair of countries.

Once we calculate the phase differences, we have to analyze who is the leader and who is the follower. For normalization, we will compute everything with respect to US, i.e. all central banks' dynamics would be benchmarked with respect to that of US. From the x-axis, one can measure the angle of the point with coordinates $x_k^{\pi},\ y_k^{\pi}$ in the anti-clockwise direction. Let us call it θ_k. If $\pi > \theta_k > 0$, then the corresponding country is lagging US. Conversely, if $-\pi < \theta_k < 0$ (measured clockwise from the x-axis), then the country is leading US. In order to see it through a simple example consider the following.

A simple example with sinusoids: Consider Fig. 1. In the left panel, we have plotted time evolution of three simple sinusoidal curves, viz. $sin(t - \frac{\pi}{4})$, $sin(t)$ and $sin(t + \frac{\pi}{4})$. From the nature of the curves, it is evident that there are pairwise phase differences of $\pm\pi/4$ from the

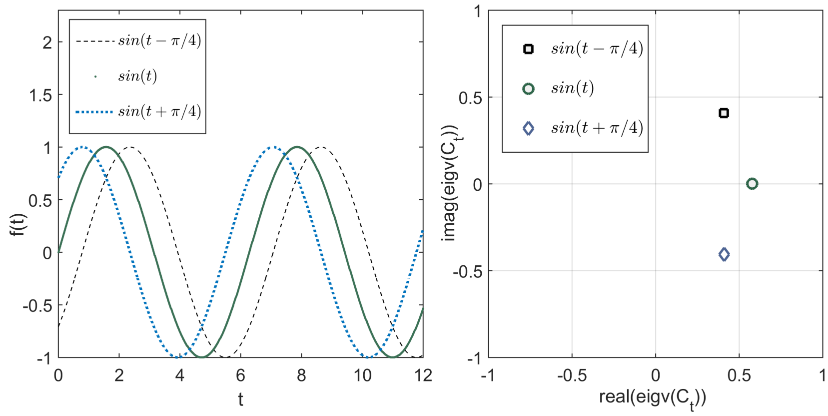

Figure 1: (color online) *Left panel*: Three simple sinusoidal curves have been drawn, viz. $sin(t-\pi/4)$, $sin(t)$ and $sin(t+\pi/4)$. Clearly the phase difference between consecutive pairs are exactly $\pi/4$. *Right panel*: After application of complex Hilbert principle component method, we recover the phase difference from the cross-correlation matrix of the three time series. The empirically found phase difference is very close to the theoretical value, i.e. $\pi/4$. See text for details.

$sin(t)$ curve. If one considers the cross-correlation of these curves (with suitable discretization), the leading and the lagging behavior cannot be inferred. So we have complexified the three series using Hilbert transformation and conducted eigendecomposition of the complex correlation matrix as stated above. The dominant eigenvector has three complex elements which are plotted on the right panel. The x-axis denotes real part of the elements of the dominant complex eigenvector and the y-axis denotes the imaginary parts of the same. The phase difference on the complex plane is clearly $\pm\pi/4$. Thus this technique is evidently useful to elicit leading and lagging behavior from multiple time series through phase differences.

2.3.2 Magnitude of the Components of the Dominant Eigenvector

The leading eigenvector (say, e_1^π) is an N-dimensional complex vector. We can express the elements in polar coordinates as

$$e_{1i}^\pi = r_i.exp(i\theta_i) \tag{13}$$

for $i \in N$, where

$$r_i = \sqrt{(x_i^\pi)^2 + (y_i^\pi)^2} \quad \text{and} \quad \theta_i = arctan\left(\frac{y_k^\pi}{x_k^\pi}\right). \tag{14}$$

Therefore, distance of points from the origin also dictates magnitude of components in the dominant eigenvector. Note that since of eigenvector is the result of decomposition of the complex correlation matrix, the distance of each point from the origin also dictates the contribution of the corresponding country to the aggregate correlation structure. Intuitively, countries lying closer to the origin contribute less the aggregate correlated movements of the times series, than the ones which are far from the origin. We will show below that the developing countries had much less presence in the central banks' reactions to tackle inflation, than the developed ones (Fig. 3).

2.4 Description of the Algorithm

Now, we provide an intuitive construction of the methodology described above through Fig. 2. To fix ideas, let us consider US. We observe three time series, viz. US gdp growth rate, US inflation rate and the policy rate. Panel (a) plots the three time series. By applying the state space model

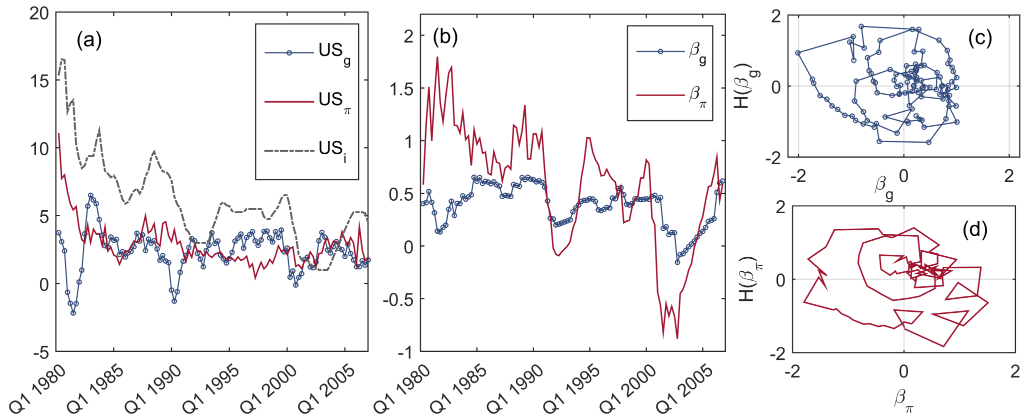

Figure 2: (color online) *Panel (a)*: Three time series of GDP growth rate, inflation and policy rate of US (quarterly data; source: OECD). *Panel (b)*: After application of Kalman filter, we extract the evolution of the *unobserved* variables β_π and β_g. *Panel (c) and (d)*: Dynamics of estimated β_π and β_g in the complex space. On the y-axis, we have plotted Hilbert transformation of the underlying time series.

in the form of Kalman filter, we extract the inferred time series $\{\beta_{\pi t}\}$ and $\{\beta_{gt}\}$ for US. These two series are plotted in panel (b). For each of the series, we construct the complexified version in the form of Eqn. 9 and 10. In panels (c) and (d), we have plotted the Hilbert components of β_{gt} and $\beta_{\pi t}$ as functions of β_{gt} and $\beta_{\pi t}$. Another way to visualize it would be to consider the time evolution of the imaginary component $imag(\gamma^x_{it})$ with respect to the real component $Re(\gamma^x_{it})$ for $x = \pi,\ g$.

3 Phase Difference in the Central Banks' Reaction Functions

Now we are in a position to analyze the leading and lagging relationships between all pairs of countries. First, we consider a set of countries with representation from developed as well as developing countries. In the later sections, we consider other sets of countries as well. Before getting into the analysis, we note that from the period of the *great recession*, many developed countries started having close to zero policy rates. However, it is evident that output growth rate and inflation was not necessarily close to zero. Hence, such a period is not good candidate for studying the central banks' behavior through Taylor rule. This is also the period when the central banks started looking for unconventional monetary policy measures (Gertler and Karadi (2011)). Hence, our first analysis is conducted on the data before the crisis period.

3.1 The Era of Great Moderation

We analyze the data of a set of countries from 1981 to 2007. This timing covers the so-called period of *great moderation* which spans roughly from mid-1980s to the crisis period. During this time, volatility of the business cycles decreased substantially and the application of Taylor rule-type behavior has been argued to have played a role in following macroeconomic stabilization policies (Bernanke (2004)).

As has been described in Sec. 2, we carry out a state-space estimation of the time varying coefficients (Sec. 2.2) of the monetary policies of different countries from 1981 to 2007. Then we analyze the relative dynamics of the $\{\beta_{\pi it}\}$ and $\{\beta_{git}\}$ coefficients for all $i \in N$. After we employ

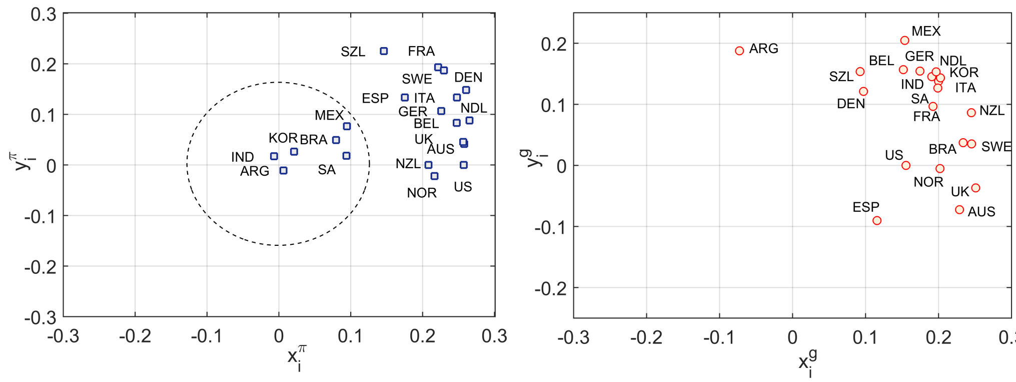

Figure 3: (color online) Plots of the elements of the dominant eigenvector of the complex correlation matrix during the period of great moderation (1981-2007). The elements have been rotated such that US has only real value and hence, lies on the x-axis. The phase difference can be measured from the x-axis. *Left panel*: Components for inflationary weights (β_π) of different countries. Clearly, developing countries have lower presence in the dominant eigenvector as indicated by clustering closer towards the origin. *Right panel*: Components for output gap weights (β_g) of different countries. *Source of data*: Datastream.

the complex Hilbert principle component analysis (Sec. 2.3), we plot the elements of the dominant eigenvector constituting the principle component of the complex correlation matrix, in Fig. 3. The *left panel* shows phase differences of $\{\beta_{\pi it}\}$ and the *right panel* shows phase differences of $\{\beta_{git}\}$. In both diagrams, we have normalized the data for US such that US lies on the x-axis. All other points have also been rotated by the same angle. Therefore, the diagrams allows us to analyze the relative phase differences with US as a benchmark. Note that since all points have been rotated by the same angle, the relative phase differences remain unaffected.

Two features are prominent through the analysis of the inflationary coefficient $\{\beta_{\pi it}\}$ in Fig. 3. First, the developing countries have much smaller presence in the joint dynamics of the $\{\beta_{\pi it}\}$ as is indicted by the low magnitude of all developing countries. Practically, they form a separate cluster close to the origin, completely segregated from the developed countries. Second, we see that almost all countries have a positive phase difference with respect to US. The implication is that the dynamics of $\{\beta_{\pi USt}\}$ leads those of all other countries. In other words, for inflationary reactions, the US Fed moves ahead of almost all other countries.

In contrast, the diagram for $\{\beta_{git}\}$ does not show either of the two features. Developed and developing countries can not be differentiated as separate clusters and the lead-lag behavior also does not show any clear pattern. From the analysis, we see that Spain is the leader and Argentina has the highest lag.

3.1.1 Robustness: OECD Countries with Yearly Data

For checking robustness of the methodology with respect to data, we considered only OECD countries with a different set of variables. The results are provided in Fig. 4. As in the earlier case, the *left panel* shows phase differences of $\{\beta_{\pi it}\}$ and the *right panel* shows phase differences of $\{\beta_{git}\}$.

We see a behavior of the phase plot of $\{\beta_{\pi it}\}$ series consistent with that in Fig. 3. Here all countries are more or less equally distant from the origin. For a clearer comparison, we have plotted a dotted circle around the origin. Again US takes the leading role, closely being followed by Australia, Canada and so on. In the *right panel*, there does not seem to be any discernible pattern. Therefore, even with this set of countries, relative dynamics of $\{\beta_{\pi it}\}$ is more informative than that of $\{\beta_{git}\}$.

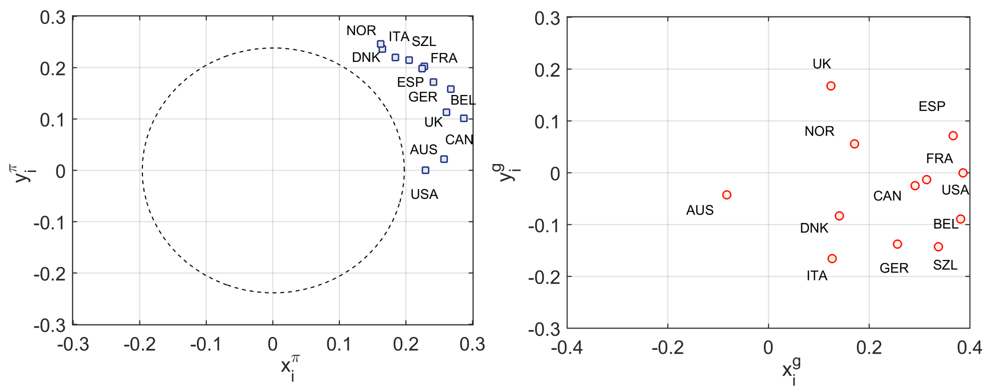

Figure 4: (color online) Plots of the elements of the dominant eigenvector of the complex correlation matrix during the period of great moderation (1981-2007), only for the OECD countries. The elements have been rotated such that US has only real value and hence, lies on the x-axis. The phase difference can be measured from the x-axis. *Left panel*: Components for inflationary weights (β_π) of different countries. The presence of the circle shows similar contribution of all countries. *Right panel*: Components for output gap weights (β_g) of different countries. *Source of data*: OECD.

Here we should note one curious case of Norway. In terms of $\{\beta_{\pi it}\}$, Norway seems to almost completely switch its behavior (leading US in Fig. 3 and lagging behind almost every country in Fig. 4). This difference can be attributed to low correlation of the Norwegian variables used in these two different analysis.

3.2 Post-Crisis Dynamics of Hawkish Stance

Finally, we analyze how the central banks' behavior evolved in the aftermath of the great recession. During this period, a number of developed countries had close to zero or even less than zero policy rates. We discard all such countries hitting the zero bound. In the remaining sample, we consider mostly developing countries along with a few developed countries. The analysis is shown in Fig. 5. As above, the *left panel* shows phase differences of $\{\beta_{\pi it}\}$ and the *right panel* shows phase differences of $\{\beta_{git}\}$. For the purpose of standardization, we have rotated all points such that India is on the x-axis and hence, the phase differences can be measured with respect to India.

Interestingly, within this set of countries, the presence of different countries in the dominant eigenvector measured through the magnitudes, are very similar to each other. For inflationary coefficients, China is leading everyone else, followed by Norway, Australia, New Zealand and India. Other countries are lagging behind India with Brazil having the largest phase difference. For output gap coefficients, Mexico is leading country with Turkey having the largest phase difference with India indicating biggest lag.

3.3 Discussion of the Modeling Methodology and Data Selection

Here we briefly discuss the modeling specification that we utilized. We first note that for estimating the central banks' behavior we have utilized a very simple backward-looking Taylor rule. In the literature on time-varying coefficients of Taylor rule, often the estimation is carried out on a more involved specification of the decision rule (see the References for a collection of such work). We have refrained from doing that for two reasons. One, the main reason is data limitation. Since we are analyzing a large and diverse set of countries, getting perfectly comparable data without missing entries is difficult. Hence, we have chosen a simple but standard specification so that we can estimate

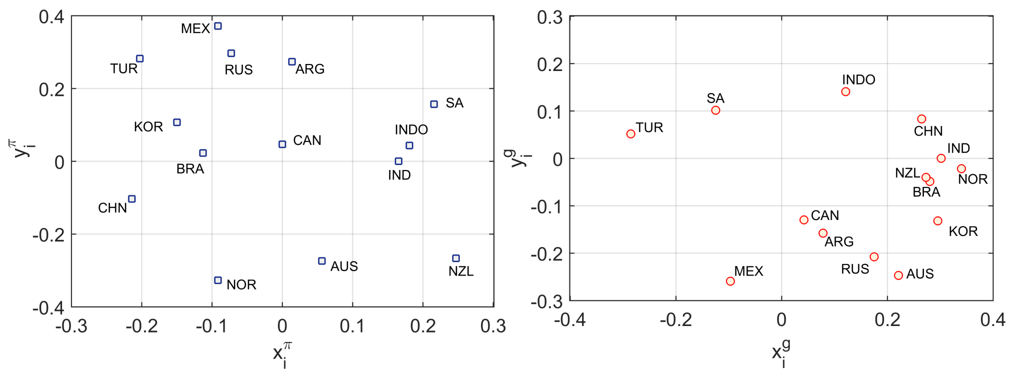

Figure 5: (color online) Plots of the elements of the dominant eigenvector of the complex correlation matrix after the great recession (2010-18). The elements have been rotated such that India has only real value and hence, lies on the x-axis. The phase difference can be measured from the x-axis. *Left panel*: Components for inflationary weights (β_π) of different countries. *Right panel*: Components for output gap weights (β_g) of different countries. *Source of data*: Datastream.

the model consistently across all countries and compare results. Second, the main contribution of the paper is to introduce the complex Hilbert PCA algorithm on the inferred $\beta_{\pi it}$ and β_{git} for all $i \in N$. Therefore, we discuss this method and emphasize its applicability in the present context.

We take this opportunity to also discuss the value addition of complexification over regular spectral analysis of correlation matrices. We note that with standard spectral decomposition, one can only get the magnitude of contribution of different countries in the cross-correlation matrix. Through complexification, we get not only magnitude, but also the phase difference. This is actually a non-trivial point to note that the phase difference is elicited from the equal-time cross-correlation matrix. Thus this methodology is superior in providing a more complete picture of relative dynamics of multiple time-series evolving in parallel.

4 Summary and Conclusion

Taylor rule is a very useful reduced form representation of a central bank's policy rule that relates the dynamics of the nominal interest rate to fluctuations in output and inflation. After the publication of the seminal article by Taylor (1993), a large literature has been developed that studied the scope and applicability of the Taylor rule in various forms across different countries (although, mostly concentrating on US). A consensus has developed that there exists clear empirical evidence of time-varying behavior of the central bank with respect to the reaction towards inflationary and output fluctuations. After the success of the developed world in having a low inflationary regime, some developing countries, e.g. India, adopted a similar approach towards monetary policy. In this paper, we propose an empirical and non-parametric method to analyze the joint evolution of the time-varying behavior of the central banks of a set of countries and infer a general ordering of adoption of policy responses by central banks.

In this paper, we utilize a state-space model to infer time-varying behavior of the coefficients assigned by a central bank from observed data on interest rate, output growth and inflation. In particular, following the literature we use a Kalman filter to get the time series of the coefficients reflecting policy stances of the central bank of a set of countries comprising both developed and developing ones. The we employ complex Hilbert principle component analysis (Aoyama et al. (2017)) that was originally developed in the geophysics literature. Through this method each time series is complexified and then the time-domain information is mapped into a complex plane in terms of rotation. By conducting eigendecomposition of the equal-time comovements of the rotational dynamics, we can extract relative presence of countries in the aggregate comovoments and through phase differences, we can find leading and lagging behavior.

Thus, we provide a methodology of finding relative dynamics of central banks' policy shifts from a purely data-driven approach. This might have important policy consequences by providing a way to find comparative dynamics of central banks. As Mishra and Rajan (2016) pointed out in the aftermath of 2007-09 crisis, aggressive monetary policies pursued by central banks will have spillover effects of other countries central banks' policy responses. Our method provides a way to quantitatively analyze such behavior. In particular, they state "*Aggressive monetary policy actions by one country can lead to significant adverse crossborder spillovers on others, especially as countries contend with the zero lower bound. If countries do not internalize these spillovers, they may undertake policies that are collectively suboptimal*" and argue for global coordination as an welfare enhancing tool for monetary policies. The method proposed in this paper allows us to quantify the leading and lagging behavior of the central banks in terms of policy responses and therefore, quantifies the lack of coordination across central banks. Therefore, this tool can be applied to analyze coherence in monetary policies across countries to achieve coordination at a global level.

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Appendix

4.1 Hilbert transform and the complex plane

Here we briefly motivate the discussion on Hilbert transform using Cauchy integration. The following discussion is partially based on Pipes and Harvill (2014) and we review the description of Hilbert transformation. This appendix solely discusses the context and existence of the transform.

Consider an *analytic* function $f(z)$ defined over region Θ. Due to the analytic nature of the function, it is single-valued, continuous and there exists definite derivative at every point in R. The Cauchy integral theorem asserts that

$$\oint_C f(z)\,dz = 0, \tag{15}$$

where C is an arbitrary closed path (piecewise smooth) within Θ. This can be used to find the value of an analytic function at a point from the path which surrounds it. In particular, let us continue with the example. So $f(z)$ is analytic in Θ including a point $z = a$. As in the earlier case, C is a closed path around it. The Cauchy integral result is that

$$f(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{z-a}\,dz. \tag{16}$$

A generalization of this result is given as follows:

$$\oint_C \frac{f(z)}{z-a}\,dz = \begin{cases} 2i\pi f(a) & \text{for } a \text{ inside closed path } C \\ 0 & \text{for } a \text{ outside closed path } C. \end{cases}$$

If a is exactly on the path C, one can consider consider a new path C', with a simple pole at a and a path C_ϵ with distance ϵ from a. Then one can have

$$\lim_{\epsilon\to 0}\oint_{C'} \frac{f(z)}{z-a}dz = PV\oint_C \frac{f(z)}{z-a}dz + \lim_{\epsilon\to 0}\oint_{C_\epsilon} \frac{f(z)}{z-a}dz, \tag{17}$$

which can be shown to yield the result

$$PV\oint_C \frac{f(z)}{z-a}dz = i\pi f(a). \tag{18}$$

In order to link it to Hilbert transform, we consider a closed path comprising the x-axis and a semicircle in the upper half-plane. Under the conditions that $f(z)$ is analytic and the contribution of the semi-circle tends to zero when the radius tends to infinity, we have

$$PV\int_{-\infty}^{\infty} \frac{f(z)}{z-a}dz = i\pi f(a). \tag{19}$$

Finally, if we write $f(z) = r(z) + ic(z)$, then

$$r(z) = \mathscr{H}(c(z)) \quad \text{and} \quad c(z) = -\mathscr{H}(r(z)). \tag{20}$$

4.2 Details of the data

Table 1: Countries and their corresponding codes

Country	Code	Country	Code	Country	Code
Argentina	ARG	Indonesia	INDO	Spain	ESP
Australia	AUS	India	IND	South Africa	SA
Belzium	BEL	Italy	ITA	Sweden	SWE
Brazil	BRA	Korea	KOR	Switzerland	SZL
Canada	CAN	Mexico	MEX	Turkey	TUR
China	CHN	Netherland	NDL	United Kingdom	UK
Denmark	DEN	Norway	NOR	USA	US
France	FRA	New zealand	NZL		
Germany	GER	Russia	RUS		

Table 2: Datastream (1981-2007)

Countries	ARG, AUS, GER, BEL, BRA, DEN‡, SPN, FRA, IND, ITA, KOR, MEX‡, NDL, NOR, NZL‡, SA, SWE, SZL‡, UK, US		
Variable	Source	Description	Frequency
Policy rate	Datastream (Oxford economics)	Central Bank policy rate	Quarterly
Inflation	Datastream (Oxford economics)	Price deflator GDP(QOQ)	Quarterly
GDP growth rate	Datastream (Oxford economics)	$g_t = \frac{y_t^q}{y_{t-4}^q} - 1$ where y_t^q is the quarterly constant price seasonally adjusted GDP	Quarterly

‡ For these countries, Datastream provides policy rate data from their respective government sources.

Table 3: OECD (1981-2007)

Countries	AUS, CAN, USA, FRA‡, GER‡, ITA‡, UK, BEL, SZL, ESP‡, DNK, SWE, NOR‡		
Variable	Source	Description	Frequency
Policy rate	BIS, OECD‡	Central Bank policy rate	Quarterly
Inflation	OECD	Consumer price index (CPI) inflation measured in terms of the annual growth rate in the index	Quarterly
GDP growth rate	OECD	$g_t = g_t^q + g_{t-1}^q + g_{t-2}^q + g_{t-2}^q$ where g_t^q is the percentage change in GDP (seasonally adjusted) from the previous quarter	Quarterly

‡ For these countries, BIS does not provide the policy rate data for the 1981-2007 period. We use short term interest rate data (provided by OECD) as an alternative for the policy rate for these countries.

Table 4: Datastream (2010-2018)

Countries	ARG, AUS, BRA, CHN‡, CAN, INDO, IND, KOR, MEX‡, NOR, NZL‡, RUS, SA, TUR		
Variable	Source	Description	Frequency
Policy rate	Data stream (Oxford economics)	Central bank policy rate	Quarterly
Inflation	Data stream (Oxford economics)	Price deflator GDP (Quater on Quarter)	Quarterly
GDP growth rate	Data stream (Oxford economics)	$g_t = \frac{y_t^q}{y_{t-4}^q} - 1$ where y_t^q is the quarterly constant price seasonally adjusted GDP	Quarterly

‡ For these countries, Datastream provides policy rate data from their respective government sources.