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Computerised decision aid for timetabling - a case analysis

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Abstract

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Timetabling is a large and complex problem. In the absence of a precise objective function, the solution process mostly aims at obtaining a feasible solution. There is a quality aspect of timetabling which is often not taken care of explicitly. This is due to the fact that the quality is largely the perception of the decision maker. This necessitates interaction of the decision maker during the generation of timetable through a computer. Case data of a timetabling problem have been analysed.

The special characteristics of the problem is the highly dense conflict matrix. The computer is used in aiding the decision maker to arrive at a desirable timetable.

Keywords. Timetabling, conflict matrix, multiple section grouping, class schedule.

1. Introduction

Timetabling is a regular activity to be performed prior to the commencement of each academic term or year. Studies on timetabling and related areas have been reported for the last forty years or so. There is a lot of interest in this subject even today [1,3,9]. The timetabling problem is of interest mainly due to its large size, complex requirements and varied nature [7]. A few survey papers dealing with studies on timetabling have also appeared [2,4-6].

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In simple words, timetabling is a class of scheduling problem, which deals with assigning a certain number of meetings of students and teachers to periods. Each of these meetings, apart from being attended by specific groups of students and a teacher (or teachers) also requires certain resources (commonly a meeting room and increasingly other resources like a projection system, computer lab, etc.). The scheduling has to be done considering the availability of resources and the achievement of certain other objectives. Approaches for the solution of the timetabling problem have been from pure simulation of the hand construction of a schedule to graph theoretic approaches, three-dimensional transportation models, quadratic assignment models etc. The complex requirements of timetabling often result in a large size problem when formulated mathematically. This becomes a great hindrance to solving the timetabling problem employing mathematical programming techniques. Due to this mathematical programming techniques were rarely used for timetabling till the 1970's. Availability of faster and larger computers along with better algorithms have resulted in some approaches to solving timetabling problems using mathematical programming techniques [8].

2. Nature of the timetabling problem

The timetabling problem has many conflicting requirements. Most of the modelling exercises try to take care of as many of the requirements as possible. In general the aim is to get a feasible solution without violating the constraints. In many cases the requirements are so conflicting that it is not possible to get a feasible solution without violating some of the constraints. In such a case some of the constraints are relaxed and are taken care of externally. Furthermore there is rarely any objective function in timetabling to be optimised, except the preferred periods for the meetings. However it can always be said whether one timetable is better than another one for the same given situation. This necessarily indicates some measure of desirability. But it is very difficult to quantify this aspect of desirability. There has been some attempt to partially quantify the desirability aspect of timetabling. This has been done by giving high weightage to the most desirable meeting-period combination, high weightage for preferred periods and the like. This is possible only to a very limited extent. As a result there is a quality aspect of timetabling which has not been fully considered in many of the modelling exercises of timetabling. This is largely due to the difficulty in explicitly expressing the measures of quality. The quality aspect of the timetabling has by and large remained the judgement of the decision maker. This calls for involvement of the decision maker at the scheduling stage. Accordingly in the present study the help of a micro computer based user friendly data base and other softwares have been taken to provide aid to the decision maker in arriving at a desirable timetable.

3. Description of the problem

The problem discussed here deals with the timetabling for a postgraduate programme in management. The programme is organised into three terms over the year. Each term is completely independent of each other so far as scheduling is concerned. The usual duration of a term is between ten and twelve weeks. In each term a certain number of elective courses are offered. The students have choice in terms of selection of the elective courses. There are about 180 students and they have complete freedom in terms of the selection of courses. A sample of the choices exercised by students is shown in Table 1. The timetabling is done after the students have completed the selection of elective courses.

The timetable has to take care of the following requirements (amongst others):

- Conflicts between the courses due to course choice exercised by the students,
- teachers preferences,
- availability of class rooms, etc.

Some of the courses have a very heavy registration and may have to be offered in multiple sections of 2 or 3. There are also some courses which have to be scheduled over two consecutive sessions. Similar additional requirements add to the complexity of the problem and make it more difficult to define quality explicitly.

4. Timetable problem formulation

The mathematical programming formulation of timetabling problems are in most cases a variant of the formulation by Tripathy [8]. The variations reflect the differing natures of the timetabling problems and the way the problem parameters are defined. In a special case of a timetabling problem we get a generalised assignment problem. We also get a 0-1 assignment problem with side constraints by suitably

Table 1. Student subject choice (0: subject not selected by the student; 1: subject selected by the student)

Student serial no.	Subject serial number													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
81	1	1	0	0	1	0	1	1	1	0	1	0	0	0
82	1	0	0	0	1	0	1	1	1	0	1	0	0	0
83	1	0	0	0	0	0	1	0	1	1	1	1	0	0
84	1	1	0	0	1	0	1	1	1	1	0	0	0	0
85	1	1	0	0	0	0	0	0	1	1	1	0	1	0
86	1	0	0	0	1	1	0	0	0	1	1	1	0	0
87	1	0	0	1	0	0	0	0	1	1	1	0	1	1
Total number of students registering in the subject	169	41	30	28	135	14	70	33	144	132	164	79	50	21

partitioning the constraint sets. The problem discussed in this study can be represented in the following way.

$$\text{(P)} \qquad \text{Maximise} \quad \sum_{i=1}^{NS} \sum_{j=1}^{NP} C_{ij} X_{ij},$$

$$\text{subject to} \quad \sum_{j=1}^{NP} X_{ij} = NSP_i, \quad i = 1, \ldots, NS, \qquad \text{(P1)}$$

$$\sum_{i=1}^{NS} X_{ij} \le NR, \quad j = 1, \ldots, NP, \qquad \text{(P2)}$$

$$\sum_{i \in T_l} X_{ij} \le 1, \quad j = 1, \ldots, NP, \quad l = 1, \ldots, \text{total no. of students}, \qquad \text{(P3)}$$

$$X_{ij} = \begin{cases} 1, & \text{if subject } i \text{ is scheduled in period } j, \\ 0, & \text{otherwise,} \end{cases}$$

where

- NS = total number of subjects,
- NP = total number of periods in a week,
- NSP_i = total number of sessions to be scheduled per week for subject i,
- NR = total number of classrooms available,
- T_l = subset of subjects chosen by student "l".

The constraint sets (P1) and (P2) can be suitably modified to represent a pure transportation problem. In this case the problem (P) becomes a transportation problem with side constraints. The constraint set (P3) is essentially the conflict matrix. The level of conflict in the problem discussed here is very high. This has resulted in the conflict matrix being very dense. Therefore no attempt has been made to solve problem (P) through mathematical programming employing Lagrangean relaxation or similar other techniques. Further such an approach does not take care of quality aspect of timetabling effectively. Accordingly in the present study the timetabling has been carried out in three steps.

Step 1. Generation of the conflict matrix.
Step 2. Multiple section grouping.
Step 3. Generation of a class schedule.

5. Generation of the conflict matrix

The conflict matrix essentially represents the courses which are in conflict and cannot be scheduled simultaneously in the same period. The constraint set (P3) takes care of not scheduling the conflicting subjects. The subset of subjects chosen by student "l", which is represented by T_l, are in conflict. The subsets for all the students (T_l for $l = 1$ to total number of students) together define the complete conflict matrix. The conflict matrix is symmetrical about the diagonal. Accordingly, usually only half the matrix is presented. The conflict matrix for the problem discussed here is presented in Table 2.

The registration data of each of the students containing the subject choice of the students, similar to Table 1, are used to generate the conflict matrix. Suppose a student has opted for subject no. 1, 2, 9, 10, 11 and 13 (student serial number 85 of Table 1), then these courses cannot be scheduled in parallel (or simultaneously) as the particular student (in this case student serial number 85) has to attend all these subjects. Accordingly all these subjects are in conflict amongst themselves. The matrix in Table 2 captures this data considering the subject choices of all the students. Any nonzero entry in the matrix indicates the number of students, who have chosen both the subjects. A zero indicates that no student has opted for both the subjects together and accordingly these subjects are not in conflict and these can be scheduled in parallel.

As can be seen from Table 2 the matrix is highly dense. Only three pairs of subjects are not in conflict and can be scheduled in parallel out of a possible 91 pairs of subjects being candidates for parallel scheduling.

A test of feasibility is carried out following the steps outlined in Appendix. In this particular case no more than three subjects are scheduled in parallel due to the limitation of availability of classrooms. Therefore the test of feasibility is carried out considering parallel scheduling of two subjects and parallel scheduling of three subjects. The conflict matrix being very dense, rarely feasibility is indicated by the test of feasibility. Two courses of actions are pursued in case of infeasibility before proceeding to generation of class schedule. The first course of action deals with multiple section grouping. This essentially is Step 2 of the timetabling. If a feasible solution is not indicated even after multiple section grouping, the second course of

Table 2. Conflict matrix (m_{ij} = entry in the cell (i,j); $m_{ij} = 0$, the subjects i and j are not in conflict; $m_{ij} \neq 0$, the subjects i and j are in conflict and the figure indicates the number of students selecting both the subjects; m_{ij} for $i = j$ indicates the total number of students selecting the subject)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	Subject
169	41	30	28	135	14	70	33	144	132	164	79	50	21	1
	41	3	9	34	1	19	7	34	32	39	15	9	3	2
		30	8	14	0	4	1	27	24	28	11	12	5	3
			28	16	0	5	1	22	20	28	9	11	4	4
				135	12	66	29	111	103	133	69	35	12	5
					14	6	4	10	7	14	8	0	1	6
						70	27	58	51	68	21	7	4	7
							33	28	21	31	6	2	2	8
								144	113	140	63	43	16	9
									132	128	61	37	15	10
										164	79	49	20	11
											79	21	10	12
												50	10	13
													21	14

action is undertaken. This action is to restrict the selection of courses by the students. In such a case efforts are made to see that the total number of students affected by this action is the least. This is identified by the lowest entry in the conflict matrix (modified after multiple section grouping) and the respective students are advised to alter their selection of courses, so as not to give rise to a new conflict between the subjects.

6. Multiple section grouping

The maximum size of a class section has to be restricted either due to the physical facilities or due to the nature of the subject requiring active interaction between the instructor and the students.

If the number of students opting for a subject is more than the maximum size of the class section, the subject is to be offered in multiple sections. The number of sections for a subject is decided on the basis of the number of students opting for the subject and the maximum permissible size of the class section. The formation of the multiple sections is done in a way that facilitates parallel scheduling of different subjects. For instance, subject 5 has a registration of 135 students and subject 10 has a registration of 132 and there are 103 students who have registered for both the courses. So subject 5 and subject 10 are in conflict and these cannot be scheduled in parallel. However due to the large registration both these subjects are to be offered in two sections, say 5A, 5B and 10A, 10B. The allocation of students to these sections are made in such a way that the students in section 5A and opting for subject 10 also will necessarily be assigned to section 10B. Similarly students in section 5B and opting for subject 10 also will necessarily be assigned to section 10A. It will now be possible to schedule 5A in parallel with 10A and 5B with 10B, thereby requiring two periods less for timetabling. A maximum of three subjects are scheduled in parallel due to class room limitations. Similarly no subjects are offered in more than three sections. The conflict matrix is reconstructed with each section being treated as a subject. This will increase the number of subjects and the size of the conflict matrix. The feasibility test discussed in Appendix is carried out with the new conflict matrix. The test is carried out with a maximum of three subjects being scheduled in parallel.

The multiple section grouping is continued till feasibility is achieved. If it cannot be achieved, the next course of action of restricting the students' choices is pursued. However, it may not always be possible to do parallel section grouping. Some of the necessary conditions for parallel section grouping, which are trivial are stated below.

(1) For two conflicting subjects to be scheduled in parallel, at least one of the subjects must be offered in two or more sections.

(2) For three subjects, to be scheduled in parallel which are conflicting amongst themselves with some students taking all the three subjects, at least three periods are to be assigned for scheduling the three subjects.

Once the feasibility expression is satisfied through multiple section scheduling with or without restricting the students choice, the generation of the class schedule can be undertaken.

In practice it has been observed that in most of the terms choices of students could be accommodated through the multiple section grouping. In some terms, two or three students have been asked to change one of their subject choices.

7. Generation of a class schedule

At this stage actual scheduling of subjects to the periods is carried out. Decisions on combinations of subjects which are to be scheduled in parallel are taken during multiple section grouping, Step 2 of timetabling. In this particular case some of the combinations for parallel scheduling are:

subject 1, subject 5, subject 7;
subject 10, subject 11;
subject 8, subject 9;
etc.

There is no precise objective function to be optimized in this case of generating a class schedule. Efforts are made to find a feasible schedule subject to various requirements. The decision maker is provided with facilities to use the schedule generation process as an aid to generate alternate schedules and evaluate them from the quality aspect as perceived by him. However, in order to generate a schedule, an objective function is explicitly defined as follows:

$$\text{Max} \sum_{i=1}^{\text{NS}} \sum_{i=j}^{\text{NP}} \text{NC}_{ij} W_j,$$

where NC_{ij} is the number of students of subject "i" attending the class in period "j". This also takes care of multiple sections, as in case of multiple sections NC_{ij} for any i, j will be less than the total number of students opting for subject i; and where W_j is the weightage of period "j" in terms of its desirability.

There are seven periods in a day. Some of these periods are more desirable compared to the others. The late afternoon periods, the period after lunch etc. are less desirable. Accordingly the weightages have been assigned to different periods. The weight assignments are presented in Table 3. The period with highest weight is the most desirable and so on. In order to achieve the above objective the subjects are sorted in the decreasing number of students opting for the subject. The subject with the largest number of students is on the top of the sorted list. The sorted list with the data of Table 2 is presented in Table 4. The scheduling algorithm schedules the subjects to periods as per the sorted list one by one. This helps achieving the objective stated earlier. Accordingly subject 1, which is at the top of the list is scheduled first. This subject has three sections and has been scheduled on Mondays, Tuesdays

Table 3. Weight assignment to periods

Period	Time	Weight
1	0830–0940	5
2	0950–1100	7
3	1125–1235	6
4	1330–1440	2
5	1450–1600	4
6	1625–1735	3
7	1745–1855	1

and Wednesdays during periods 1, 2 and 3 with the highest weightages. Since it has been decided to schedule subject 5 and subject 7 in parallel with subject 1; these subjects are also scheduled along with subject 1. Next in the sorted list is subject 11. This subject has been scheduled on Thursdays and Fridays during periods 2 and 3 and on Wednesdays periods 5 and 6. This is so as no other periods with higher weightages (weightages of 7, 6 and 5) are available for scheduling subject 11. (Period 1 with weightage 5 on Thursdays and Fridays cannot be used as two sessions of subject 11 have already been scheduled on those days. Moreover, as far as possible subjects are scheduled on continuing days.) Subject 10 is also scheduled as it has been decided to schedule it in parallel with subject 11. In this way subjects are scheduled serially as per the priority in Table 5. Subjects which are scheduled in parallel with other subjects, having higher priority, are passed over when these become candidates for scheduling. A partial class schedule generated on this basis is presented in Table 4. The decision maker at this stage takes care of various externalities. These externalities could be scheduling certain subjects continuously over a long duration, scheduling over weekends etc. The externalities could also be in terms of preferred periods by visiting faculty. Further, it is not always the case that every subject has

Table 4. Class schedule (partial)

Period	Weight	Day				
		Mon	Tue	Wed	Thu	Fri
1	5	*S*1(A)	*S*1(A)	*S*1(A)	*S*12	*S*12
2	7	*S*1(B) *S*5(A) *S*7	*S*1(B) *S*5(A) *S*7	*S*1(B) *S*5(A) *S*7	*S*10(A) *S*11(A)	*S*10(A) *S*11(A)
3	6	*S*1(C) *S*5(B)	*S*1(C) *S*5(B)	*S*1(C) *S*5(B)	*S*10(B) *S*11(B)	*S*10(B) *S*11(B)
4	2	*S*12	*S*13 *S*6	*S*9(A) *S*8		
5	4	*S*9(A) *S*8	*S*9(A) *S*8	*S*10(A) *S*11(A)	*S*13 *S*6	*S*13 *S*6
6	3	*S*9(B) *S*8	*S*9(B) *S*8	*S*10(B) *S*11(B)		
7	1			*S*9(B)		

Table 5. Priority of subjects for scheduling

Priority	Subject	No. of students opting for the subject
1	1	169
2	11	164
3	9	144
4	5	135
5	10	132
6	12	79
7	7	70
8	13	50
9	2	41
10	8	33
11	3	30
12	4	28
13	14	21
14	6	14

three sessions every week. Sometimes a subject may have two sessions in a week, three sessions in another week, but not more than three in any week. These factors are taken care of by the decision maker in generating a variant of the basic weekly schedule (similar to Table 4) for different weeks of the term. The quality aspects are also considered by the decision maker at this stage to arrive at the implementable timetable. The decision maker pre-assigns certain subjects (or subject combinations for parallel scheduling) to certain periods in order to take care of some externalities and/or to improve the quality of the schedule. A variant of the timetable is generated with the pre-assignments taking help of the scheduling algorithm. About two or three such iterations with pre-assignments are made before arriving at an acceptable timetable.

8. Conclusion

The timetabling problem varies widely in its nature from one situation to another. The constraints to be considered are of different nature and many times it may not be possible to define these explicitly. There is a quality parameter for a timetable, which is reflected by the preference for one timetable over the other. It is often difficult to express the quality parameter precisely. This remains largely the perception of the decision maker. Accordingly it is desirable that the decision maker is involved actively in the timetable generation process. This has been achieved by providing a decision aid in the form of schedule generation to the decision maker. The requirements of the timetabling problem have been expressed mathematically and these have been taken care of at the various stages of timetabling.

Appendix

Quick test of feasibility

A quick test of feasibility can be carried out once the conflict matrix is prepared. If the following expression is not satisfied, then we can say that it will not be possible to get a feasible solution.

Sum of the number of periods required per week by the subjects ($=\sum_{i=1}^{\mathrm{NS}} \mathrm{NSP}_i$) − reduction in the required number of periods due to parallel scheduling of subjects (= REDUCE) ≤ NP, where

- NSP_i = total number of periods to be scheduled per week for subject i,
- NS = total number of subjects,
- NP = total number of periods in a week.

REDUCE for parallel scheduling of two subjects is given by

$$\mathrm{REDUCE} = \sum N_{ij}\delta_{ij},$$

where

$$N_{ij} = \min(\mathrm{NSP}_i, \mathrm{NSP}_j),$$

$$\delta_{ij} = \begin{cases} 1, & \text{if the entry in the } i\text{-}j \text{ cell of the conflict matrix is zero,} \\ 0, & \text{otherwise,} \end{cases}$$

$$i = 1, \ldots, (\mathrm{NS}-1),$$
$$j = (i+1), \ldots, \mathrm{NS}.$$

While carrying out the summation to obtain REDUCE, care has to be taken that the subjects 1, 2, ..., NS do not appear more than once for the parameters i and j in the summation terms $N_{ij}\delta_{ij}$ as parallel scheduling of one subject can be considered only once with another subject. E.g. according to Table 2 the subject combinations for which $\delta_{ij} = 1$ are (3-6), (4-6) and (6-13). Since subject 6 is common in all the combinations, only one of the combinations can be considered to arrive at REDUCE. In this case the combination with highest N_{ij} will be considered.

However, theoretically it is possible to schedule subject 6 in parallel with more than one subject under certain situations. Suppose subject 6 is required three times per week, subject 3 once per week and subject 4 twice per week. In this situation subject 6 can be scheduled once per week in parallel with subject 3 and twice per week in parallel with subject 4. Now for each combination N_{ij} has to be computed by the expression shown earlier and will be considered for computing "REDUCE". So N_{ij} for the combination (6-3) is 1 and that for the combination (6-4) is 2. Both these are to be considered for computing "REDUCE". Similar situations are also to be considered for parallel scheduling of three subjects discussed below. But such situations do not occur in the case considered here.

In the timetabling problem under study a maximum of three subjects are scheduled in parallel. REDUCE for parallel scheduling of three subjects can be arrived at as follows:

$$\text{REDUCE} = \sum N_{ijk}\delta_{ijk} + \sum N_{ij}\delta_{ij},$$
$$i = 1, \ldots, (\text{NS}-2), \qquad i = 1, \ldots, (\text{NS}-1),$$
$$j = (i+1), \ldots, (\text{NS}-1), \quad j = (i+1), \ldots, \text{NS},$$
$$k = (j+1), \ldots, \text{NS}.$$

The first term represents the reduction due to parallel scheduling of three subjects and the second term due to the parallel scheduling of two subjects.

$$\delta_{ijk} = \delta_{ij} * \delta_{ik} * \delta_{jk},$$

$$N_{ijk} = \text{NSP}_i + \text{NSP}_j + \text{NSP}_k - \max(\text{NSP}_i, \text{NSP}_j, \text{NSP}_k).$$

As in the case of scheduling of two subjects in parallel, in this case also care has to be taken that the subjects 1, 2, ..., NS do not appear more than once for the parameters i, j and k in the two summation terms considered together. In case of more than one combinations involving the same subject, the combination with highest N_{ijk} or N_{ij}, as the case may be, is considered. However, special situation as discussed for two subjects in parallel scheduling, may also arise here and has to be dealt with in a similar way. But such situations do not arise in the case considered here.

The summation terms are combinatorial in nature. Since the conflict matrix is highly dense and only the combinations with $\delta_{ij} = 0$ in the conflict matrix are to be considered, the number of candidate combinations for parallel scheduling are very limited.

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