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Fuzzy Transfer Pricing World: On the Analysis of Transfer Pricing with Fuzzy Logic Techniques

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W.P. No. 2005-12-03 December 2005

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INDIAN INSTITUTE OF MANAGEMENT AHMEDABAD-380 015 INDIA

Fuzzy Transfer Pricing World

On the Analysis of Transfer Pricing with Fuzzy Logic Techniques

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Abstract

The arm's length analysis of international transfer prices of multinational firms lacks sound methodological approach of the so-called function and risk analysis. In practice, such analyses are descriptive. Derived from Zadeh's mathematical theory of fuzzy sets, this paper investigates a quantitative approach to identify the function and risk pattern of related parties of multinational companies. We illustrate our fuzzy logic approach with a simple case.

Key Words: Function and risk analysis, fuzzy logic theory, multinational company, transaction costs, transfer pricing. *JEL: C69, D49, L29, M21*

1. Introduction

The analysis of transfer prices for related-party transactions within the multinational company is a very complex decision-making process. In many jurisdictions which imposed documentation requirements, the taxpayer is requested to substantiate that transfer prices meet the so-called Arm's Length Principle (ALP; comp. Art. 9(1) OECD Model Tax Convention¹). Then, the tax authority may have to judge this assessment of the taxpayer. ALP as applied by most national tax jurisdictions dictates that transfer prices between related parties of a multinational group have to be set as if the business had taken place between independent parties under similar conditions. The challenge for the business decision maker (and the tax authorities) lies in the appropriate judgement of circumstances and conditions which may determine prices – be they for transactions between independent parties or between related parties.² As it is between independent parties, many of these decision-making features behind transfer pricing within the multinational group have a very vague nature and, hence, are unlikely to be driven through any exact mathematical calculation. However, it is exactly this mathematical approach which many mainstream transfer pricing studies seek to achieve, neglecting at the same time that most of the decisions on transfer pricing are vague with degrees of tendency.

This paper develops an analytical framework to do transfer pricing analysis along the notion of real-world fuzziness and vagueness. For that we use fuzzy logic theory. We indicate what type of decisions in the context of (transfer) pricing are fuzzy and, hence, should be modelled to establish the degree of the arm's length nature of a given transfer price.

1 On Transfer Pricing

The judgement on conditions and the arm's length nature of transfer prices is determined by a wide range of micro- and macro-economic factors which also independent parties take into account in the pricing process. In transfer pricing language, these factors (variables) are used as comparison criteria to assess as to whether and, if so, to what degree, the tested party (analysed party) of the multinational company is comparable with independent parties (comparables). The comparison criteria determining (transfer) prices have the nature of legal, political, and economic conditions, and, to a large extent, transaction-specific conditions such as the type of transaction considered (e.g. goods, services, licences, BPO, etc.), functions performed, risk assumed, and assets deployed.

Figure 1 provides a pattern of factors which can be referred to regarding pricing decisions. Derived from industrial organization theory on the Structure-Conduct-Performance-Approach,³ the table is a first attempt to categorize such comparison criteria and to indicate what data features are necessary to establish a sound economic analysis of arm's length transfer prices.

¹ In transfer pricing, the arm's length principle (ALP) is the standard to assess whether transfer prices were manipulated for shifting income in low-tax jurisdictions. In practice, ALP results either in a direct or indirect arm's length comparison and in the distinction between a real and hypothetical arm's length comparison. Our paper refers to the hypothetical comparison used for many arm's length analyses. This type of comparison is borne by a high degree of comparability and reliability of data regarding the tested party of a multinational group and the third-party comparables. Normally, a full degree of comparability does not exist for which reasons degrees of comparability are supposed to be determined in a transparent way. For an overview of the types of arm's length comparison, see Bick/Kotschenreuther (1997: 307).

² In addition, transfer prices are subject to country-specific accounting principles and other legal provisions.

³ The "Structure-Conduct-Performance-Approach" was developed by Mason (1939) and Mason (1949); see also Emmanuel/Mehafdi (1995: 154) and Tucha (2001: 747).

	Structure			Con	duct	Perfor	mance
Political and Social Factors		Economic and	Market factors	Strategie		Performance	
Regulation	Labour market	Growth rate	Interest rates	Company strategy	Company philosophy	Set of objectives	Value added
Anti-trust policies	Marcro-economics	Economic situation	Inflation	Administrative strategy	Management	Remuneration of principal	Value stability
Economic law	Political stability	Unemployment	Price structures	Operational stratetgy	Collusion	Conflicts	Prices
Taxes and tariffs	Social stability	Foreign trade	Capital markets	Price strategy	Strategic planning	Production efficiency	Profits
Investment subsidies	Associations	Exchange rates	Stock exchanges			Allocative efficiency	Market development
		Tariffs	Infrastructure			Technological progress	(capital) costs
		Monetary system				Company performance indicators	

Figure 1: Data structure to model pricing decisions as possible for transfer pricing

Exogeneous factors		Endogene	ous factors	Economic decisions		Divisional results	
Industry	Sites	Organization	Information system	Global strategy	Diversification	Set of objectives	Value added
Market participants	Technology	Number of employees	Degree of technology	Integration	Differentiating	Employments of agents	Value stability
Supply elasticity	Product cycles	Sortiment	Company functions	Marketing	MIS	Conflicts	Transfer prices
Demand elasticity	Seasonality	Product specifics	Cost Accounting System	R & D	Company financing	Product efficiency	Profits
Substitutes	Unit sizes	Sales volumes	Market penetration	M&A		Allocative efficiency	Product quality
Market forms	Consumer behavior	Value step	Contracts			Technological progress	Customer list
Market penetration	Sub-contracting	Terms and conditions	Incentive systems			Divisional performance indicators	
End-market	Economy of scale	Terms of payment	Accounting Information System				
Procurement	Economy of scope	Warranties	Company indicators				

Notes: the abbreviations AIS, F&E, KRS, M&A, MAS, and MIS refer to Accounting Information System, Research and Development, Cost Accounting, Merger and Acquisition, Management Accounting System and Management Information System.

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Patents

Source: comp. Tucha (2001: 747).

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Risks

A different perspective on describing a company is given in Figure 2. Compared with **Error! Reference source not found.**, the features in Figure 2 address to a large degree the information as required – or at least preferred – for the third-party arm's length comparison. Note that the features given should be placed in the context of cross-border related-party transactions. The exogeneous environment of such transactions is determined by political-social, market-economic, and structural factors. Compared to Error! Reference source not found., in Figure 2 they are positioned at the first level of company analysis, which itself represents an endogenous perspective on pricing and comparison.

In the transfer pricing context, the analysis of various facets of a business unit can be split up into the analysis of the overall company (its position within the multinational group) and the more detailed analysis of various sub-units of this company (e.g. function and risk units).⁴ The former analysis primarily refers to the "conduct-performance" features of a related-party company, whereas the latter part reveals indicators on endogeneous "structure". This translated into transfer pricing language, the in-depth analysis of sub-units of the multinational firm is normally labelled as function and risk analysis. The analysis of functions performed, risks assessment and assets deployed is, by convention, the basis for justifying the allocation of costs and profit pies to such functions and, respectively, their legal entities residing in different tax jurisdictions. In other words, the function and risk analysis reveals to be a valuation issue in transfer pricing, both in principle and in size. While many transfer pricing analysts deploy a narrow focus of function and risk analysis in a broader sense to provide for a high degree of comparability and sufficient quality of the arm's length analysis.

	Arm's Length Analysis			
Analysis of company A				
Environment (exogeneous)Company analysis (endogeneous)				
Politics and policies	Overall company Company-Subunits			
Society	Assets	Functions	Risks	
Legal	Capital	Production	Production	
Economy	Expenses	Procurement	Procurement	
Technology	Profit	Sales	Sales	
Ecology	Costs	Marketing	Marketing	
	Finance	R & D	R & D	
	Strategy	Administration	Administration	
	Conduct			

Figure 2: Features of transfer p	pricing analysis relating to ALP
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Function and risk analysis (primary scope)

Function and risk analysis in the broader sense

Source: Brem and Tucha 2006, forthcoming.

⁴ See Brem and Tucha (2005) for a proposal as to how a multinational group can be featured along "layers".

Unfortunately, to our knowledge, transfer pricing provisions around the globe and the OECD Transfer Pricing Guidelines do not really provide for a detailed guidance of what aspects and comparability criteria should be deployed for a transfer pricing analysis, in general, and the function and risk analysis in specific.⁵ In other words, almost all national provisions and international guidelines lack a clear distinct guidance which of the factors indicated above – or any other factors – are the "appropriate" ones to describe, and to value, the function and risk of a company. On the other hand, it is accurately this choice of factors which may deem a certain business unit with this functional pattern as more or less valuable than another one (say, the third-party comparable) with the consequence that the former one should receive a higher profit markup (or even a residual income) as compared to the latter one.

No clear-cut statements are really given in such provisions whether, which, and to what degree qualitative and/or quantitative indicators should be deployed for function and risk analysis and how each of these indicators contributes to the value of a function. On the other hand, the mainstream approach as used in most of today's transfer pricing practices seeks to identify and to reference such indicators in the transfer pricing analysis with erroneous degree of accuracy of the second, or even third, digit after the comma of quantitative cost or profit margin figures – let aside the highhandedness of using any qualitative indicator. Given the individualized analysis approaches as practised by various transfer pricing expert teams⁶, today's transfer pricing practice is stuck in a stalemate. Each expert team deployes its own concepts, indicators, and ways of analysis resulting in a situation that three different experts are most likely to generate three different results. Even worse, the approaches within teams are subjective without clear and transparent approaches of decision-making in assessing information collected for transfer pricing.

Put it differently, the assessment of information on transfer pricing features, especially the function and risk analysis and the selection of comparable criteria and comparables, lacks a sound transparent and traceable model. In practice, each individual case of transfer pricing is treated heuristically using qualitative analysis and, most surprisingly, qualitative and quantitative information without methodological judgement on the relationship between the two. Normally, questionnaires are designed case-specifically in order to collect information on functions and risks. The evaluation of these questionnaires is performed heuristically and strongly discretionary with random intervals along the scale of a variable.

From a theoretical perspective, it is undiscovered so far how the criteria picked to compare (e.g. comparable criteria for function and risk analysis) as well as the factors at the various levels of analysis (e.g. exogeneous environment, endogeneous factors) are in relation to each other and how they determine an arm's length behaviour. From the overview above, following issues arise:

- How to deal with the interdependencies between qualitative and quantitative variables used for comparison?
- Is it possible at all to combine quantitative and qualitative variables for comparison?
- How can we, or do we even have to, weigh the single factors and variables determining arm's length behaviour?
- Is there room for generalizing rules and procedures in order to treat each case with the same "methodology" of information analysis?

Given this unanswered key questions in transfer pricing, an even more basic question may arise: how should be the arm's length analysis conducted at all, notwithstanding the outstanding lack of comparable data in most non-simple transfer pricing cases? Databases and their applicability for

⁵ See Tucha (2001) and Brem (2004) for a detailed discussion.

⁶ For example, KPMG, PWC, Ernst & Young, Deloitte & Touche, Haarmann & Hemmelrath, etc.

transfer pricing analysis purposes are extensively discussed in Tucha (2001) and Oestreicher and Vormoor (2004). Given the open questions and the huge relevance in daily decision-making on transfer pricing, it is really surprising that the function and risk analysis lacks a profound methodological concept. In other words, as the basis for any profit allocation, the function and risk analysis requires a profound concept which is not case-specific but transparent in terms of comparability and valuation.

The lack of a model to the decision-making problems in transfer pricing astonishes even more if one considers the economic relevance of transfer prices in our globalizing world: OECD reports in its statistics on 2004 for the OECD region a cross-border trade of about US\$ 16.5 trillion (US\$ 7.7 trillion exports; US\$ 8.8 trillion imports). Estimates claim that about half of this trade is subject to transfer pricing, i.e. within the boundaries of multinational groups. In some cases of cross-border trade, this intra-group trade goes up to 60 per cent (Neighbour, 2002).

The next section illustrates the limitations of using "traditional" qualitative scoring models for function and risk analysis. It provides the basis for a discussion of the questions raised above from which we derive a systematic approach of function and risk analysis on the basis of fuzzy logic techniques for transfer pricing purposes. The final section provides an outlook.

2. Function and Risk Analysis: Mainstream Approaches

In general, valuation and company comparisons are complex decision-making issues because of the large number of possible variables to be considered and the need to break this number down to a size which can be handled and understood. Such variables can be of qualitative and quantitative nature, they can be interdependent or not, and they can have varying degrees of relevance (weight) for a given fact pattern. Likewise relevant, they are very likely understood differently by different analysts. Hence, unless a clear methodological approach is in place providing for a standardized and coherent application, fact pattern descriptions and comparisons are personspecific and highly subjective.

In the 1950s, decision-making problems of high degree of complexity were considered to be only solvable through mathematical formulas. As the following example shows, exact mathematical calculation for decision-making is only possible in very rare and rather simple cases. Given today's understanding and need for complex decisions, the large expectations in mathematically exact formulas to support or to (re-)construct decision-making are not met in many cases. Real-world applications which support decision-making are rather looking for alternatives in resolving complex decision-making on analysis and valuation. Transfer pricing is at large one of these real-world problems which seeks better applications than available today.

Unfortunately, the transfer pricing consulting industry has been failing to truly develop a consistent and transparent approach of function and risk analysis and company comparison. So has the academic community dealing with transfer pricing. For example, the Anglo-American standard approaches of database-driven screenings and identification of appropriate cost plus markups and so-called profit level indicators prevail in transfer pricing discussions.⁷ In such approaches, the problems of database availability, lack of comparable information out from these databases, including the error probability that data available in databases is not correct (as one of our practical cases in August 2004 revealed), and missing concepts of when and how to use quantitative and qualitative comparison data are mitigated by using large ranges of prices or profit

⁷ For an overview of approaches to do database screenings, see Tucha (2001: 745).

margins (so-called arm's length range).⁸ However, as to whether a transfer price derived from a given transfer pricing method – or more often practised the profit margin in the case of an indirect arm's length comparison – should be assessed against the upper or the lower benchmark point or anywhere in between is solely subject to individual judgement of the analyst, often disregarding further information potentially available in the field. Hence, such approach, in our mind, is purely heuristic and lacks transparency and methodology. And it might be such approach which results in the mess and controversy in today's transfer pricing.

Likewise, the function and risk analysis as such (primary scope), which forms the basis of any screening approach, can be unmasked as a simplified collection of information and data by means of case-designed questionnaires which are supposed to determine the true angle of informational scope and depth (comp. Figure 2). In practice, if two analysts have the task to do a transfer pricing analysis on the same fact pattern, they are likely to draft two different questionnaires. In addition, they will reveal two different information sets and, hence, two different results on function and risk analysis. The collected information is normally neither aggregated on any pre-defined concept of rules, nor are the interdependencies between the variables accounted for in a transparent and model-based way. A standardized model which would reproduce – at least – similar results if applied by two different analysts does not exist to our knowledge.

Likewise, translating information on the function and risk into the next step of the transfer pricing analysis, i.e. the comparables search step (screening) and the determination of any arm's length range is a game of dice – rather than a coherent appraoch. This, however, implies that such "function and risk analysis in principle" (in the broader sense) is most likely simply some additional information to describe the fact pattern for documentation purposes. What they however lack is a link to the economic arm's length analysis. The task to quantitatively determine transfer prices ex ante remains unresolved as long as merely descriptive information is generated in the course of function and risk analysis.

This problem of establishing a link between a fact pattern profile, the function and risk analysis, and the value assessment of such function through screenings and arm's length ranges of comparable companies can be deemed even more strange if not only one single function is of relevance but a complex functional pattern determines the profit of a company unit. This is because in such cases, without a function and risk valuation model and the corresponding indicator model, the appropriateness of a transfer price cannot be defined. This even in such cases where the arm's length transfer price (or profit markup) is known for each individual function performed and the risk assumed.

Practitioner transfer pricing mitigates this shortcoming of today's transfer pricing concepts by means of a very artful shift of argumentation: transfer prices related to standard tasks (so-called routine functions) can be assessed on the basis of a database-driven comparison and its screening steps, whereas non-routine functions which represent entrepreneurial tasks are not comparable.⁹

This argument represents the residual profit split: the non-routine functions are valued through residual profit split, after the routine functions have been remunerated on the basis of routine markups. The approach behind the Residual Profit Split Method¹⁰ requires to classify the

⁸ The US Regulations provide for the so-called Interquartile Range (i.e. 25%-75% of observations) as a more reliable benchmark range for transfer pricing purposes; comp. IRC, 1996, Sec. 1.482-1 e 2 iii; see also <u>http://www.intltaxlaw.com</u>.

⁹ Compare, for example, the new German administrative principles on documentation and procedures (VWG Verfahren IVB4 - S 1341-1/05). For the term "entrepreneur", see also Langlois (2005).

¹⁰ For the residual profit split approach, comp. OECD Transfer Pricing Guidelines 1995, Tz. 3.5. This approach enjoys an increasing relevance from both theoretical and regulatory perspective (for example,

functions involved as routine and non-routine. However, again the question remains open how the functions and risks can be related to each other and whether an identified function and risk belongs to the routine or the non-routine class.¹¹

In addition, the recent years have seen the advent of simplified scoring models to resolve the decision-making problems on the assessment of what unit is routine and is non-routine. However, existing scoring models are weak and often insufficient. For example, models which produce an arithmetic mean value of indicators chosen lack both a sufficient argument on comparable valuation and the weighting of comparison criteria (Biethahn et al., 1997). We deem exactly this choice on the appropriate weighting as one of the basic decision-making issues in the course of a transfer pricing analysis. This is because the relative weight of individual criteria chosen towards the accumulated judgement on routine or non-routine (or anything in between) is normally not a discrete either-or decision rather than a choice along a continuum of degrees of the factor considered, subject to the parameter values of the variables and the objectives set by the companies.

The following example in Table 1 may illustrate the assessment problem. We refer to a functional pattern of research and development in a multinational company and the question whether this function is more routine or more non-routine. In our example, the degree is assessed by using two variables: "required know how" and "developing costs". It illustrates the shortcomings of existing approaches, scoring information on these two variables is added up without further weighting.¹² Cases 2 and 3 result in the same assessment regarding the routine degree of this function, i.e. the score value 5.

However, since developing costs would include COGS and labour costs and, in general, a high degree of know-how normally comes with a high labor cost, this can be questioned at a certain parameter value. Hence, our simple example does not reflect the potential interdependency between both variables. The assessment on the degree of routine, given the scoring value, actually would have to be adjusted in favour of the variable "required know how".¹³

Another issue to qualify such approach as appropriate refers to the scaling of 0.5 scores [0/2; 20/2] regarding the possible degrees of non-routine. Neither the equi-distant room of assessment results nor the rules to equally weighting the variables is provided for in mainstream transfer pricing with arguments.¹⁴ In other words, the scoring model is too abstract in terms of its linearity to describe real-world phenomena so that mathematical modelling can be deployed.

- ¹³ Alternative variables would be the variable "labour costs" instead of know-how, and costs of material. On the other hand, it can be questioned whether labor costs would be an appropriate measure of the scope of development function in a company, as the variable "know-how" represents a higher aggregation of information regarding the feature to be considered.
- ¹⁴ For example, it is questionable whether the ten scores of case 2 to describe developing costs have the same value as the ten scores of case 3 describing know how, allowing for a distinct assessment of the degree of non-routine. Similarly, case 4 in Table 1: the degree of non-routine of 10 has been generated by adding the cases 2 and 3 an unlikely situation in real-world situations.

the new Administrative Principles of Germany explicitly refer to this approach if so-called entrepreneur functions with non-routine characteristics are involved in the transfer pricing fact pattern; see VwG Verfahren, Par. 3.4.10.3 c).

¹¹ In the framework of a transfer pricing analysis, the problem of not having a model of valuing functions and risk is simply relabelled with different terms. The valuation itself is not really achieved (Brem, 2004).

¹² In this approach it is assumed that the degree of non-routine of the function considered increases as the employee know-how and the developing costs increase. The variables are scaled along a scale with 9 intervals [0;10] with a simple arithmetic mean of the degree of non-routine.

Scoring the Degree of Routine				
Case	"Required know how"	"Developing costs"	Scoring the non-routine degree	
1)	0	0	0	
2)	0	10	5	
3)	10	0	5	
4)	10	10	10	

Table 1: Scoring the non-routine degree of functions by means of mainstream scoring models

One way out to derive of what is a relevant variable and how interdepenent it is could be the design of aggregation rules. Such rules guide establishing an aggregrate assessment derived from individual variables. The aggregation rules could be developed by (transfer pricing) experts – as it is, for example, with the internationally widely accepted OECD Transfer Pricing Guidelines. Examples of such aggregration rules are given in Table 2.¹⁵ In such sets of rules, the features "required know how"¹⁶ and "developing costs" are not assessed by means of a scoring model but through verbal terms – so-called linguistic variables – with the values "little" (1), "medium" (m), and "high" (h). In addition, valuation steps can be introduced such as "+" and "-". The linguistic variables can be based on disjunct intervals as given in Table 3 and Table 4.

Table 2: Assessing the degree of non-routine by means of set of rules

Asses	sing the degree of routine fur	nction by means of set of ru variables	les and, respectively, linguistic
Case	"Required know how"	"Developing costs"	Assessing the non-routine degree
1)	1	1	1
2)	1	m	1
3)	1	h	m-
4)	m	1	1
5)	m	m	m
6)	m	h	m
7)	h	1	m-
8)	h	m	h-
9)	h	h	h

¹⁵ The example is derived from Nolte-Hellwig et al. (1991) referring to a static valuation approach. He also discusses dynamic approaches which assume time-lags between criteria and, respectively, indicators (e.g. costs, profits). The basis of such dynamic valuations are deviations of means over the total observation period.

¹⁶ The variable "required know-how" is here measured as share of employees with a training specifically designed for the project, compared to all employees assigned to this project.

Assessi	Assessing the linguistic variable "required know how"				
Rules	Share of employees with specific education	Assessment of the "required know how"			
1)	< 0%	1			
2)]0%; 20%]	1			
3)]20%; 40%]	m			
4)]40%; 60%]	m			
5)]60%; 80%]	h			
6)	> 80%	h			

Table 3: Assessing the linguistic variable "required know how"

Table 4: Assessing the linguistic variable "developing costs"

Assessi	ng the linguistic variable ,,developing cost	is"
Rules	Developing costs in million €	Assessment of developing costs
1)	< 1	1
2)]1; 5]	1
3)]5; 10]	m
4)]10; 20]	m
5)]20; 40]	h
6)	> 40	h

It can be shown that the shortcomings of describing verbal assessments by means of intervals is that such intervals are often large and, hence, assess different functions as they were similar. On the other hand, such approaches assess functions which differ only to a tiny degree would be characterized as significantly different, as Table 5 indicates.

Assessing the R&D function across companies				
Company	Share of employees with specific education	Developing costs in million €		
A)	21%	5,2		
B)	59%	11,6		
C)	19%	5,2		

Source: own presentation.

Based on the set of rules in Table 2, one would assess the R & D functions of companies A and B as non-routine to a medium degree, though company A performs R & D to a significantly closer degree of routine (only 21% specifically educated employees as compared to 59% in company B). On the other hand, the same set of rules would assess company C as routine function, though – given same developing costs in absolute terms – it appears to have only slightly less share of specifically educated employees (variable "required know how") compared with company A.

This illustrative example clearly shows that small variances in the variables used in the function and risk analysis can result in significantly different assessment results whereas large variances may not be reflected in the assessment if the pre-defined category is not left.¹⁷ Further, the

¹⁷ A larger number of intervals could mitigate some of the shortcomings, however with the negative consequence of a multiple increase in the number of rules. If no combination of criteria can be excluded

separating borders of distinct categories may not be sufficient to describe (or reconstruct) human decision-making processes and the ability to distinguish between options. The expertise knowledge accumulated in the set of rules is shrinked down in the saltus between the neighboring intervals of a given variable, whereas, the description of real-world phenomena would rather benefit from a soft transition between categories and, respectively, a membership of an observation into two categories. The mainstream transfer pricing analysis approaches can be interpreted as follows: experts intuitively solve the analysis case, given their decision-making pattern and experience. The intervals of valuation are introduced ex post, after data have been generated and assessed.

Zadeh's theory of fuzzy sets (fuzzy logic theory) offers a view which significantly differs from classical set theory with its distinct membership functions and which provides a more realistic approach in transfer pricing. In fuzzy logic theory, linguistic variables verbally describe the parameter value of the indicator considered in a mathematical way as an expert can catch the issue and describe in the given context. As shown below, the determination of linguistic variables is construed on the basis of fuzzy membership functions with degrees of membership into sets. This allows overlapping of otherwise distinctively separated intervals of the variable. In addition, a combination of qualitative and quantitative information from the variable chosen is possible.

In our mind, fuzzy logic theory suits well the practical challenges in transfer pricing analysis, especially with respect to function and risk analysis and the ever more integrating global business model of multinationals. Compared to mainstream approaches, fuzzy logic theory allows for mitigating the dilemma that inexact data and/or non-linear rules prevent from a clear-cut assessment as to whether a bunch of functional activity can be deemed routine or non-routine. According to Zadeh's fuzzy set theory, the transition between both stati is flowing and is defined not as an exact jump but a fuzzy issue. We assume that this brings the function and risk analysis closer to reality and, hence, accurateness.

3. Fuzzy Logic in Function and Risk Analysis

In the fuzzy logic approach, the evaluation scale and the underlying decision rules depend upon the degree of determinateness of data and information deployed in the planning process (decisionmaking problem). Two types of degree of accurateness can be distinguished (Schneeweiß, 1991): unknownness given a lack of information, and unknownness given a lack of clearness in terms describing the status considered. The latter one is characterized as fuzziness and can be specified as follows (Rommelfanger, 1994):

- The *intrinsic fuzziness* is caused by the vagueness of human perception. For example, terms used in transfer pricing are not unambiguously defined such as "high risk", "functionally strong", "appropriate transfer price" or "functional comparability".
- The *informational fuzziness* refers to the problem that, even though a term can be exactly defined, its practical application remains difficult because all relevant information is not available or cannot be identified. An example is "profitability of a company" or "value chain" or "functional contribution to a value chain".
- *Fuzzy relations* are statements on interdependencies between indicators such as "not larger as" or "approximately similar as".

for the reasons of logic in the economics of the fact pattern, then the number of rules R with k criteria of parameter values a is: $R = a^k$, hence in case of two criteria with five clusters, we would have 25 rules to provide for.

The fuzzy set theory as developed by Zadeh (1965) represents a generalization of the traditional set theory with bipolar membership (Zadeh, 1965). It provides for modelling the three types of fuzziness as given above. Subject to the degree of distinctiveness, the objectives, alternatives and their assessment, the relationships, and their dependencies are modelled fuzzy and vague rather than deterministically and/or stochastically.

Three steps are normally taken into model (Kuhl, 1996: 31; Grauel, 1995: 58 and 82): Fuzzifying the variables, inferencing of fuzzy variables, and defuzzifying (Figure 3). In the first step, distinct features separated by defined intervals are redesigned into fuzzy linguistic variables. Examples are given in Table 3 and Table 4 whereas normally in fuzzy logic approaches specific and standardized transformation functions are deployed (so-called membership functions). In the inference step, these fuzzy variables are linked – using specified operators – with a set of rules provided by an expert panel (Table 2). The outcome of the rule-based link is another fuzzy variable which then needs to be defuzzified to result in a clear-cut variable value. The defuzzifying step is a densening of information generated in the fuzzy assessment. A real number is one of the possible outcomes and, given its loss of information, should be only generated if devices (computers) need to process the information generated in the inference step.

To illustrate this basic process of a fuzzy logic based expert system to assess real-life phenomena, the following generalized example in graphical and analytical form may be useful.

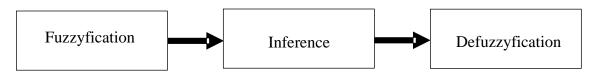


Figure 3: Basic process of a rule-based fuzzy system

Source: own presentation.

The following illustrations refer to the example of ,,developing costs". Given the fuzzy model of how the level of ,,developing costs" determine the degree of non-routine of an R & D function of the company, the classical set approach with "yes-or-no" categories (either-or) appears to be inappropriate to judge "high developing costs" because an element can simply belong to a given set or not with a distinct criteria to determine this bipolar membership (Popp et al., 1993) disregarding information which describes the degree of membership. Rather, elements can belong to a fuzzy set to a certain degree in a way that a sliding transition from "full membership" to "non-membership" can be construed. A fuzzy set is defined as (Zadeh, 1975: 220; Tilli, 1993: 15, and Zimmermann, 1991: 11):

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x) | x \in X\} \text{ with } \mu_{\widetilde{A}} : X \to \Re$$

Definition 1: Fuzzy Set

Source: Zadeh (1975: 220).

The larger the value $\mu_{\tilde{A}}(x)$ at position x, the larger the membership of element x to \tilde{A} . The membership normally is scaled to the interval [0;1], i.e. a normalized membership (Bellman/Zadeh, 1970: 57). The membership function $\mu_{\tilde{A}}: X \to \Re$ can have various forms. Typically, the graphical forms are triangular for a fuzzy number (Figure 4) and the trapezoidal forms for a fuzzy interval (Figure 5) (Bandemer/Gottwald, 1993: 42; Schoppe, 1991: 51). Given

the example of categories of developing costs, verbalization can be identified such as *"developing costs of about* \in 10 million" or "average developing costs of about \in 10 to 20 million".

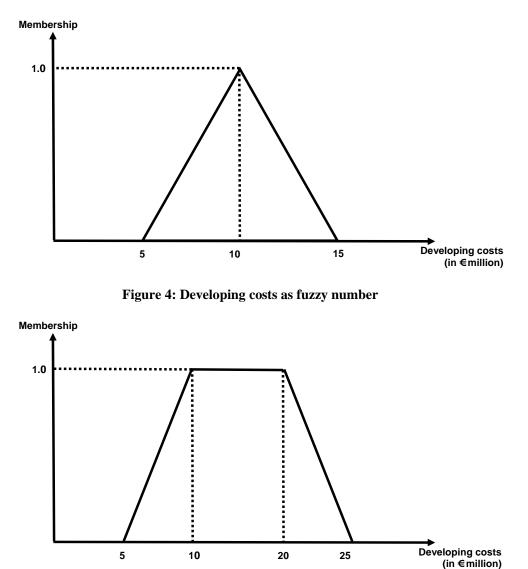


Figure 5: Developing costs as fuzzy interval

Under the perspective of a fuzzy logic based expert system to analyse functions and risk in transfer pricing cases, the variables chosen to determine the degree of routine and non-routine are defined by using linguistic variables which are based on fuzzy sets of membership. The parameter values of the variables are words of human or artificial language used to process knowledge in computers with the lowest degree of information loss. Figure 6 shows how "developing costs" of the research and development unit of a multinational company can be measured as a linguistic variable of three fuzzy sets.¹⁸ The parameter values of the variable are represented by the three membership functions, the so-called fuzzy sets "low", "medium", and "high". Normally, if fuzzy logic techniques are deployed in practical cases, experts in the respective field of application

¹⁸ Basically, more than one fuzzy set can be used to describe linguistic variables, which, however, results in more complex sets of rules (Altrock, 1996: 43).

classify the fuzzy sets of each variable. In the field of transfer pricing, the stock of expertise knowledge would easily allow to establish such classification of fuzzy set memberships for a large number of variables.

On the basis of Figure 3, developing costs of, say, $\in 8$ million (exact parameter value) with a degree of membership of 0.33 would be classified as "low" and with a degree of membership of 0.66 as "high". This statement, or assessment, is relative to the intervals of company size. The value Euro 8 million is different to assess for small companies compared to large companies. One way out of this problem is the use of relative size indicators such as "developing costs by revenues". However, because this brings a different problem, i.e. the aggregation of size into a relative number, a transfer pricing analysis would have to account for this.

The exact figure value $\in 8$ million can be fuzzified with the membership vector (0.33; 0.66; 0). For transfer pricing purposes, a similar translation of information can be proceeded for the linguistic variable "know-how" and for many other qualitative and quantitative factors influencing the degree of non-routine of a functional unit of an organization. As Figure 4 indicates, fuzzifying the linguistic variable "know-how" generates the membership vector (0; 0.40; 0.60).

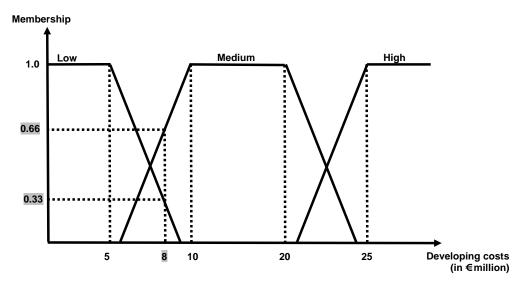


Figure 6: Developing costs as linguistic variable

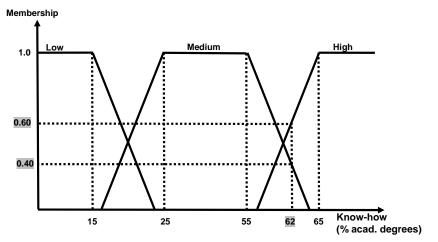


Figure 7: Know How as Linguistic Variable

The result of fuzzifying linguistic variables is a type of information which computers can process. Note that the result of fuzzification does not yet represent an assessment of the variable's parameter value. It does not yet indicate the degree of non-routine (target value). Rather, the gradual membership of the variable into fuzzy sets has been determined so far. In the next step, the inference using linguistic rules (Kuhl, 1996: 35), the relative impact of each parameter value on the degree of routine is assessed on the basis of expert opinions regarding fuzzy relationships between linguistic variables. A complete set of rules to determine the target value "degree of non-routine of a function" is given in Table 2.¹⁹

The inference process only refers to such rules whose conditions show a degree of fulfillment (DOF) larger than zero. As long as the conditions of a rule are undoubtedly true, such processing of set of rules is easy and unproblematic. However, a situation may arise in which the parameter values of one or more linguistic variables lie in the transition period between two fuzzy sets which indicates membership degree different from $1.^{20}$ In such a case, more than one rule is used to determine inference. Then, a rule is only deployed if the conditions show a positive DOF. Such "fuzzy conclusion" logic is based on the notion that any conclusion can only be true to the extent to which the underlying rule-conditions are true.²¹ The DOF of a single rule is then derived from the degree of membership of its propositions. The propositions can be linked with a logical AND operator from which follows that all constraints have to be met so that any logic conclusion can be used. Basically, it can be shown that the DOF corresponds with the lowest degree of membership μ of the rule's conditions (for the derivation of this relationships, see Bandemer/Gottwald, 1993: 124). Consequently, the DOF is defined using a Minimum Operator²² (see Definition 2).

$$DOF_{Rule} = Min[(\mu_{\tilde{A}_{1}}(x_{1}), \mu_{\tilde{A}_{2}}(x_{1}), \mu_{\tilde{A}_{3}}(x_{1})); (\mu_{\tilde{A}_{1}}(x_{2}), \mu_{\tilde{A}_{1}}(x_{2}), \mu_{\tilde{A}_{1}}(x_{2}))] = Min[\mu(x_{1}); \mu(x_{2})]$$

Definition 2: Minimum operator to process rules (AND-linked rules)

It is helpful to illustrate the fuzzy inference by means of an example. The set of rules deployed are given in Table 2. The parameter values of the linguistic variables "developing costs" (Figure 6) and "know-how" (Figure 7) results in rule which are indicated in Table 6 with underlying grey colour:

¹⁹ One way to logically conclude is the classical Modus Ponens, i.e. the rule "if A is true, then B is true" and the observation "A is true", then the conclusion is that "B is true" (Harmon/King, 1989: 56).

²⁰ If the presumptions of a rule are fulfilled to a certain degree, then the Degree Of Fulfillment ("DOF") is smaller than 1; see Biethahn et al. (1997: 185).

²¹ This is one of the strong assets of fuzzy logic techniques. Though an expert may have his own generalized model of typical parameter values in his set of rules, strong and stabile conclusions can be made (Spies (1993: 237) and regarding fuzzy reasoning. see Negoita (1985: 95).

²² Repeatedly used inference-operators (also called aggregation operators) are the Minimum Operator as logic AND operator for the rules and the Maximum Operator (OR operator). More complex systems of logical conclusions using a multiple of single sets of rules are determined by so-called Accumulation Operators. See also Hofer (1993: 42), Schulte (1993: 31), Tilli (1993: 141) and Zimmermann/Angstenberger (1993: 96).

Assessi	Assessment of the Degree of Routine using Sets of Rules from Linguistic Variables				
Rules	"Required know-how"	"Developing costs"	Assessment of Non-Routine		
1)	1 (0,00)	1 (0,33)	1 (0,00)		
2)	1 (0,00)	m (0,66)	1 (0,00)		
3)	1 (0,00)	h (0,00)	m (0,00)		
4)	m (0,40)	1 (0,33)	1 (0,33)		
5)	m (0,40)	m (0,66)	m (0,40)		
6)	m (0,40)	h (0,00)	m (0,00)		
7)	h (0,60)	1 (0,33)	m (0,33)		
8)	h (0,60)	m (0,66)	h (0,60)		
9)	h (0,60)	h (0,00)	h (0,00)		

Table 6:Activited Rules of a given set of rules and results derived from the
Minimum-Operator

The next step is now to elaborate how an overall assessment can be aggregated from these results generated by using the Minimum Operator and the activited rules. Empirical analysis has shown that the algebraic sum of the degrees of memberships satisficingly reconstructs the human decision-making pattern and, hence, can be used for the assessment of the overall parameter of the target value (here: degree of non-routine) (Rommelfanger, 1994; Zimmermann, 1993). The algebraic sum represents the membership functions which itself are logic conclusions derived from the activated rules, proportionally in the overall assessment (Rommelfanger, 1994).²³ Consequently, the so-called *Max-Prod-Inference* is defined as follows (Grauel, 1995):

$$DOF_{overall,assessment} = 1 - \prod_{i} \left[1 - DOF_{Rule,i} \right]$$

Definition 3: Max-Prod-Inference – aggregation of the rules used for the overall assessment

Given the example above, following results of Max-Prod-Inference can be shown:

	Asses	sment of memberships of target variable	
	"Degree of	Non-Routine", derived from activitated rules	
Term	Linguistic Term	Computational Term	Result
		MaxProd-Inference	
1)	l (Low)	1 - (1 - 0.00) (1 - 0.00) (1 - 0.33)	0.330
2)	m (Medium)	1 - (1 - 0.00) (1 - 0.40) (1 - 0.00) (1 - 0.33)	0.598
3)	h (High)	1 - (1 - 0.60) (1 - 0.00)	0.600

Figure 8 shows the output vector (0.330; 0.598; 0.600) of the degree of non-routine (*Output Fuzzy Set*, see Hofer, 1994). In more comprehensive approaches, there could be a hierarchical inference. This is a situation where the factors influencing the degree of non-routine of a given function can be identified with a multiple qualitative and quantitative indicator hierarchy. In such cases, the

²³ As an alternative, the Max-Min-Inference can be also chosen which is easier to handle from a computational point of view. The Max-Min-Inference does not reduce the membership functions proportionally but it cuts off the degree of membership (see also Rommelfanger, 1994: 160)

first partial result can be transferred to the superior sets of rules which an expert team can define regarding their hierarchical structure.

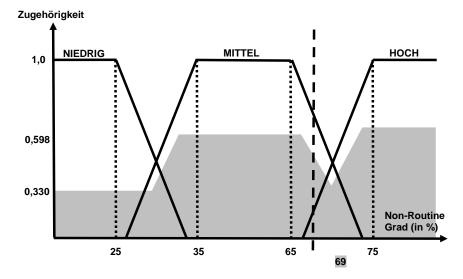


Figure 8: Overall assessment of the degree of non-routine of the research and development function

In some cases of transfer pricing issues, instead of generating an output fuzzy set the situation may require to produce an exact total output of a distinct yes-or-no type or an exact number output. Referring to our example, a certain number value regarding the degree of non-routine could be used. For that, it is necessary to defuzzify the overall result on the basis of the expert system.²⁴ Different approaches exist for the transformation of the fuzzy output set into a discrete, exact result value.²⁵ The two most often used approaches are the Centre of Gravity method and the Bisection Method (also called Planar Bisection Method). The former identifies the area of centre of the fuzzy output set; the latter calculates in a first step the size of area by means of an exact measure of area size and in a second step does align, by bisecting, the output fuzzy sets to the area.

In our example, the Centre of Gravity Method produces the exact overall result of 69% as the degree of non-routine for research and development activies (Figure 8). Hence, the function can be classified as non-routine, rather than routine. In transfer pricing using a value chain approach, this information can then be used, for example, to allocate residual profit to this non-routine function, after having remunerated routine functions. Also, fuzzy logic approaches can be used to determine other non-routine functions along the value chain and to split the residual profit into these non-routine functions.

²⁴ This is often necessary in technical systems (e.g. temparature control systems). In our example, the determination of the degree of non-routine would benefit from an exact degree measure of non-routine, if the results do not have to be processed further in the context of setting the hierarchy of criteria. However, to avoid loss of information, in such a situation it is better to use the fuzzy total result. In other words, defuzzifying should be part of the last step of the decision-making process modelling.

²⁵ For an overall review and discussion of various types of defuzzifying, see Zimmermann/Angstenberger (1993), Traeger (1993), and Graham (1988). In Figure 8, the resulting value of 69 per cent is shown by the dash line. This value is the area centre of gravitity, as indicated by grey coloured area, of the given fuzzy output set of inference.

4. Outlook

Our example shows that, though various challenges to implement fuzzy logic lie ahead, it can be a very helpful technique for decision-making tasks in transfer pricing analysis, especially when the function and risk analysis is to be done. This paper is a first step to provide an overview of the methodological approach behind fuzzy logic, which is in our mind a very promising one to mitigate the many pitfalls and shortcomings behind heuristic methods to determine routine and non-routine functions and to assess the arm's length nature of transfer prices in multinational companies. While these heuristic models lack standardization and transparency in terms of theoretically sound valuation, fuzzy logic theory is a worthy candidate for that. To further promote this approach, other papers will follow.

By making use of available theoretical and practitioner know-how, the next steps is to provide for the comprehensive and deterministic hierarchy of qualitative and quantitative indicators which can be used for function and risk analysis. For that, we use information from practitioner literature, expert interviews, and theories to establish the distinction between routine and nonroutine. The challenge will be the collection of such available know-how.

In a second step, the indicators considered to be relevant for transfer pricing analysis will be transformed into linguistic variables. Here, the challenge is to identify adequate membership functions. The quality of developing fuzzy sets primarly depends upon the degree to which the membership functions reflect the view of experts. Hence, a membership function is conditioned by subjective opinion. Also, the shape of the membership function is subject to approximation and establishment of "landmarks" – by the way to set landmarks is a very common feature of mainstream transfer pricing analysis. In our fuzzy logic perspective, the objective behind the determination of landmark features (shape, position) of simple membership functions is a high degree of standardization regarding the overall pattern of the transfer pricing case. We suggest to do so because we believe that the acceptability and transparency of a given methodological approach for transfer pricing purposes depends upon such standardization: fuzzy logic would be one such approach.

Our current project deals with the definition of a hierarchy of qualitative and quantitative indicators to describe the function and risk value behind a given transfer pricing case. The determination of membership functions on the basis of quantitative features out of company databases appears to be very promising. We produce typical landmark parameter values for trapezoidal and triangle shapes of membership functions as derived from the statistic variables of subsamples in such databases. The benefit using such approach to generate membership functions can be derived from its objective, standardized, and transparent approach and its comparatively easy accomplishment and implementation.

In this paper, we have discussed the merits of fuzzy logic approaches for the function and risk analysis. We believe, however, that fuzzy logic not only serves for function and risk analysis issues but can also be used to a much larger extent in transfer pricing. For example, fuzzy sets may allow to compare products, to screen for comparable companies (database screening), or even to produce a benefit analysis in the context of cost sharing problems. Also, in addition to valuation issues, the fuzzy set approach in the sense of fuzzy clustering can be used for catergorization questions such as establishing patterns of similar companies. In our words, whenever vague decision-making problems appear– and this in transfer pricing is the rule rather than the exeption, including in cases of characterizing related-party business with legal terms – fuzzy logic theory appears to be a promising tool for the next generation of transfer pricing.

In our opinion, the shortcomings of mainstream transfer pricing approaches has to do with fact that the key decision-making processes including valuation issues (e.g. selection of related-party

transaction, function and risk analysis, arm's length analysis, valuing intangibles) are erroneously based on exact distinctions of artificial categories. As a result, if two intependent analysts assess the same fact pattern in transfer pricing, they are very likely to produce significantly different outputs, since each person defines his own criteria catalogue and, within that, its own intervals of each criteria for assessing a given case.

Currently, we use fuzzy logic theory to develop an index measure (Brem-Tucha-Ratio) which describes the relative advantageousness of hybrid governance forms (inside the multinational firm) compared to market governance on the one hand and hierarchical governance on the other (Brem and Tucha, 2006). Transaction cost economics forms the theoretical economic basis to justify why transfer prices between related parties within the multinational group can be different in dollar terms from market prices (if existent at all) and costs of internal production (hierarchy). Fuzzy logic is the technique to model the decision-making process. Given the shortcomings of mainstream transfer pricing, we assume that this approach will lead to transfer pricing results more consistent with the nature of the arm's length principle.

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