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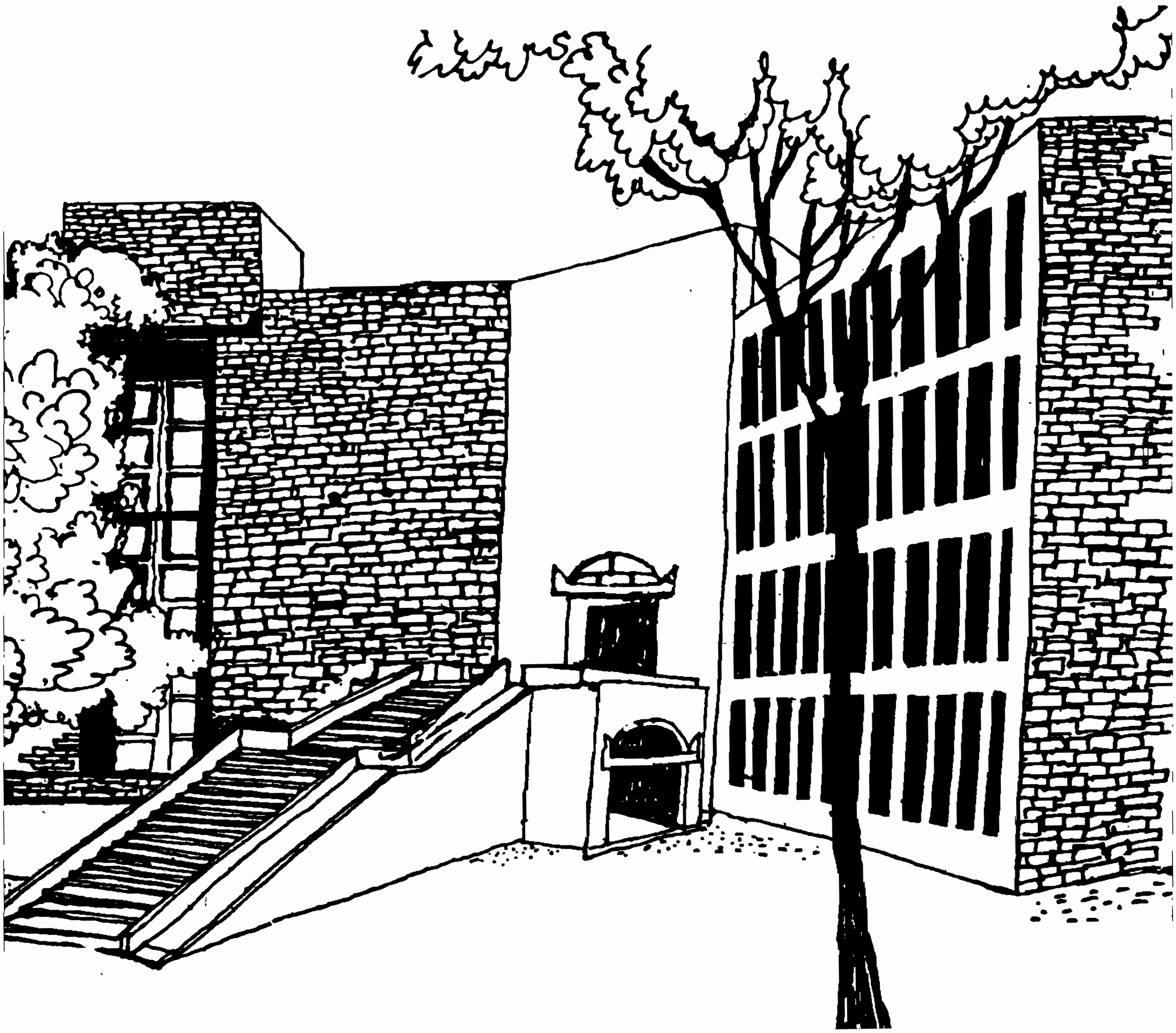

SHIFTS IN CHOICE PROBLEMS

By

Somdeb Lahiri

W. P. No. 1175
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INDIAN INSTITUTE OF MANAGEMENT
AHMEDABAD - 380 015
INDIA

Abstract

In this paper we consider choice problems which are bounded both above and below and provide a new axiomatic characterization of the equal loss choice function. We subsequently turn to a study of various properties implied by shifts in the choice problem. one of which was used in characterizing the equal loss solution. Then we characterize rational choice behaviour when a decision maker is confronted with a choice problem. Finally we turn to an axiomatic characterization of a particular rationalizable choice function - the utilitarian choice function - using a shift invariance property.

1. Introduction :- **A choice problem is a feasible set of attribute vector contained in the nonnegative orthant of a finite dimensional Euclidean** space, together with a target point which is also contained in the same orthant. A choice function defined on a set of choice problems, assigns to each problem a feasible attribute vector. A shift in a choice problem moves the choice problem inwards in a specified direction. The theory of choice problems as studied here has its origin in a series of papers by Chun (1988), Chun and Thomson (1992), Chun and Peters (1989), Bossert (1992a,b), Lahiri (1993a,b) and Abad and Lahiri (1993).

In Yu (1985), can be found a theory of multiattribute choice problems and a statement of the equal loss choice function for such problems. There a number of properties of this and other compromise solutions has been discussed. In Lahiri (1993b) the framework was partially extended to study a certain "monotonicity with respect to the target point" property of the entire family of compromise solutions suggested by Yu. In Chun and Thomson (1992) and Lahiri (1993a) different axiomatic characterizations have been provided for a different choice function to multiattribute choice problems which satisfies a property called "restricted monotoniciity with respect to the target point". An axiomatic characterization of a choice function is a statement of some properties which uniquely characterizes the choice function. In this respect the earliest axiomatic characterization of the equal loss solution is due to Chun (1988). Subsequently Bossert (1992b) provided a different axiomatic characterization of the same solution. However, both these studies considered domains which admitted unbounded choice problems.

In this paper, we consider choice problems which are bounded both above and below. This is more in keeping with the spirit of multiattribute choice theory as enunciated for instance in Keeney and Raiffa (1976) and Yu (1985). As a result of this modification, the earlier characterization results break down and what replaces it is completely original both in content and style. Application of this choice theory to production planning problems can be found in Abad and Lahiri (1993) and Lahiri (1993a).

We subsequently turn to study in this paper monotonicity (of choice functions) with respect to unilateral shifts. This adapts to our chosen domain the concept of monotonicity with respect to the disagreement point due to Thomson (1987). On domains similar to those studied by Thomson (1987), we obtain similar results with slightly modified proofs. On a

...what extended domain studied in Lahiri (1993a,b) we obtain the result that the equal loss choice function and the choice function which select the unique efficient point on the straight line connecting the origin to the target point both satisfy monotonicity with respect to unilateral shifts.

Then, we proceed to a study of a property called concavity with respect to shift, which adapts to our framework a concept due to Chun and Thomson (1990a,b). We show that concavity with respect to shifts imply a certain stability property of the choice function.

In Peters and Wakker (1991) can be found a theory of rational choice in choice problems, which answers the question: When is a choice function a maximizer of a real valued function defined on the nonnegative orthant of an Euclidean space. They draw on the consumer demand theory in microeconomics and answer the question along the lines of Richter (1971), Varian (1982) and Pollack (1990).

However, Sondermann (1982) has provided an elementary treatment of revealed preference and a purpose of this paper is to obtain a similar treatment for multiattribute choice problems.

Then, we turn to a study of the utilitarian choice function and obtain a new characterization result for the same. This solution is well-known in the literature on social choice and independent characterizations of the same can be found in Myerson (1981) and Thomson (1981). We propose yet another characterization in this paper.

. Choice Problems :- A choice problem is an ordered pair (S,c) where $\emptyset \in S \subseteq \mathbb{R}^n_+$ and $c \in \mathbb{R}^n_+$ for some $n \in \mathbb{N}$ (the set of natural numbers). S is called the feasible set of attribute vectors and c is called the target point. We shall following Moulin (1988) consider the following class $\mathcal{C}$ of admissible choice problems: $(S,c) \in \mathcal{C}$ if and only if

i) S is nonempty, compact and convex;

ii) S satisfies <u>minimal transferability</u> : $\forall x \in S$ $\forall i \in \{1,...,n\}$, if $x_i > 0$, there exists $y \in S$ with $y_i < x_i$ and $y_j > x_j \forall j \neq i$.

iii) S is comprehensive: $0 \leq y \leq x \in S \Rightarrow y \in S$.

A <u>domain</u> is any subset of $\mathcal{C}$.

Let D be a domain. A choice function is a function $F: D \to \mathbb{R}^n_+$ such that $F(S,c) \in S$ $\forall$ $(S,c) \in D$.

We shall consider two important domains apart from $\mathcal{C}$ itself in the subsequent analysis.

$\mathcal{B}_u$ = {(S,c)∈$\mathcal{B}$/c=u(S) where u_i(S)=max {x_i /x∈S}}.

A problem (S,c) in $\mathcal{B}_u$ is denoted simply by S.

$\mathcal{B}^0$ = {(S,c)∈$\mathcal{B}$/S ≠ {0} =>c>>x(P(S)} where P(S)={x∈S/y≧x,y∈S=>y=x}.

The domain $\mathcal{B}_u$ is referred to in the literature as the class of bargaining problems. We shall refer to $\mathcal{B}^0$ as the class of proper choice problems. It is easy to verify that $\mathcal{B}_u \subseteq \mathcal{B}^0$. (In the above for x,y∈$\mathbb{R}^n$,x≧y<=>x_i ≧y_i ∀i=1,...,n;/x>y<=>x≧y and x ≠ y;x>>y<=>x_i >y_i ∀i=1,...,n).

On $\mathcal{B}_u$ we consider the following two choice functions:

(1) F_N :$\mathcal{B}_u$ ->$\mathbb{R}^n_+$ is called the Nash (1950) choice function and defined as

$$F_N(S) = \arg\max_{x\in S} (\pi^n_{i=1} x_i)$$

(2) F_E :$\mathcal{B}_u$ ->$\mathbb{R}^n_+$ is called the egalitarian choice function and defined as

F_E (S) = $\bar{\lambda}$e where $\bar{\lambda}$=max{λ≧0/λe∈S}.

On $\mathcal{B}^0$ we consider the following choice function

(3) F_{RE} : $\mathcal{B}^0$ ->$\mathbb{R}^n_+$ is called the relative egalitarian choice function and defined as

F_{RE} (S,c) = $\bar{\lambda}$c where $\bar{\lambda}$=max {λ≧0/λc∈S}.

To define our final choice function we consider a subdomain $\tilde{\mathcal{B}}$ of $\mathcal{B}^0$:

$\tilde{\mathcal{B}}$ ≡ (S,c)∈$\mathcal{B}^0$ /c-($\min_i c_i$)e∈S} where e is the vector in $\mathbb{R}^n$ with all coordinates equal to 1.

(4) F_{EL} : $\tilde{\mathcal{B}}$ ->$\mathbb{R}^n_+$ is called the equal loss choice function and defined as

F_{EL} (S,c) = [c-($\min_i c_i$)e]+$\bar{\lambda}$($\min_i c_i$)e

where $\bar{\lambda}$ = max(λ≧0/[c-($\min_i c_i$)e+λ($\min_i c_i$)e∈S}.

This solution is originally due to P.L. Yu.

Let D be a domain and (S,c)∈D. If given a∈$\mathbb{R}^n_+$,(S(a),c-a)∈D where S(a)={x-a/x∈S}∩$\mathbb{R}^n_+$, then we say that (S(a),(c-a)) is a shift of (S,c)∈D.

Let $F:D \to \mathbb{R}^n_+$ be a choice function. We say that F satisfies monotonicity with respect to unilateral shifts if $\forall$ $i\in\{1,\ldots,n\}$ $\forall$ $\alpha_i \geq 0, a=\alpha_i e, (S,c), (S(a),c-a)\in D$ implies $F^i(S(a),c-a)+\alpha_i \geq F^i(S,c)$. We say that F satisfies strict monotonicity with respect to unilateral shifts if $\forall$ $i\in\{1,\ldots,n\}$ $\forall\alpha_i > 0, a=\alpha_i e, (S,c), (S(a),c-a)\in D$ implies $F^i(S(a),c-a)+\alpha_i > F^i(S,c)$.

We say that $F:D \to \mathbb{R}^n_+$ is concave with respect to shifts if $\forall(S,c)\in D, (S(a),c-a), (S(a'),c-a'), S(ta+(1-t)a'), c-ta(1-t)a')\in D \forall t\in[0,1]$ implies $F(S(ta+(1-t)a'),c-ta-(1-t)a') \geq tF(S(a),c-a)+(1-t)F(S(a'),c-a')$

3. An Axiomatic Characterization of the Equal Loss Choice Function :-

Let $F:D \to \mathbb{R}^n_+$ be a choice function. Three important properties often required of a choice function are the following:

(P.1) Efficiency :- $\forall$ $(S,c)\in D$, $x\in S$, $x \geq F(S,c) \Rightarrow x=F(S,c)$

(P.2) Symmetry :- If $\forall$ permutation $\sigma:N\to N$, $\sigma(S)=S$ and $\sigma(c)=c$, then $F_i(S,c)=F_j(S,c)$ $\forall$ $i,j\in\{1,\ldots,n\}$.

Here for $x\in\mathbb{R}^n_+$, $\sigma(x)$ is the vector in $\mathbb{R}^n_+$ whose ith coordinate is $x_{\sigma(i)}$ and $\sigma(S)=\{\sigma(x):x\in S\}$.

(P.3) Restricted Monotonicity :- $\forall(S,c)$, $(S',c)\in D$, $S \subseteq S'$ $\Rightarrow F(S,c) \leq F(S',c)$.

In order to characterize F_{EL} axiomatically we will require the following property:

Let $(S,c)\in D$ and $a\in\mathbb{R}^n_+$. Then if $a\leq c$ and $(S-\{a\}) \cap \mathbb{R}^n_+ \neq \emptyset$, we have the choice problem $(S(a),c-a)\in D$ where $S(a)=(S-\{a\})\cap\mathbb{R}^n_+$.

(P.4) c-Shift Invariance :- $\forall(S,c)\in D$ $\forall$ $a\in\mathbb{R}^n_+$ such that $a\leq c-[\min_i(c_i)]e$.

Now we show that the above four properties characterize F_{EL} on $\bar{\mathcal{Q}}$.

Theorem 1 :- The only choice function on $\bar{\mathcal{Q}}$ to satisfy properties (P.1), (P.2), (P.3) and (P.4) is the equal loss choice function.

Proof :- It is easily verified that F_{EL} satisfies the above properties. Hence let $F:\bar{\mathcal{Q}} \to \mathbb{R}^n_+$ be any choice function satisfying the above properties. If $S=\{0\}$, then by the definition of a choice function $F(S,c)=0=F_{EL}(S,c)$. Hence suppose $S \neq \{0\}$ and let $a=c-[\min_i(c_i)]e$. Then $F_{EL}(S(a),c-a)=\bar{\lambda}.e$ for some $\bar{\lambda} \geqq 0$.

If $S(a)=\{0\}$, then $\bar{\lambda}=0$ and $F(S(a),c-a)=0$, so that by appealing to (P.4) we get $F(S,c)=F_{EL}(S,c)$. Hence suppose $S(a) \neq 0$, so that $\bar{\lambda}>0$. By minimal transferability, $\forall$ $i\in\{1,\ldots,n\}$, there exists $v^i \in S(a)$, such that $v^i_i < \bar{\lambda}$ and $v^i_j > \bar{\lambda}$ if $j \neq i$. Let $\alpha=\min_{1\leqq i \neq j\leqq n} v^i_j$.

Clearly $\alpha> \bar{\lambda}$. For $i\in\{1,\ldots,n\}$, let $a^i \in \mathbb{R}^n_+$ such that $a^i_i=0$, $a^i_j=\alpha$ for $j \neq i$. Clearly $a^i < v^i$ and by comprehensiveness, $a^i \in S$ $\forall$ $i\in\{1,\ldots,n\}$. Let T=convex hull $\{0,a^1,\ldots,a^n,\bar{\lambda}e\}$. T is symmetric, $\bar{\lambda}e$ is efficient in T, $\bar{\lambda}e$ has all coordinates equal to $\bar{\lambda}>0$ and $c-a$ has all coordinates equal to $\min_i(c_i)$.

Hence by (P.2) and (P.1), $F(T,c-a)=\bar{\lambda}e$. Now, $T\subseteq S(a)$. Hence by (P.3) $F(S(a),c-a)\geq F(T,c-a)=\bar{\lambda}e$. However $\bar{\lambda}e=F_{EL}(S(a),c-a)$ is an efficient point in $S(a)$. Thus $F(S(a),c-a)=\bar{\lambda}.e=F_{EL}(S(a),c-a)$. By (P.4) applied to both F and F_{EL} we get since $a\leqq c-[\min_i(c_i)]e$,

$F(S,c)=F_{EL}(S,c)$.

4. Monotonicity with respect to unilateral shifts on $\mathcal{C}_u$:-

A choice function $F:D \to \mathbb{R}^n_+$ is said to be scale independent if $(S,c)\in D,(a.S,a.c)\in D$ for $a\in\mathbb{R}^n_{++}$ implies $F(a.S,a.c)=a.F(S,c)$. (Here for $x\in\mathbb{R}^n$, $y\in\mathbb{R}^n$, $x.y\equiv(x_1 y_1,\ldots,x_n y_n)$ and for $S\subseteq\mathbb{R}^n_+$ $x.S=\{x.y/y\in S\}$. $\mathbb{R}^n_{++}\equiv\{x\in\mathbb{R}^n / x_i>0\forall i=1,\ldots,n\}$.) A choice function $F:D \to \mathbb{R}^n_+$ is said to satisfy strict individual rationality if $\forall(S,c)\in D$, $F(S,c)\gg0$.

<u>Theorem 2</u> :- Both $F_N : \mathcal{C}_u \to \mathbb{R}^n_+$ and $F_E : \mathcal{C}_u \to \mathbb{R}^n_+$ satisfy monotonicity with respect to unilateral shifts. In fact they do so strictly.

<u>Proof</u> :- F_N satisfies efficiency, scale independence and strict individual rationality. Let $S\in\mathcal{C}_u$, $S \neq \{0\}$, $a=\alpha_i e$, $\alpha_i>0$ $S(a)\in\mathcal{C}_u$. By scale independence we may assume, $F_N(S)=e$.

Since $e-a\in S(a)$, $1-\alpha_i < \pi^n_{j=1} x_j$. Since $x+a\in S$, $(x_i+\alpha_i)\pi_{j\neq i}x_j < 1$. By strict individual rationality, $x_i>0$, if $S(a) \neq \{0\}$.

$\therefore (x_i+\alpha_i)\,\pi^n_{j\neq i}\, x_j < x_i$. Similarly $(1-\alpha_i)(x_i+\alpha_i)$ $<(x_i+\alpha_i)\pi^n_{j=1}\, x_j$. Thus $x_i > (1-\alpha_i)(x_i+\alpha_i)$. Thus $x_i > x_i - \alpha_i x_i + \alpha_i - \alpha^2_i$. This implies since $\alpha_i>0$, $x_i+\alpha_i>1$. If $S(a)=\{0\}$, then $1-\alpha_i<0$ i.e. $\alpha_i+0>1$ and hence $F^i_N(S(a))+\alpha_i>F^i_N(S)$.

Now let us consider $F_E:\mathcal{L}_u \to \mathbb{R}^n_+$. F_E is efficient. Let $S\in\mathcal{L}_u$, $\alpha_i>0$, $a=\alpha_i e$, $S(a)\in\mathcal{L}_u$. Suppose $F^i_E(S(a))+\alpha_i<F^i_E(S)$. Thus $F^i_E(S(a))\leq F^i_E(S)-\alpha_i<F^i_E(S)$. Since $F^j_E(S(a))=F^i_E(S(a))$ $\forall$ $j=1,\ldots,n$, $F^j_E(S)=F^i_E(S)$ $\forall$ $j=1,\ldots,n$ and $F_E(S)-a\in S(a)$, we get $F_E(S)-a>F_E(S(a))$, contradicting the efficiency of F_E.

Q.E.D.

5. Monotonicity with respect to unilateral shifts on other domains :-

<u>Theorem 3</u> :- (i) On $\mathcal{L}^0$, F_{RE} satisfies monotonicity with respect to unilateral shifts. In fact, F_{RE} satisfies strict monotonicity with respect to unilateral shifts

(ii) On $\bar{\mathcal{L}}$, F_{EL} satisfies monotonicity with respect to unilateral shifts.

<u>Proof</u> :- (i) F_{RE} satisfies scale independence and efficiency. Let $\{0\}\neq(S,c)\in\mathcal{L}^0$ be such that $c=e$. We can do this by scale invariance. Let $F_{RE}(S,c)=\bar{\lambda}e$, $\bar{\lambda}>0$. Let $\alpha_i>0$, $a=\alpha_i e$, $(S(a),c-a)\in\mathcal{L}^0$ and $F_{RE}(S(a),c-a)=\bar{\mu}(c-a)$. Assume towards a contradiction that $\alpha_i+\bar{\mu}(1-\alpha_i)\leq\bar{\lambda}$. Since the point x with $x_i=\bar{\lambda}-\alpha_i$, $x_j=\bar{\lambda}$ $\forall$ $j\neq i$ belongs to $S(a)$, we must have that $\bar{\mu}\geq\bar{\lambda}$ otherwise we would be contradicting the efficiency of $F_{RE}(S(a),c-a)$. But then $\alpha_i(1-\bar{\mu})\leq 0$. Since $\alpha_i>0$, we must have $\mu\geq 1$, which is impossible. Hence $\alpha_i+\bar{\mu}(1-\alpha_i)>\bar{\lambda}$ and F_{RE} satisfies strict monotonicity with respect to unilateral shifts.

(ii) Let $(S,c)\in\bar{\mathcal{L}}$, $\alpha_i>0$, $a=\alpha_i e$. It is easy to see that F_{EL} satisfies the following property discussed in Lahiri (1993c):

<u>c-Shift Invariance</u> :- $\forall (S,c)\in\bar{\mathcal{C}}\ \forall b\in\mathbb{R}^n_+$ such that $b \leqq c-[\min_i(c_i)]e$,

$F(S(b),c-b)=F(S,c)-b$.

Hence F_{EL} satisfies monotonicity with respect to unilateral shifts.

Q.E.D.

6. Concavity With Respect to Shifts :- Let D be a domain such that if $(S,c)\in D$, $(S(a),c-a),(S(a'),c-a')\in D$ for some $a,a'\in\mathbb{R}^n_+$, then $(S(ta+(1-t)a'),c-ta-(1-t)a')\in D\ \forall t\in[0,1]$.

<u>Theorem 4</u> :- Let $F:D\rightarrow\mathbb{R}^n_+$ be an efficient choice function, which satisfies $F(S,c)\leqq c$. Then if F satisfies concavity with respect to shifts, then given $(S,c)\in D\ \forall a=tF(S,c)$, $t\in[0,1]$, $F(S(a),c-a)=F(S,c)-a$

<u>Proof</u> :- Let $a'=F(S,c)$ and $b=0$.

Then $(S(a'), c-a')=(\{0\},c-a')$, so that $F(S(a'),c-a')=0$.

$(S(b),c-b)=(S,c)$.

Thus

$F(S(ta'),c-ta')\geqq(1-t)F(S,c)\quad\forall\ t\in[0,1]$ (which follows from concavity). The efficiency of $(1-t)F(S,c)$ in $S(ta',c-ta')$ implies $F(S(ta'),c-ta')=(1-t)F(S,c)\ \forall\ t\in[0,1]$

i.e. $F(S(a),c-a)=F(S,c)-a\ \forall\ a=tF(S,c),\ t\in[0,1]$.

Q.E.D.

<u>Note</u> :- The assumption that $F:D\rightarrow\mathbb{R}^n_+$ satisfy $F(S,c)\leqq c$ implies a domain restriction. Thus for instance an $(S,c)\in\mathcal{C}$ with $x\in S$ such that $x>c$ would automatically be excluded. However the domain is large enough to include bargaining problems as a strict subset.

7. Representable choice functions on $\mathcal{C}_u$:- We now try to impose conditions on a choice function $F:\mathcal{C}_u\rightarrow\mathbb{R}^n$ such that there exists a real valued function $f:\mathbb{R}^n_+\rightarrow\mathbb{R}$ and $\forall S\in\mathcal{C}_u$,

$$F(S)=\arg\max_{x\in S} f(x).$$

Such choice functions will be called representable choice functions.

Given a choice function $F:\mathcal{C}_u \to \mathbb{R}^n$ we define a binary relation R_F on $\mathbb{R}^n_+$ as follows : $xR_F y$("x is directly revealed preferred to y") if there is an $S\in\mathcal{C}_u$ with $x=F(S), y\in S$.

We now postulate the following axiom which is essentially sufficient for the representability of choice functions:

Strong Axiom of revealed Preference (SARP) :- R_F is acyclic; i.e. $x^1 R_F x^2 R_F \ldots R_F x^k$ implies not $x^k R_F x^1$ where $x^1,\ldots,x^k \in \mathbb{R}^n_+$ and are necessarily distinct.

Let H_F denote the <u>transitive hull</u> of R_F that is, $xH_F y$ if and only if $xR_F x^1 R_F x^2 \ldots R_F y$ for some finite (possibly empty) sequence $x^1,\ldots,x^k$ in $\mathbb{R}^n_+$. Then the Strong Axiom is equivalent to : H_F is irreflexive.

We now prove the following theorem, which is essentially a slight modification of Sondermann (1982).

<u>Theorem 5</u> :- Let $F:\mathcal{C}_u \to \mathbb{R}^n$ be a choice function satisfying the following connectedness property:

$\forall x, y\in\mathbb{R}^n_+$ such that $x, y\in\{F(S) : S\in\mathcal{C}_u\}$ and $xR_F y$ there exists $U\in\mathcal{C}_u$ and $t\in(0,1)$ such that $tx+(1-t)y=F(U)$, and $\{x,y\}\cap$ interior $(U) \neq \emptyset$. Then the Strong Axiom of Revealed Preference implies that F is a representable choice function.

<u>Proof</u> :- (Almost as in Sondermann (1982)) : The topology of $\mathbb{R}^n_+$ has a countable base of open sets, say $\{O_k\}_{k\in\mathbb{N}}$ For $x=F(S), S\in\mathcal{C}_u$ define $N(x)=\{k\in\mathbb{N} : x\in O_k$ or $wH_F x$ for some $w\in O_k\}$ and $f(x) =\Sigma_{k\in N(x)} 2^{-k}$. For $x\in\mathbb{R}^n_+ \setminus \{F(S):S\in\mathcal{C}_u\}$ set $f(x)=-1$. For $x \neq y$, let $xR_F y$. Clearly, by transitivity of H_F, $N(x)\subseteq N(y)$, hence $f(x)\geq f(y)$. If $y \notin \{F(S):S\in\mathcal{C}_u\}$, then $f(x)\geq 0>-1=f(y)$. Otherwise by hypothesis there exists $t\in(0,1)$, $U\in\mathcal{C}_u$ such that $z=tx+(1-t)y=F(U)$ and $\{x,y\}\cap$ interior $(U) \neq \emptyset$.

Let $x=F(S)$. $xR_F y \Rightarrow y\in S$; thus by convexity of S, $z\in S$. Further $z \neq x$ since $t\in(0,1)$. Thus $xR_F z$. By the Strong Axiom of Revealed Preference we have not $zR_F x$ i.e. $x \notin U$. Thus $y\in$ interior (U). Hence $\exists\, k\in\mathbb{N}$ such that $y\in O_k \subseteq$ interior (U) and $z \notin$

O_k i.e. $xR_F\ zR_F\ w\ \forall w \in O_k$. By the Strong Axiom of Revealed Preference $\forall w \in O_k$, not $wH_F x$ i.e. $k \in N(y) \setminus N(x$. Thus $f(x)>f(y)$. This completes the proof.

Q.E.D.

The above result could have been proved on any domain $D(\subseteq \mathcal{L}_u)$, so long as it is closed under the operation required in the hypothesis of Theorem 1. Such a domain is $\mathcal{L}^0_u$ defined as follows :

Given $S \subseteq \mathbb{R}^n$, let $P(S)=\{x \in S / y \geq x, y \in S \Rightarrow y=x\}$. P(S) is called the <u>set of efficient points of S</u> . We define

$$\mathcal{L}^0_u = \{S \in \mathcal{L}_u / x,y \in S,\ t \in (0,1) \Rightarrow tx+(1-t)y \notin P(S)\}.$$

8. The Utilitarian Choice Function :- On $\mathcal{L}^0_u$ the utilitarian choice function $F_{ut} : \mathcal{L}^0_u \to \mathbb{R}^n$ is defined as follows :

$$F_{ut}(S) = \arg\max_{x \in S} \left(\Sigma^n_{i=1} x_i\right)$$

It is easy to see that on $\mathcal{L}^0_u$ the utilitarian choice function is well defined. Myerson (1981) has established the additivity of the utilitarian choice function i.e. if one adds two sets in $\mathcal{L}^0_u$, then the utilitarian choice for the sum is the sum of the utilitarian choices for each. Myerson proceeds to show that additivity, anonymity and efficiency essentially characterize the utilitarian choice function on $\mathcal{L}^0_u$. It can also be observed that F_{ut} satisfies the conditions of theorem 1.

Moulin (1988) proposes a characterization of the utilitarian choice function by considering a class of unbounded choice problems. We propose a characterization on $\mathcal{L}^0_u$ itself, since as much of multiattribute choice theory suggests, a domain should necessarily be a subset of $\mathcal{L}$. We consider the following property for a choice function $F: \mathcal{L}^0_u \to \mathbb{R}^n$.

(P.5) <u>Shift Invariance</u> :- For $S \in \mathcal{L}^0_u$ and $a \in \mathbb{R}^n_+$, let $S(a)=\{x-a : x \in S\} \cap \mathbb{R}^n_+$. ($S(a) \in \mathcal{L}^0_u$ since $a \in \mathbb{R}^n_+$). If $S \in \mathcal{L}^0_u$ and $0 \leq a \leq F_{ut}(S)$, then

$$F(S(a)) = F(S)-a.$$

A property that turns out to be important now is the following:

(P.6) Nash's Independence of Irrelevant Alternatives (NIIA) :- $\forall S, S' \in \mathcal{L}_u$: $(S \subseteq S'$ and $F(S') \in S) \Rightarrow (F(S) = F(S'))$.

Theorem 6 :- There is only one choice function on $\mathcal{L}^0_u$ which satisfies anonymity, efficiency NIIA and shift invariance. It is the utilitarian choice function.

Proof :- Let $S \in \mathcal{L}^0_u$. That F_{ut} satisfies the above properties is clear. If $S = \{0\}$, then $F(S) = 0 = F_{ut}(S)$. Hence suppose $S \neq \{0\}$ and let $x^* = F_{ut}(S)$. Let $\bar{\lambda} = \sup\ \{\lambda \geq 0 / x^* - \lambda e \in \mathbb{R}^n_+\}$ where e is the vector in $\mathbb{R}^n_+$ with all coordinates being equal to unity. Let $a = x^* - \bar{\lambda} e \leq x^*$ $a \in \mathbb{R}^n_+$ by the definition of $\bar{\lambda}$. It is easily observed that $F_{ut}(S(a)) = \bar{\lambda} e$. Let T be the smallest symmetric set in $\mathcal{L}^0_u$ containing S(a). Clearly $\bar{\lambda} e \in T$; in fact $\bar{\lambda} e \in P(T)$. By efficiency and anonymity, $F(T) = \bar{\lambda} e \in S(a)$. (Observe $\bar{\lambda} e \in P(S(a))$). By NIIA, $F(S(a)) = \bar{\lambda} e$, thus completing the proof.

Q.E.D.

9. Conclusion :- As discussed in the introduction to the paper, we have here dwelt on introducing and analyzing the concept of shift invariance in the context of choice problems, in order to axiomatically characterize existing choice functions. The proof of Theorem 2 is almost identical to the proof of the corresponding theorem in Thomson (1987). It has been provided primarily for completeness and whatever ingenuity there exists in the first half of the proof.

The properties we invoke to characterize the various choice functions are both elegant and concise. The meaningfulness of the properties characterizing the choice functions should enhance their appeal in group decision making.

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