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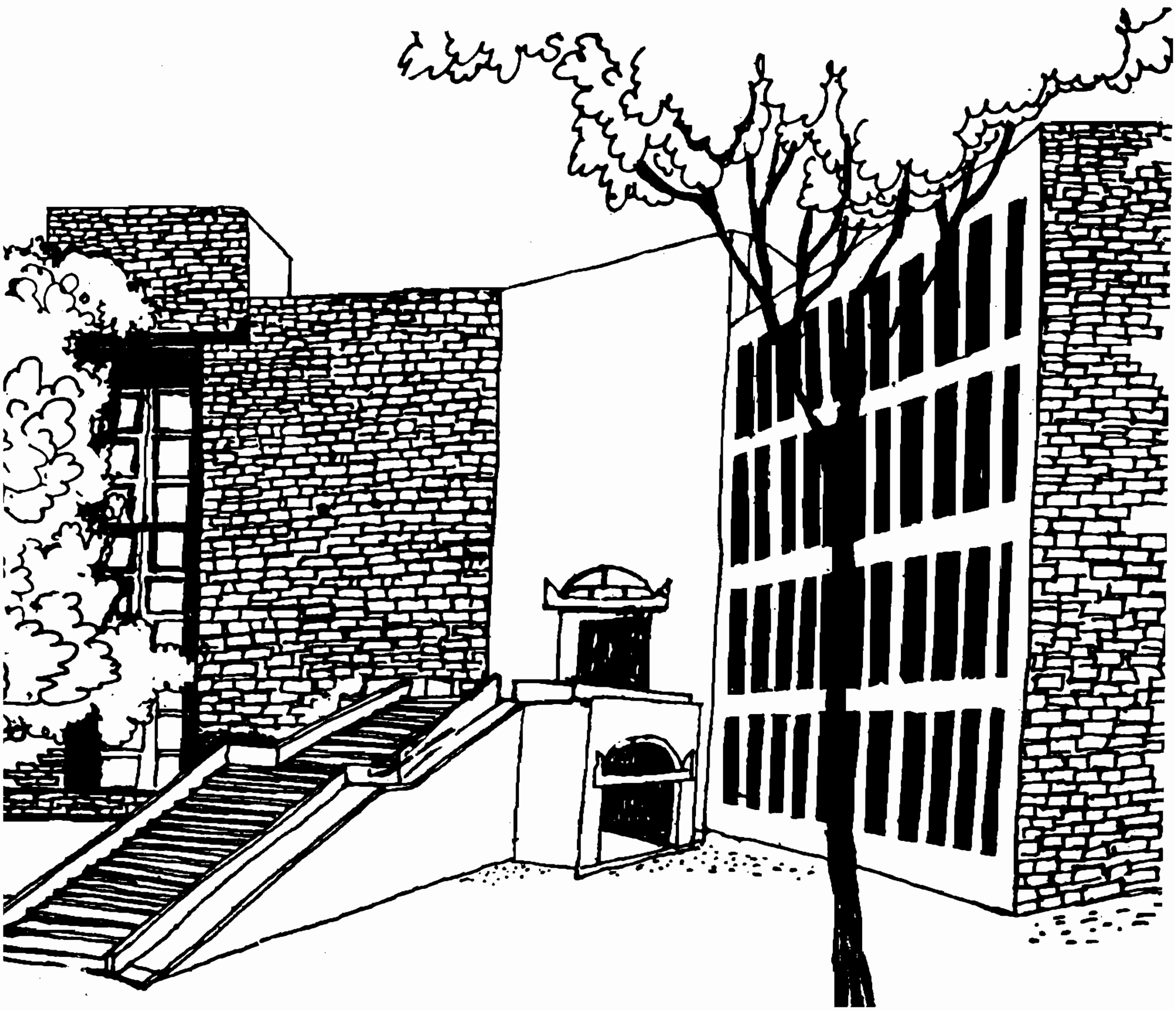

RESOURCE MONOTONICITY OF BARGAINING SOLUTIONS

By

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Abstract

In this paper we establish that the main solutions to bargaining problems display a resource monotonicity property in bilateral monopoly situations when preferences exhibit consumption externalities. Suitable assumptions are invoked to establish the results.

1. **Introduction** : The purpose of this paper is to establish (under suitable assumptions) a resource monotonicity property of the main bargaining solutions in a bilateral monopoly, when the preferences of the agents exhibit consumption externalities.

Following Chatterji (1986), Lahiri (1991a), Lahiri (1991b), we reduce the problem as in Roth (1979), to a simple problem of dividing a fixed amount of money between two agents.

Consider a situation with two agents 1 and 2 whose initial wealth (for the sake of notational simplicity) is set equal to zero. Suppose they bargain over the division of Q units of money. We assume that each agent i has preferences over possible distributions of money between the two agents which are summarized by a utility functions $u_i : \mathbb{R}^2_+ \to \mathbb{R}$, where $\mathbb{R}^2_+$ is the non negative orthant of two dimensional Euclidean space. A <u>feasible proposal</u> is a proposed split (c_1, c_2), such that $c_1 + c_2 \leq Q$ and $c_1 \geq 0$, $c_2 \geq 0$.

In this paper we assume that preferences of each agent displays the following property :

<u>Property 1</u> : (i) $\forall c_2 \geq 0$, $u_1(., c_2) : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing

$\forall c_1 \geq 0$, $u_2(c_1, .) : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing

(ii) $\forall c_1 \geq 0$, $u_1(c_1, .) : \mathbb{R}_+ \to \mathbb{R}$ is non-increasing

$\forall c_2 \geq 0$, $u_2(., c_2) : \mathbb{R}_+ \to \mathbb{R}$ is non-increasing

(iii) $u_1(0, c_2) = u_2(c_1, 0) = 0 \; \forall (c_1, c_2) \in \mathbb{R}^2_+$

Here $\mathbb{R}_+$ is the set of all non-negative reals.

We propose to study three main solutions to bargaining solutions :

(1) <u>The Egalitarian Solution</u> :- A <u>Feasible proposal</u> (c_1^*, c_2^*) such that $c_1^* + c_2^* = Q$ is called an egalitarian solution if

$u_1(c_1^*,c_2^*) = u_2(c_1^*,c_2^*)$.

(2) The Nash Solution :- A feasible proposal (c_1^*,c_2^*) is called the Nash Solution if it solves the following problem

$$u_1(c_1,c_2)\ u_2(c_1,c_2) \to \max$$

(3) The Kalai-Smorodinsky Solution :- A feasible proposal (c_1^*,c_2^*) is called the Kalai-Smorodinsky Solution if $c_1^*+c_2^* = Q$ and

$$\frac{u_1(c_1^*,c_2^*)}{u_1(Q,0)} = \frac{u_2(c_1^*,c_2^*)}{u_2(0,Q)}.$$

For an exhaustive discussion of the above solutions one may refer to Roth (1979) or Thomson (forthcoming).

It is easily verified using Property 1 (i), that given u_1,u_2 and $Q>0$, the egalitarian solution and the Kalai-Smorodinsky solution are uniquely defined. The same property reveals that for the Nash solution the budget constraint is satisfied with equality and it is uniquely defined.

For the purpose of this paper let us fix u_1 and u_2 and allow $Q>0$ to vary. Let $\bar{N} : \mathbb{R}_{++} \to \mathbb{R}_+^2$ denote the Nash solution, $\bar{E} : \mathbb{R}_{++} \to \mathbb{R}_+^2$ denot the egalitarian solution and $\bar{K} : \mathbb{R}_{++} \to \mathbb{R}_+^2$ denote the Kalai-Smorodinsky solution. Here $\mathbb{R}_{++}$ is the set of positive real numbers.

The relevant property of the above solutions (in this extended framework) that we want to establish is the following :

Resource Monotonicity :- $Q',Q \in \mathbb{R}_{++}$, $Q' \geqq Q \Rightarrow \bar{F}(Q') \geqq \bar{F}(Q)$ where $\bar{F}: \mathbb{R}_{++} \to \mathbb{R}_+^2$ is a bargaining solution (satisfying $\bar{F}_1(Q)+\bar{F}_2(Q) \leqq Q \ \forall Q>0$.)

2. The Egalitarian Solution :- The egalitarian solution satisfies resource monotonicity under Property 1.

Theorem 1 :- Under Property 1, $\bar{E}:\mathbb{R}_{++}\to\mathbb{R}^2_+$ satisfies resource monotonicity.

Proof :- Let $Q',Q\in\mathbb{R}_{++}$ with $Q'>Q$ and suppose towards contradiction (and without loss of generality), $E_1(Q')<E_1(Q)$. Since $\bar{E}_1(Q')+\bar{E}_2(Q') = Q'>Q = \bar{E}_1(Q)+\bar{E}_2(Q)$, $\bar{E}_2(Q')>\bar{E}_2(Q)$.

But this implies, (by Property 1),

$$u_1(\bar{E}_1(Q'),\bar{E}_2(Q'))<u_1(\bar{E}_1(Q),\bar{E}_2(Q'))\leq u_1(\bar{E}_1(Q),\bar{E}_2(Q))$$
$$= u_2(\bar{E}_1(Q),\bar{E}_2(Q))\leq u_2(\bar{E}_1(Q'),\bar{E}_2(Q))<u_2(\bar{E}_1(Q'),\bar{E}_2(Q')),$$

contradicting that $\bar{E}$ is the egalitarian solution.

Q.E.D.

3. **The Nash Solution** :- Under additional assumptions the Nash solution also satisfies resource monotonicity. We assume the following :

Property 2 :- $u_i: \mathbb{R}^2_+\to\mathbb{R}$ is continuous for $i=1,2$

Property 3 :- (i) $u_1(.,c_2):\mathbb{R}_+ \to\mathbb{R}$ is concave $\forall c_2\in\mathbb{R}_+$

$u_2(c_1,.):\mathbb{R}_+ \to\mathbb{R}$ is concave $\forall c_1\in\mathbb{R}_+$

(ii) $u_1(c_1,.):\mathbb{R}_+ \to\mathbb{R}$ is convex $\forall c_1\in\mathbb{R}_+$

$u_2(.,c_2):\mathbb{R}_+ \to\mathbb{R}$ is convex $\forall c_2\in\mathbb{R}_+$.

Theorem 2 :- Under properties 1,2 and 3, $\bar{N}$ satisfies resource monotonicity.

Proof :- For simplicity, consider the situation where the utility profile $u=(u_1,u_2)$ is differentiable. (The other cases can be dealt with by an approximation argument making use of the fact that the Nash solution is a continuous function of the utility profile). $\bar{N}(Q)$ is obtained by maximizing $u_1(c_1,c_2)u_2(c_1,c_2)$ in $(c_1,c_2)\in\mathbb{R}^2_+$ subject to $c_1+c_2=Q$. This problem is solved by requiring

$$\frac{1}{u_1(c_1^*,c_2^*)}\left[\frac{\partial u_1 (c_1^*,c_2^*)}{\partial c_1} - \frac{\partial u_1 (c_1^*,c_2^*)}{\partial c_2}\right]$$

$$= \frac{1}{u_2(c_1^*,c_2^*)}\left[\frac{\partial u_2 (c_1^*,c_2^*)}{\partial c_2} - \frac{\partial u_2 (c_1^*,c_2^*)}{\partial c_1}\right]$$

with $c_1^*+c_2^* = Q$.

Now suppose $Q'>Q>0$, $\bar{N}_1(Q')<\bar{N}_1(Q)$. So, $\bar{N}_2(Q')>\bar{N}_2(Q)$.

By Properties 1,2 and 3,

$$\frac{1}{u_1(\bar{N}(Q'))}\left[\frac{\partial u_1(\bar{N}(Q'))}{\partial c_1} \quad \frac{\partial u_1(\bar{N}(Q'))}{\partial c_2}\right] > \frac{1}{u_1(\bar{N}(Q))}\left[\frac{\partial u_1(\bar{N}(Q))}{\partial c_1} - \frac{\partial u_1(\bar{N}(Q)}{\partial c_2}\right.$$

$$= \frac{1}{u_2(\bar{N}(Q))}\left[\frac{\partial u_2(\bar{N}(Q))}{\partial c_2} - \frac{\partial u_2(\bar{N}(Q))}{\partial c_1}\right]$$

$$> \frac{1}{u_2(\bar{N}(Q'))}\left[\frac{\partial u_2(\bar{N}(Q'))}{\partial c_2} - \frac{\partial u_2(\bar{N}(Q'))}{\partial c_1}\right]$$

contradicting that $N(Q')$ is the Nash bargaining solution for Q'.

Q.E.D.

4. <u>The Kalai-Smorodinsky Solution</u> :- Under assumptions similar to that in Theorem 2, the Kalai-Smorodinsky Solution also satisfies resource monotonicity. This is the essence of the next theorem.

<u>Theorem</u> <u>3</u> :- Suppose the utility profile $u=(u_1,u_2)$ is differentiable and $\frac{\partial u_i}{\partial c_i}(c_i,.):\mathbb{R}_+\to\mathbb{R}$ is a nonincreasing function. Then under Properties 1,2 and 3, $\bar{K}$ satisfies resource monotonicity.

<u>Proof</u> :- For simplicity, consider a utility profile $u=(u_1,u_2)$ which is twice differentiable. (The general result follows by an approximation argument and by appealing to the fact that $\bar{K}$ is a continuous function of the utility profile). Then $\bar{K}(Q)$ satisfies,

(i) $\dfrac{u_1(\bar{K}(Q))}{u_1(Q,0)} = \dfrac{u_2(\bar{K}(Q))}{u_2(0,Q)}$.

Since this equation is satisfied for all $Q>0$, we get by differentiating with respect to Q :

(ii) $f_1(Q) \equiv f_2(Q)$ where

$$f_1(Q) = \frac{\dfrac{\partial u_1(\bar{K}(Q))}{\partial c_1} \cdot \dfrac{d\bar{k}_1(Q)}{dQ} + \dfrac{\partial u_1(\bar{K}(Q))}{\partial c_2} \dfrac{d\bar{k}_2(Q)}{dQ}}{u_1(Q,0)} - \frac{u_1(\bar{K}(Q))}{[u_1(Q,0)]^2} \frac{\partial u_1(Q,0)}{\partial c_1}$$

and,

$$f_2(Q) = \frac{\dfrac{\partial u_2(\bar{K}(Q))}{\partial c_1} \dfrac{d\bar{k}_1(Q)}{dQ} + \dfrac{\partial u_2(\bar{K}(Q))}{\partial c_2} \dfrac{d\bar{k}_2(Q)}{dQ}}{u_2(0,Q)} - \frac{u_2(\bar{K}(Q))}{[u_2(0,Q)]^2} \frac{\partial u_2(0,Q)}{\partial c_2}$$

Now suppose that $\dfrac{d\bar{k}_1(Q)}{dQ}<0$ for some Q. Since $\bar{K}_1(Q)+\bar{K}_2(Q) = Q \; \forall Q$, we must have $\dfrac{d\bar{k}_1(Q)}{dQ} + \dfrac{d\bar{k}_2(Q)}{dQ} = 1$. Therefore $\dfrac{d\bar{k}_2(Q)}{dQ} > 1$.

Now,

$$f_2(Q) = \frac{u_2(\bar{K}(Q))}{u_2(0,Q)} \left[\frac{\dfrac{\partial u_2(\bar{K}(Q))}{\partial c_1} \dfrac{d\bar{k}_1(Q)}{dQ}}{u_2(\bar{K}(Q))} + \left\{ \frac{\dfrac{\partial u_2(\bar{K}(Q))}{\partial c_2} \dfrac{d\bar{k}_2(Q)}{dQ}}{u_2(\bar{K}(Q))} - \frac{\dfrac{\partial u_2(0,Q)}{\partial c_2}}{u_2(0,Q)} \right\} \right]$$

Since u_2 is concave in c_2 and $0 \leqq \bar{k}_1(Q) \leqq Q$, $0 \leqq \bar{K}_2(Q) \leqq Q$, we get

$$\frac{\frac{\partial u_2(\bar{K}(Q))}{\partial c_2}}{u_2(\bar{K}(Q))} \geq \frac{\frac{\partial u_2(\bar{K},(Q),Q)}{\partial c_2}}{u_2(\bar{K}_1(Q),Q)} \geq \frac{\frac{\partial u_2(0,Q)}{\partial c_2}}{u_2(0,Q)}$$

This, in conjunction with $\frac{d\bar{k}_2(Q)}{dQ} > 1$ yields that $f_2(Q) > 0$.

But if $\frac{d\bar{k}_1(Q)}{dQ} < 0$, then $f_1(Q) < 0$. These two statements on the signs of $f_1(Q)$ and $f_2(Q)$ are incompatible with (ii).

Q.E.D.

In this theorem we required the utility profile to satisfy an additional property in order that resource monotonicity continues to hold. This property states that the marginal utility of an agent's income be a decreasing function of the other agents income. The concept of envy introduced in Property 1 (ii) is reinforced by this property; an additional unit of income to an agent gives greater additional utility if his opponent is richer, than if he were poor.

Conclusion :- In this paper we have extended results obtained by Thomson (forthcoming), to the situation where preferences display consumption externalities. This is an important step forward, as it highlights some realistic considerations that bilateral monopoly has so far ignored.

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