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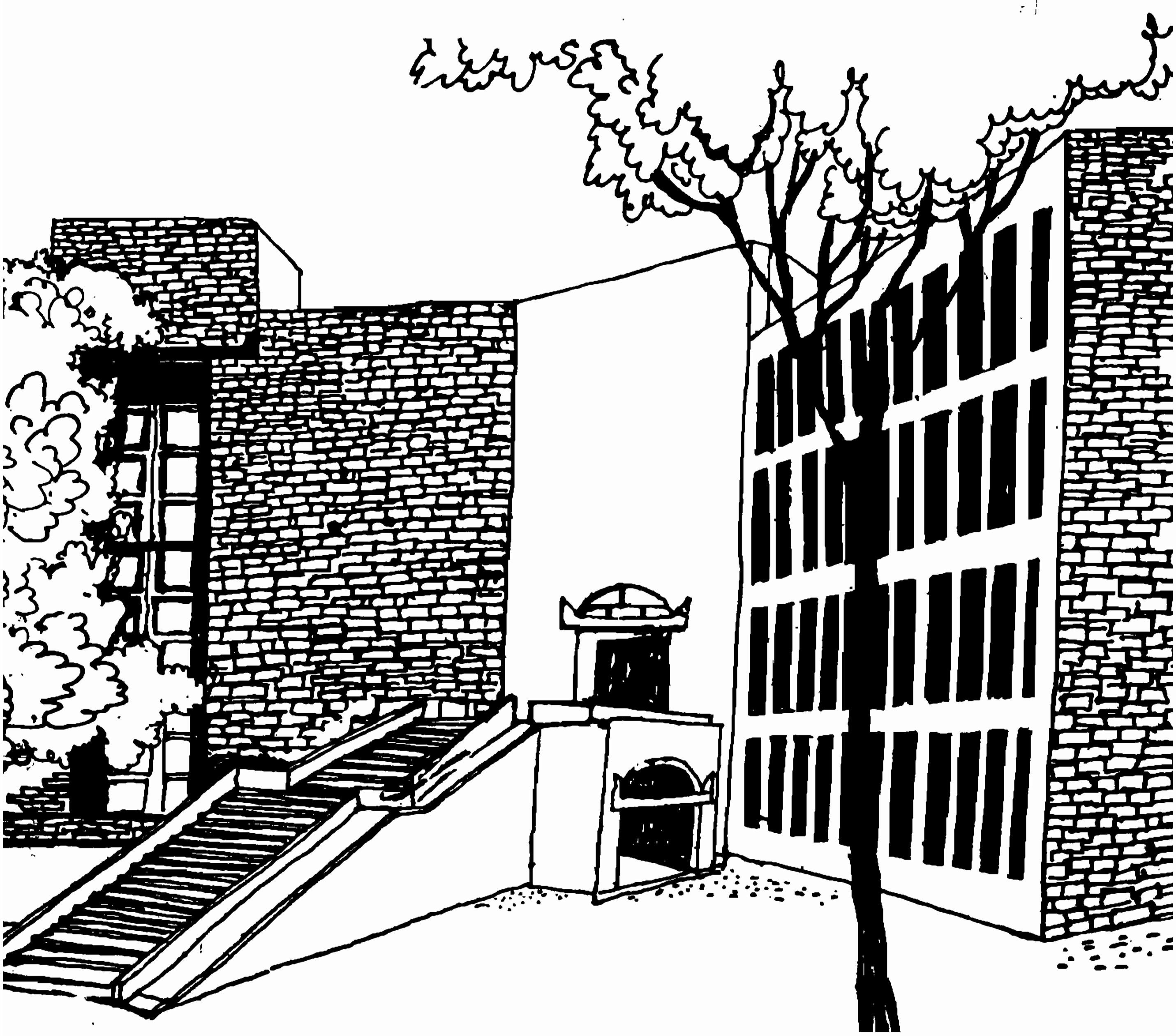

SUFFICIENT CONDITION FOR THE BACKWARD BENDING SUPPLY CURVE

By

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Sufficient Condition for the Backward Bending Supply Curve

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Whether an individual's supply curve is backward bending or not is a relevant question to investigate while fixing or revising the price of the product. The possibility of the backward bending supply curve arises only when a producer or a group of producers of the product also self-consumes a part of the production. This way of conceptualizing the problem would encompass a wide range of phenomena including the case of a country like India which is one of the largest producers of agricultural products but self consumes most of them to remain an insignificant player in the international trade or financing the investment in business through own funds, etc. The sufficient condition for the backward bending supply curve is considered to be a positive price elasticity of self-consumption of the product. Professor Kothari (1998) has recently examined the condition under which the price elasticity of the self-consumption becomes positive. He has shown that the sufficient condition for the price elasticity of the self-consumption to be positive is much more stringent than the traditional condition of the income elasticity of demand exceeding the elasticity of substitution in consumption. According to him this would happen in a situation where the consumer derives only a part of his income from the production of the product in question. As a result, he argues, the posititive price elasticity and hence backward bending supply curve becomes a remoter possibility. In the present note, we examine the sufficient condition for the backward bending supply curve in a more general case.

The whole problem can be analysed in the standard Slutsky-Hicks-Allen mathematical formulation. Let $U = f(X,Y)$ be the utility function of the producer-cum-consumer of good X. He is producing X* and self-consumes X. Therefore, his income constraint is:

1. $M' + P_X (X^* - X) = P_Y.Y$ (where M' = Income from other sources)

   i.e. $M = M' + P_X X^* = P_X X + P_Y Y$

Applying Lagrangian Multiplier method for constrained maximization.

2. $V = f(X,Y) + \lambda (M' + P_X.X^* - P_X.X - P_Y.Y)$

For maximization, all the first partial derivatives have to be equated to zero.

$$\therefore \quad \frac{\partial V}{\partial X} = f_X - p_X \lambda = 0$$

$$\frac{\partial V}{\partial Y} = f_Y - p_Y \lambda = 0$$

$$\frac{\partial V}{\partial \lambda} = M' + P_X X^* - P_X.X - P_Y.Y = 0$$

Taking differential of these 3 equations:

3. $f_{XX}dX + f_{XY}dY - P_X d\lambda = \lambda dP_X$

4. $f_{YX}dY + f_{YY}dY - P_Y d\lambda = \lambda dP_Y$

5. $-P_X dX - P_Y dY = -dM' - P_X dX^* - (X^*-X)\, dP_X + YdP_Y$

Taking dX, dY and $d\lambda$ as variables and solving for them in terms of dP_X, dP_Y, dM', and dX*, we get

6.
$$dX = \frac{\lambda D_{11}dP_X + \lambda D_{21}dP_Y + D_{31}(-dM' - P_X dX^* - (X^* - X)\,dP_X + YdP_Y)}{D}$$

[Solving through Determinants and Cramer's Rule]

7.
$$\therefore \quad \frac{\partial X}{\partial P_X} = \frac{\lambda D_{11}}{D} - (X^* - X) \cdot \frac{D_{31}}{D}$$

[This is the revised version of Slutsky Equation. D_{11} and D_{31} are cofactors]

$$\therefore \quad \frac{\partial X}{\partial P_X} \cdot \frac{P_X}{X} = \frac{\lambda D_{11}}{D} \cdot \frac{P_X}{X} + \frac{P_X(X^* - X)}{X} \left(\frac{\partial X}{\partial M}\right)_{\text{prices constant}}$$

$$\text{Since} \left(\frac{\partial X}{\partial M}\right)_{\text{prices constant}} = \frac{(-)\,D_{31}}{D}$$

$$\therefore \quad e_P = (-)\,\frac{P_Y.Y}{M} \; \frac{\lambda P_Y P_X.M}{Y.X.D.} + \frac{P_X(X^* - X)}{M} \; \frac{\partial X}{\partial M} \; \frac{M}{X}$$

8. i.e. $e_P = -K_Y\,\sigma + (m_X - K_X)\; e_I$

(where σ = elasticity of substitution, e_I = income elasticity. For the derivation and the formula of σ, see Allen, 1966, Ch. 13)

This is Professor Kothari's basic result in equation (2) in his paper (1998). Important point here is that the concept of income (M) is based on what the consumer produces of X and not on what he sells in the market. Professor Kothari also recognised this. However, the main advantage of the mathematical formulation is that it brings to the focus the critical assumption behind this result. X^* is assumed fixed and independent of P_X, i.e. supply response of the producer is assumed to be completely price inelastic. In other words, Professor Kothari's analysis is restricted to Marshallian market period. The problem in its most general form need not be constrained by such a strong and often unrealistic assumption. Only in some extreme situations like time available for labour supply, one can try to justify such assumptions. When we relax this assumption, the case becomes more general and interesting. First of all, it makes X^* a variable dependent directly on P_X, i.e. $X^* = \phi\,(P_X)$, with $\phi'\,(P_X) > 0$. Secondly, it genuinely modifies the differential in the equation (5) above, viz.

9. $$-P_X dX - P_Y dY = -dM' - P_X\ \phi'\,(P_X)\,dP_X - (X^* - X)\,dP_X + YdP_Y$$

This results in the change in the Slutsky equation:

10. $$\frac{\partial X}{\partial P_X} = \frac{\lambda D_{11}}{D} - (P_X\phi'(P_X) + X^* - X)\ \frac{D_{31}}{D}$$

Now, $$\frac{-D_{31}}{D} = \left(\frac{\partial X}{\partial M'}\right)_{\text{prices constant}}$$ (As can be seen from the solution of dX above)

$$\therefore\ \frac{-D_{31}}{D} \cdot \frac{M'}{X} = \frac{\partial X}{\partial M'} \cdot \frac{M'}{X} = e_I$$ (For e_I, it may be noted that P_X & P_Y should be constant)

11. $$\therefore\ \frac{\partial X}{\partial P_X} = \frac{\lambda D_{11}}{D} + (P_X\phi'(P_X) + X^* - X)\ e_I \cdot \frac{X}{M'}$$

$$\text{Now,}\ \frac{\partial X}{\partial P_X} \cdot \frac{P_X}{X} = -\frac{P_Y.Y}{P_X.X + P_Y.Y}\ \sigma + \frac{P_X(X^*-X)}{M'}\ e_I + \frac{P_X^2.\phi'(P_X)}{M'}\ e_I$$

$$\text{i.e.}\quad e_P = -K_Y\sigma + \frac{m_X - K_X}{1 - m_X}\ e_I + \frac{P_X.X^*}{M'} \cdot \frac{P_X}{X^*} \cdot \phi'\,(P_X) \cdot e_I$$

$$= -K_Y\sigma + \frac{m_X - K_X}{1 - m_X}\ e_I + \frac{m_X}{1 - m_X}\ e_S \cdot e_I$$

$$= -K_Y\sigma + \frac{m_X - K_X + m_X e_S}{1 - m_X} e_i \quad \text{(where } e_S \text{ is elasticity of supply)}$$

12. $$e_P = -K_Y\sigma + \frac{m_X(1 + e_S) - K_X}{1 - m_X} e_i$$

Some important implications of this are:

(a) Price elasticity of demand for X (e_P) can be positive with less restrictions than what Professor Kothari (1998) has shown. For instance, with $m_X = 1/10$ and $K_X = 1/20$, σ has to be less than (0.12) e_i rather than (0.05) e_i even if $e_S = 0.5$. If e_S is more, e_P is more likely to be positive.

(b) The higher is the proportion of income from the product X (i.e. m_X), the higher are the chances that e_P is positive. For instance, in the above illustration, other things remaining the same, if m_X is taken to be 50%, i.e. $m_X = 0.5$, then σ has to be less than (1.47) e_i for e_P to be positive which is much less restrictive than what is considered in (a) above.

(c) The higher the proportion of income spent on the product (i.e. K_X), the lower are the chances of e_P becoming positive. To continue with the same illustration as in (b) above, consider $K_X = 0.4$ instead of 0.05. Now, σ has to be less than (1.17) e_i for e_P to be positive.

(d) However, it is no longer necessary that whenever e_P is positive, the supply curve has to bend backward. It depends on the difference X^*-X. With higher P_X, both X^* and X may increase. It depends on the magnitude of their increases. Diagramatically, it can be shown as in Figure 1. Most of the self-consumption cases would fall under this category. This is the reason why milk consumption in several famililities would have increased without any fall in their marketed surplus -- because milk production has increased due to better rewards.

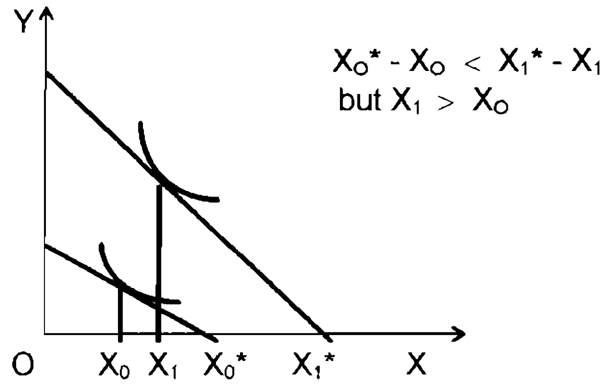

Figure 1

(e) As price of X rises, m_X (i.e. proportion of the income from the product X) would increase and therefore, as P_X increases and reaches a sufficiently high level, the e_P can become positive. Although $e_P > 0$ is not a sufficient condition for backward bending supply curve of marketed surplus, it is a necessary condition.

(f) The sufficient condition for the backward bending individual supply curve can be stated as follows using the notations of Figure 1.

$X_O^* - X_O > X_1^* - X_1$ if the supply curve is backward bending.

i.e. $X_1 - X_O > X_1^* - X_O^*$

$$\text{i.e. } \frac{X_1 - X_O}{\Delta P} \cdot \frac{P_O}{X_O} > \frac{X_1^* - X_O^*}{\Delta P} \cdot \frac{P_O}{X_O^*} \cdot \frac{X_O^*}{X_O}$$

$$13. \quad \text{i.e.} \quad e_P > \frac{m_X}{K_X} e_S$$

Thus, for the backward bending supply curve for the product X, the positive price elasticity of self-consumption (e_P) should exceed a multiple of the elasticity of supply (since m_X invariably exceeds K_X for those producers of X who have some marketed surplus). If the price elasticity (e_P) is positive but does not fulfil the inequality in (13) above, the supply curve would not be backward bending. Moreover, we can substitute equation (12) in (13) to get the following inequality as the sufficient condition for obtaining the backward bending supply curve for the product X:

$$- K_Y \sigma + \frac{m_X (1 + e_S) - K_X}{1 - m_X} e_i > \frac{m_X}{K_X} e_S$$

$$14. \quad \text{i.e.} \quad \frac{1}{1-K_X} \left[\frac{m_X (1 + e_S) - K_X}{1 - m_X} e_i - \frac{m_X}{K_X} e_S \right] > \sigma$$

It is interesting to see that with this generalized sufficient condition for obtaining a backward bending individual supply curve, it is much less likely to get such a curve than what Professor Kothari suggested. Thus, to continue with the same illustration as in (a) above, we may get the backward bending supply curve for the product X if the income elasticity is about 62.5 times the elasticity of substitution within the plausible ranges of the values or if the elasticity of substitution is less than (0.016) e_i.

The moral of the story is that the backward bending supply curve of a product even for an individual is not likely to be found in reality. Although theoreticall the possibility of an individual's supply curve bending backward cannot be ruled out, in practice, it is extremely difficult to find such cases. The market supply curve of the product X which is obtained as the horizontal summation of the individual supply curves may never be backward bending in practice. Even theoretically, the backward bending market supply curve requires many more implausible assumptions than those required for the individual backward bending supply curve. The policymakers need not get concerned about such hypothetical possibilities.

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