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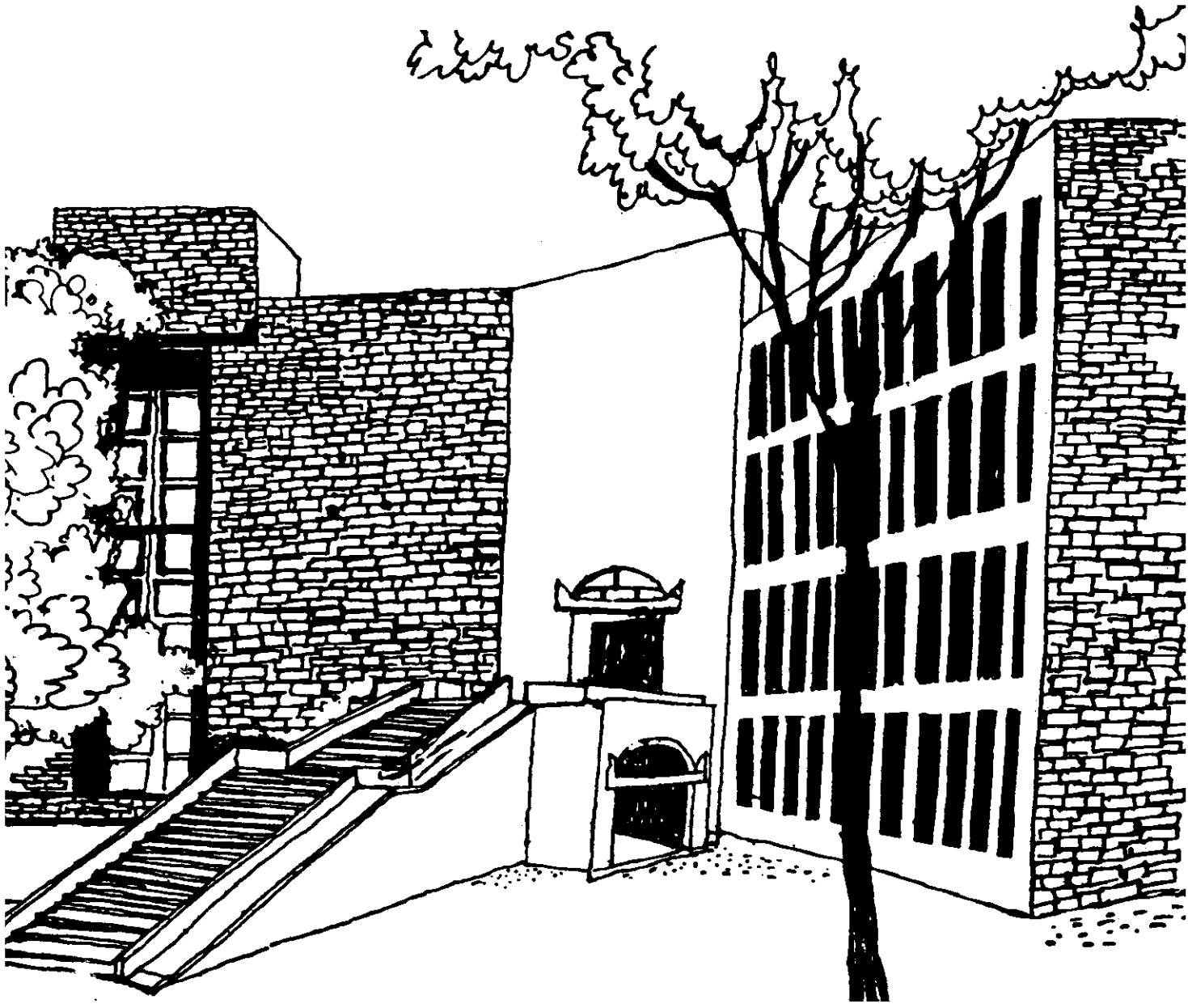

REVEALED PREFERENCE AND INDEPENDENCE OF IRRELEVANT ALTERNATIVES: A NOTE

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Abstract

Our objective here is to establish an equivalence between the IIA axiom which plays a significant role in axiomatic models of bargaining, and a generalization of WARP which plays a significant role in demand analysis.

1. Introduction :- Our objective here is to establish an equivalence between the Independence of Irrelevant Alternative Assumption (IIA), which plays a significant role in axiomatic models of bargaining, and a generalization of the Weak Axiom of Revealed Preference (WARP), which plays a significant role in the nonparametric approach to demand analysis, but which for our purposes has been adapted so as to be meaningful in bargaining contexts. Our approach to bargaining problems is that of Nash (1950) and our extension of WARP proceeds directly from Richter (1966).

2. The Model :- An n-person bargaining problem, or simply a problem, is a pair (S,d), where S is a subset of $\mathbb{R}^n$ and d is a point in S, such that (1) S is convex and closed, (2) (S,d) is comprehensive, i.e. for all $x \in S$ and for all $y \in \mathbb{R}^n$, if $d \leq y \leq x$, then $y \in S$, and (4) there exists $x \in S$ with $x > d$.

S is the feasible set. Each point x of S is a feasible alternative. The coordinates of x are the utility levels, measured in some von Neumann Morgenstern scales, attained by the n agents indexed by $i \in \{1,\ldots,n\}$ through the choice of some joint action. The point d is the disagreement point. The intended interpretation of (S,d) is as follows : the agents can achieve any point of S if they unanimously agree on it. If they do not agree on any point, they end up at d. Let Σ^n be the class of all n-person problems.

A <u>solution</u> is a function $F : \sum^n \to \mathbb{R}^n$ such that for all $(S,d) \in \mathbb{R}^n$, $F(S,d) \in S$. $F(S,d)$, the value taken by the solution F when applied to the problem (S,d), is called the <u>solution outcome</u> of (S,d).

We are interested in solutions satisfying either one of the two following axioms :

<u>Independence of Irrelevant Alternatives</u> (IIA) : If $(S,d) \in \sum^n$, $(T,d) \in \sum^n$, $T \subseteq S$ and $F(S,d) \in T$, then $F(T,d) = F(S,d)$.

<u>Weak Axiom of Revealed Preference</u> (WARP) : If $(S,d) \in \sum^n$, $(T,d) \in \sum^n$, and $F(T,d) \in S$, then $F(S,d) \notin T$ provided $F(S,d) \neq F(T,d)$.

The definition of IIA is standard. The definition of WARP is an easy extension of the conventional definition in demand analysis and says that for two different bargaining problems with the same disagreement point if the solution outcome of the second problem is feasible for the first problem, then the solution outcome for the first problem is infeasible for the second.

3. <u>The Main Theorem</u> :- In this section we establish the main conclusion of this paper.

<u>Theorem</u> :- $F : \sum^n \longrightarrow \mathbb{R}^n$ satisfies IIA if and only if it satisfies WARP.

<u>Proof</u> :- (WARP) $\Rightarrow$ (IIA) : Let the conditions of IIA be satisfied where $(S,d) \in \sum^n$, $(T,d) \in \sum^n$, $T \subseteq S$ and $F(S,d) \in T$. Suppose

$F(T,d) \neq F(S,d)$. Then since $F(T,d) \in T \subseteq S$, by WARP, $F(S,d) \notin T$ which is a contradiction.

(IIA) $\Rightarrow$ (WARP) : Let $(S,d) \in \sum^n$ and $(T,d) \in \sum^n$. Then by conditions (2) and (3) of the definition of a bargaining problem, $(S \cap T,d) \in \sum^n$. Suppose $F(T,d) \in S$, $F(T,d) \neq F(S,d)$ and towards a contradiction that $F(S,d) \in T$.

$\therefore$ $F(T,d) \in S \cap T$ implies by IIA that $F(S \cap T,d) = F(T,d)$ and $F(S,d) \in S \cap T$ implies by IIA that $F(S \cap T,d) = F(S,d)$.

However, $F(S,d) \neq F(T,d)$ leads to a contradiction and completes the proof.

References :-

1. J.F. Nash (1950) : "The Bargaining Problem," Econometrica 18, 155-162.

2. M.K. Richter (1966) : "Revealed Preference Theory," Econometrica 34, 635-645.