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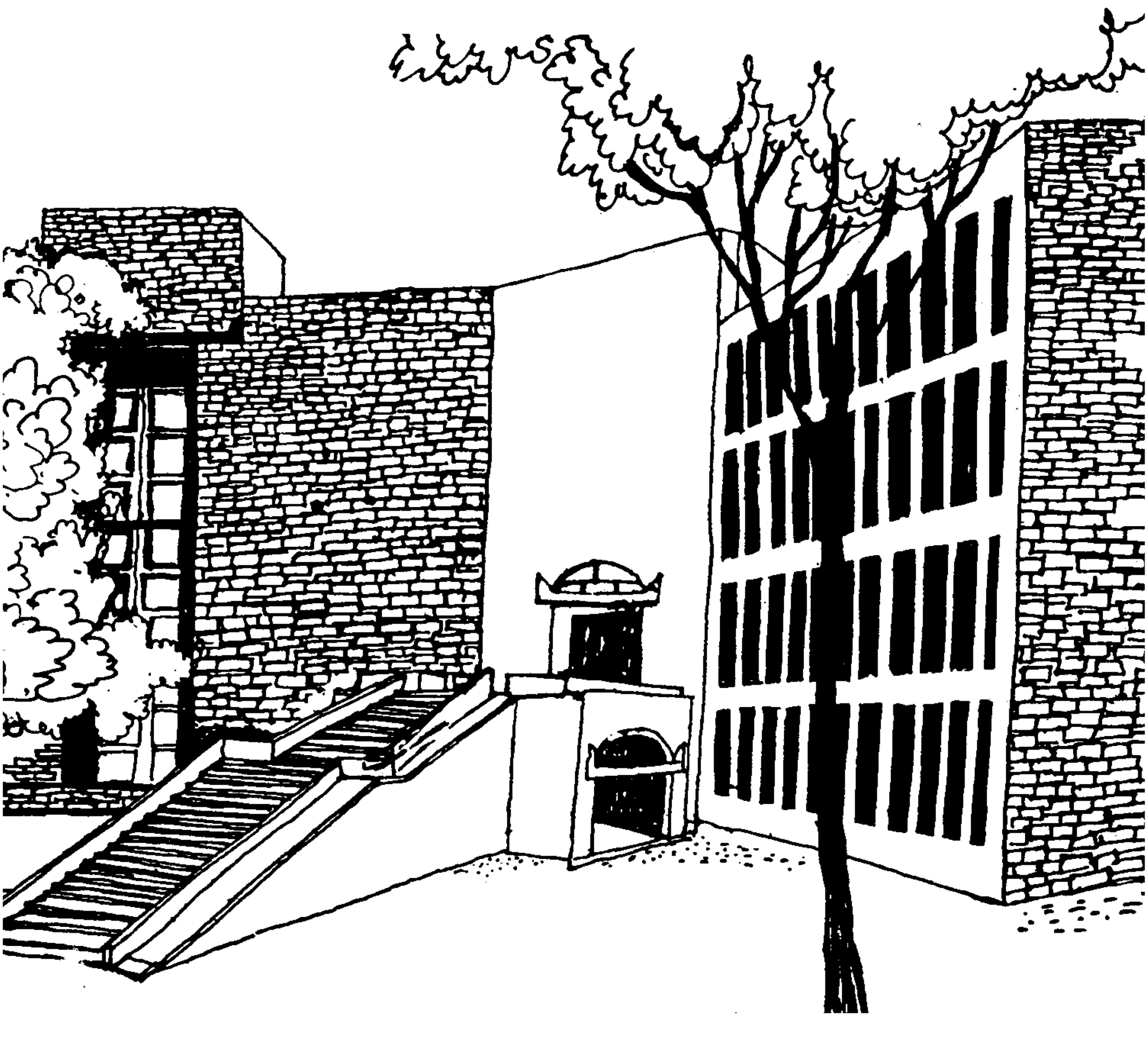

STRONGLY FAIR ALLOCATIONS IN ECONOMIES WITH PRODUCTION

By

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W P No. 1068
November 1992

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Abstract

In this paper we show that essentially the only mechanism which is strongly fair in an economy with production is the equal income marginal cost pricing (EIMCP) mechanism. A variant of the analysis would prove that the only mechanism which guarantees strongly fair net trades is the marginal cost pricing (MCP) mechanism.

1. Introduction :- Equity and efficiency, are two concepts which are at the center of many economic analysis.

In Foley (1967), originates an ordinal concept of equity, the concept of an envy-free allocation. This concept has been analyzed by Varian (1974,1975). Interesting results for pure exchange economies exist (for instance in Foley (1967)), Schmeidler and Vind (1972), Kolm (1972), Schmeidler and Yaari (1971), Goldman and Sussangkarn (1978)). Some generalizations and modifications of the above concept, in the context of pure exchange economies have been worked out (see Thomson and Varian (1975) for a summary).

In the context of economies with production, analysis of equity considerations exist; but the body of literature has grown much less compared to the analysis in pure exchange economies. This is probably because, the inputs of two different individuals are not always of the same variety.

To compare the inputs of different individuals, Mirrlees (1974), invokes the concept of productivity. Our paper analyses the equity issue in an economy with production, where productivities of the individuals may be different. We have adapted a concept of strong-fairness (due to Sato (1987)), proposed for an economy with public goods, to the present context, and we show that the equal income marginal cost pricing mechanism is essentially the only mechanism satisfying strong fairness. Conceptual difficulties about the interpretation of the results arise. That forms the subject of our discussion in the conclusion.

2. The Model :- The economy has two private goods: Labor is used as input to produce corn (the output). The production of y units of corn requires $x=c(y)$ units of standard labor. For the sake of simplicity, we assume that the function $c:\mathbb{R}_+\to\mathbb{R}_+$ is linear; i.e. $\exists c>0$ such that $c(y)=c.y, \forall y>0$.

We have n agents. Initially, agent i is endowed with w_i units of leisure (leisure can be consumed or used up as labor) and no corn. At the final allocation, he sues a portion of his leisure, say x_i, as labor and consumes y_i units of corn. His preferences are described by a utility function $u_i(w_i - x_i, y_i)$ over leisure x corn.

We assume that the productivity of agent i is given by a positive real number π_i which tells us the quantity of standard labor that one unit of his labor produces. Thus agent i is more productive than agent j is equivalent to saying that $\pi_i > \pi_j$.

An allocation $(x,y)=(x_1, \ldots, x_n; y_1, \ldots, y_n)$ is thus feasible if and only if

$$0 \le x_i \le w_i,\ 0 \le y_i,\ \forall i \text{ and } \Sigma^n_{i=1} \pi_i x_i = c(\Sigma^n_{i=1} y_i) \quad (1)$$

For the sake of simplicity we assume that for each i. $u_i : \mathbb{R}^2_+ \to \mathbb{R}$ is continuous and strictly increasing. Such is the familiar framework of an economy with production as conceived by Mirlees (1974).

A feasible allocation (x,y) is said to be efficient if there is no other feasible allocation (x',y') such that $u_i(w_i - x'_i, y'_i) > u_i(w_i - x_i, y_i)$ for every agent i.

An agent i envies an agent j at an allocation (x,y) if $u_i(w_i - x_i, y_i) < u_i(\frac{\pi_j}{\pi_i}(w_j - x_j), y_j)$. An allocation is envyfree if no agent envies any other agent.

<u>Definition</u> :- An allocation (x,y) is fair if it is both efficient and envy-free.

Throughout our analysis we will consider the preferences and the cost function as fixed and address our concerns to the following problem: There is given an aggregate amount of <u>standard</u> leisure $\bar{w} > 0$. We consider the set $\mathcal{E}(\bar{w}) = (w,(x,y)) \in \mathbb{R}^n_+ \times (\mathbb{R}^n_+)^2 / \Sigma^n_{i=1} \pi_i w_i = \bar{w}$ and (x,y) is an allocation for w}. A <u>choice function</u> is a function $F: \mathbb{R}_{++} \to (\mathbb{R}^n_+)^3$ such that $F(\bar{w}) \in \mathcal{E}(\bar{w}) \forall \bar{w} \in \mathbb{R}_{++}$. A <u>choice function</u> is said to be efficient if $\forall \bar{w} > 0$, $F(\bar{w}) = (w,(x,y))$ implies (x,y) is efficient for w. It is said to be <u>envy free</u> if $\forall \bar{w} > 0$, $F(\bar{w}) = (w,(x,y))$ implies (x,y) is envyfree for w. It is said to be <u>fair</u> if it is both efficient and envy free.

To begin with we propose the following <u>choice function</u>: $\forall \bar{w} > 0$, let $F(\bar{w}) = (w,(x,y))$ satisfy,

(i) $\pi_i \; w_i = \pi_j w_j \; \forall i, j.$

(ii) $\exists p>0$, such that (x_i, y_i) maximizes

$u_i \; (w_i - x_i', y_i')$

s.t. $\pi_i \; (w_i - x_i') + p y_i' = \pi_i \; w_i$

$0 \le x_i'$, $0 \le y_i'$, $\forall i \in \{1, \ldots, n\}$.

We call the above choice function, the equal income, marginal cost pricing choice function as p in the above definition would have to equal the marginal cost of production 'c'. It is easy to see that the equal income marginal cost pricing (EIMCP) choice function is fair.

We now propose a further strengthening of the fairness criteria by suggesting a definition of strong fairness along the lines suggested by Sato (87).

Let $(x,y) \in \mathbb{R}^n_+ \times \mathbb{R}^n_+$ be an allocation for $w=(w_1, \ldots, w_n) \in \mathbb{R}^n_+$.

If $u_i \left(\sum_{j=1}^{n} \delta_j \frac{\pi_j}{\pi_i} (w_j - x_j), \sum_{j=1}^{n} \delta_j y_j \right) > u_i (w_i - x_i, y_i)$ for

some n non-negative rational numbers $\{\delta_j\}^n_{j=1}$ whose sum, $\sum_{j=1}^{n} \delta_j = 1$,

then agent i is said to have a strongly legitimate complain. The allocation is said to be strongly envy free if there is no agent who has a strongly legitimate complaint. If in addition it is efficient for w, we say that the allocation is strongly fair.

A choice function $F: \mathbb{R}_{++} \rightarrow (\mathbb{R}^n_+)^3$ is said to be strongly envy free if $\forall \bar{w} > 0$, $F(\bar{w}) = (w, (x,y))$ implies (x,y) is strongly envy free for w. It is said to be strongly fair if it is both efficient and strongly envy free.

3. The Main Theorem :- The main theorem of this paper characterizes the equal income marginal cost pricing solution (EIMCP) uniquely in terms of strongly fair allocations under mild assumptions

Theorem 1 :- An EIMCP allocation is strongly fair. Conversely, if utility functions of the agents are quasi-concave then a strongly fair allocation is a EIMCP.

Proof :- The first part of this theorem is easily verified.

So we prove the second part. Let (x,y) be a strongly fair allocation for $w=(w_1,\ldots,w_n)$. We need to show $\pi_1 w_1=\ldots=\pi_n w_n$ since Pareto optimality of (x,y), implies that there exists $p(=c)$ with respect to which (x_i,y_i) maximizes $u_i(w_i-x_i',y_i')$ s.t. $\pi_i(w_i-x_i')+py_i'=\pi_i w_i$ $0\le x_i'$, $0\le y_i'$ $\forall i\in\{1,\ldots,n\}$.

Suppose, there exists $i,j\in N$ such that $\pi_i w_i>\pi_j w_j$

$\therefore \pi_i(w_i-x_i)+py_i>\pi_j(w_j-x_j)+py_j$

$\therefore ([\frac{\pi_i}{\pi_j}(w_i-x_i)],y_i)$ lies above the budget set of j.

By continuity and quasi-concavity of $u_j: \mathbb{R}^2_+\to\mathbb{R}$, $\exists\delta\in(0,1)$ such that,

$$u_j(\delta\frac{\pi_i}{\pi_j}(w_i-x_i),y_i)+(1-\delta)(w_j-x_j,y_j))>u_j(w_j-x_j,y_j)$$

By continuity of u_j, δ can be chosen rational.

This contradicts that (x,y) is strongly envy-free.

Thus $\pi_i x_i=\pi_j x_j$ $\forall i,j\in\{1,\ldots,n\}$.

Q.E.D.

4. Conclusion :- Much though we would like to view this as a planning problem, it is difficult to conceive of mechanisms, which allocate labor (or leisure) across individuals to begin with. Thus the significance of the above problem remains purely positive i.e. if allocations of leisure were such that $\pi_i w_i=\pi_j w_j$ $\forall i,j\in N$, then the MCP allocation would be strongly fair and conversely.

To interpret the above result normatively, we would have to conceive of the numeraire good as something other than labor, something which can be distributed by a planner across individuals, and which also enters as an input in the production process for a simple two good economy. Then our above result becomes a strong endorsement of the EIMCP mechanism. However, what different productivities of this numeraire good means, is not clear except possibly in an international trade context.

In a different context, we could have defined the concept of a strongly fair net trade and shown that it implied a marginal cost pricing (MCP) allocation. Since the analysis would be completely analogous, we rest content with the result obtained above.

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