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MEASUREMENT OF CAPITAL INPUT AND
ESTIMATION OF TIME SERIES
PRODUCTION FUNCTIONS IN
INDIAN MANUFACTURING

by

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A B S T R A C T

This study makes an attempt to examine the impact of errors in the specification and measurement of capital input on the estimates of time series production functions in Indian manufacturing. For this purpose, it examines the main alternative measures of capital input generally used in the production function exercises and also derives the corresponding set of alternative estimates of the widely used Cobb-Douglas production function for the Indian iron & steel industry, on the basis of time series data for the period 1946-1966, essentially in the form of an illustrative exercise.

The major conclusions of the study are that the estimates of time series production functions for Indian manufacturing are highly sensitive to the measurement of capital input; and the elimination of preliminary errors in the measurement of capital input alters significantly the basic structure of the estimated production function and, hence, the estimates of the relative contributions made by various factors to the growth of output in Indian manufacturing.

MEASUREMENT OF CAPITAL INPUT AND ESTIMATION OF TIME SERIES
PRODUCTION FUNCTIONS IN INDIAN MANUFACTURING

Bakul H. Dholakia*

An ever-increasing volume of quantitative research especially in the field of economic growth during the post-war period seems to have created a special interest in the analysis of the relative importance of various factors contributing to the growth of output in the organised manufacturing sector. It is hardly a matter of surprise, therefore, to find that the last few years have witnessed an increasing number of interesting studies examining the relative contributions of labour, capital and technical change to the growth of output in large-scale Indian industries based on the time-series data thrown up by the annual reports of the Census of Indian Manufactures (CMI) and the Annual Survey of Industries (ASI)*¹. Almost all of such studies use the technique of production function for estimating the factor elasticities of output and also, in some cases, the rate of technical progress.

Thus the common features of these studies are the basic sources of data and the technique of analysis. However, quite apart from these fundamental similarities, there exist considerable differences among these studies in respect of the particular industries and the specific time periods covered on the one hand, and, the specification and measurement of major variables involved on the other. Consequently, the results of the production function exercise vary considerably from one study to another. While the first source of variation noted above (i.e., the one relating to the differences in the industry and the time period covered) appears to be genuine and inevitable, the other seems to be more a source of created differences rather than the real differences that actually exist, inasmuch as it reflects the differences that are essentially conceptual and statistical in nature. It is obvious therefore that there is a need for making an attempt to examine the influence of variations, or errors as they may be called, in the measurement of major variables on the regression coefficients of the estimated production function. This becomes all-the-more interesting as well as necessary because in quite a few cases, the usual production

* The author is grateful to Ravindra N. Dholakia for his valuable comments on an earlier draft of this paper.

function exercise has yielded results which, as some of the authors have themselves pointed out, can be regarded as totally 'meaningless' in the context of the relevant economic logic.*² In what follows, we have therefore made an attempt to examine the impact of errors in the measurement of factor inputs, especially the capital input, on the estimated factor elasticities of output and also the estimated rate of 'technical progress'. For this purpose, we have followed the method of sensitivity analysis in its simplest form, and tried to examine the effect of alternative measures of one of the main factor inputs, viz., capital, on the sign and magnitude of the regression coefficients of the estimated production function. In order to conduct this sort of sensitivity analysis, we have presented the various possible alternative estimates of the widely used Cobb-Douglas production function for the Indian iron & steel industry, essentially in the form of an illustrative exercise. These estimates of the production function have been derived from the time-series data on the iron & steel industry for the period 1946 to 1966 obtained from the various reports of CMI (1946 to 1958) and ASI (1959 to 1966).

II

The crucial variable in the usual production function analysis is, of course, the capital input. The term 'capital', especially when used in the context of economic analysis, refers to one of the major factors of production conventionally distinguished in economic theory. Accordingly, the stock of capital existing at any given point of time consists of a variety of physical objects which have been produced by the economic system and which are being used for the production of other commodities at the specified point of time. The measurement and comparison of the stock of real capital existing at two different points of time would pose considerable problems, firstly because of the intricate problem of the great heterogeneity of capital goods, and secondly because of the serious problem of valuation arising out of the more or less continuously changing physical specification and form of capital goods largely on account of the changing technology. Economists have frequently tackled these problems by constructing the index of real capital stock which (a) abstracts from the phenomenon of technological progress by ignoring the quality change; and (b) approximates as closely as possible the actual changes in the various types of physical objects as such comprising the capital stock at the aggregate level. The index of real capital stock is derived by obtaining the value of the existing stock of

capital goods at any given point of time (before making any allowance for capital consumption) at some pre-determined base period prices. This, in turn, is done by measuring the aggregate amount it would have cost in the specified base period to produce the actual stock of capital goods existing at the given point of time.*3 The measure of real capital stock so obtained for different points of time over a given time period is "congenial to the production function point of view", because, "it corresponds to the essential nature of capital regarded as a factor of production."*4

In the light of the above discussion, it should be quite obvious that the data, especially relating to the fixed capital stock in Indian manufacturing, that are available from the sources like the CMI and the ASI do not correspond to the appropriate concept of capital discussed above, and, to that extent, they totally fail to provide the required measure of capital input. The reason is that the figures reported in such basic sources of data invariably indicate the so-called 'depreciated book value of capital assets', which is nothing but the accumulated annual aggregate expenditure on capital assets (in money terms, for each of the respective years) adjusted for the corresponding annual accounting depreciation charges. In other words, the reported figures of capital stock turn out to be grossly inadequate and totally misleading measure of the economically meaningful real capital stock mainly on account of two major defects that they contain. These defects are: (a) The reported figures represent a simple aggregation of the actual money values of annual additions to capital stock over a period of time without making any adjustment for price changes over the period. Thus, each annual addition to the stock of capital goods gets evaluated at different prices (i.e., at the prices prevailing in the corresponding year), and hence their simple aggregation as such does not yield any meaningful aggregate that can be directly used for economic analysis. (b) The reported figures represent the written down or depreciated book values of capital assets, and it is well known that the accounting methods of computing the depreciation charges are quite arbitrary and bear no relation whatsoever to the actual decline, if any, in the productive capacity of capital over a specified period of time.

Of these two specific problems, the first can be readily resolved by deflating the annual additions to each of the various types of capital assets, for each of the years in the period under consideration, with the help of suitable price deflators for various types of assets. It is the second problem, however, which is rather difficult to tackle. The reason is that it involves the basic question of choice of an

appropriate measure of capital input. The major problem here is to judge whether the efficiency of the fixed capital assets to contribute to production undergoes a significant decline with the passage of time or not. In other words, we have to decide whether the gross stock of real capital, i.e., undepreciated values of capital assets at constant prices, or, the net stock of real capital, i.e., depreciated values of capital assets at constant prices, would provide the best measure of capital input required for our purpose. Most economists have generally favoured the gross stock measure over the net stock measure for the purpose of production function and related analysis. The reason, as sharply pointed out by Professor Leontief, is that "Use of depreciated coefficients implies that capital stocks decrease in efficiency in exact relation to depreciation charge", whereas "most available evidence indicates that this is not a reliable assumption."⁵ Further evidence in support of this argument is provided by the findings of a fairly detailed study on the effective life of capital assets, made by Tibor Barna, which revealed that the capital goods used in the manufacturing industries in the United Kingdom are generally maintained in a pretty good conditions until a decision is made to scrap them.⁶

The use of gross stock of capital as the measure of capital input involves the assumption that fixed capital assets do not decrease in their efficiency to contribute to output with the passage of time. It may, therefore, be argued that, while the assumption implicit in the use of net stock figures (viz., that capital stock decreases in efficiency in exact relation to the depreciation charge) is undoubtedly a highly unrealistic assumption, the use of gross stock figures is also based on an extreme assumption of constant efficiency throughout the service life of fixed capital assets. However, it may be noted in this connection that, whenever a choice is to be made between the two alternative measures of capital stock, most economists usually prefer to use the gross stock of capital on the ground that a correct index of capital services would generally lie much closer to a gross stock index as compared to a net stock index.⁷

III

It is obvious from the above discussion that there exist at least three different measures of capital stock depending upon the method of derivation that is adopted. These alternative measures are:

- (1) Depreciated book value of capital assets;
- (2) Depreciated value of capital assets at some constant base-period prices; and
- (3) Undepreciated value of capital assets at some constant base-period prices.

Of these three alternative measures, the first implies a direct use of the reported figures without any adjustment; the second involves fairly detailed price adjustments for the base-period net stock and the series of annual additions to it over the entire period under consideration; while the third involves a simultaneous adjustment for both price changes as well as the arbitrary depreciation charges.

It is interesting to note that, so far as most of the recent studies on time series production functions for Indian industries are concerned, the more frequently used measures are the first two measures discussed above, whereas the third measure is found to be rarely, if ever, used in such studies.*⁸ The common features of the studies using the first two measures have generally been:

- (a) either negative or statistically insignificant factor elasticity coefficients for capital input; (b) positive, statistically significant and relatively higher factor elasticity coefficients for labour input; and (c) statistically insignificant, and in some cases even negative, coefficient for exponential trend term indicating the absence of technical change over time.*⁹

It scarcely needs to be emphasised here that the above observations have been offered more in the nature of some broad generalisations based on the findings of several studies, rather than a set of rules specifying the relationship between the measurement of capital input on the one hand and the estimates of time-series production functions on the other. It should not be surprising, therefore, if we come across some exceptions in a few particular cases.

Apart from the above-mentioned details regarding the estimated regression coefficients of the production functions, it is perhaps equally interesting to find that almost all the attempts at estimating time-series production functions have,

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Apart from the above-mentioned details regarding the estimated regression coefficients of the production functions, it is perhaps equally interesting to find that almost all the attempts at estimating time-series production functions have,

irrespective of the specific measure of capital used, invariably shown excellent overall fits with fairly high values of R^2 especially in the case of the widely used Cobb-Douglas production function. It is, therefore, not the overall explanatory power of the estimated function as such that seems to be so sensitive to the measurement of capital input, as perhaps the estimated regression coefficients of the explanatory variables involved. In other words, the error in the measurement of capital input affects more significantly the structure of the estimated functional relationship rather than its overall statistical significance as indicated by the value of R^2 . Consequently, the estimates of production functions based on any given measure of capital tend to be regarded as good enough to warrant quite fascinating 'policy implications'; whereas the negative factor elasticities observed especially in the case of capital input tend to be looked upon as incidental disturbances cropping up in the estimated relationships that better deserve to be either completely ignored, or rationalised on the basis of a set of more or less standardised explanations.*10 More interestingly, the statistical insignificance of the exponential trend term tends to be readily accepted as sufficient evidence to warrant the conclusion that "The industry (under consideration) is found to have zero neutral technical progress."*11 Obviously, such drastic conclusions and some of the policy implications drawn from the results of this kind of production function exercises could be seriously misleading especially if they are found to be highly sensitive to the measure of capital input underlying the estimates. In view of this, the remaining part of this paper is devoted to a detailed examination of this important problem. Accordingly, in the next (fourth) section, we have estimated the production function for a specific industry (Iron & Steel) over a specified time period (1946 to 1966) and tested its sensitivity to the alternative measures of capital input discussed above. The implications of this exercise have been examined in the following (fifth) section and the main conclusions of the study have been presented in the last section.

IV

The specific industry that we have selected for the purpose of our illustrative exercise is Iron & Steel (Basic Metals), which is reported as industry No. 23 under CMI and industry No. 341.1 under ASI. The main reason why we have selected this particular industry for our analysis is that it happens to be one of the few industries which are relatively less heterogeneous in nature and at the same time do not pose any serious problems of comparability between the

CMI data and the ASI data. Moreover, this industry has served as a crucial link in our industrial planning, and has, therefore, experienced a very rapid expansion along with significant changes in the production pattern, scale and also presumably the technology, especially since the commencement of the Second Plan.

The following variant of the Cobb-Douglas production function is estimated on the basis of time-series data for the period 1946-1966:

$$Y = A.L^a.K^b.e^{cT} \quad \text{-- (1)}$$

which can also be written in its logarithmic form as

$$\text{Log } Y = \text{log}A + a \text{ log}L + b \text{ log}K + cT \quad \text{-- (2)}$$

where, Y, L, K and T represent the variables output, labour capital and time respectively; while A, a, b and c are the parameters representing the constant term, elasticity of output with respect to labour, elasticity of output with respect to capital and the rate of 'neutral technical progress' respectively.

The time series of the dependent variable Y is obtained by estimating the corresponding value added at 1960-61 prices.*12 The estimates of value added at constant prices have been derived by using the method of double deflation of total output and total input with the help of appropriate price deflators for each year in the period under consideration.*13 The variable L is measured as the total man-hours worked per annum.*14 As already indicated above, three alternative measures have been used to represent the crucial variable K. The details regarding the method of estimation adopted for the derivation of different measures of capital stock are given in a separate methodological note appended at the end. The time series data on each of these variables covering the period 1946-1966 are given in Appendix Table 3; while the alternative estimates of the Cobb-Douglas production function for Indian iron & steel industry based on the same presented in Table 1.

A number of interesting observations can be made on the basis of the estimates presented in Table 1. In the first place, it can be readily seen from the table that there is a considerable difference among the three alternative sets of estimates of the production function. While the difference can be regarded as significant for almost all the regression coefficients estimated, it seems to be particularly marked in the case of the coefficients of capital and labour inputs, where we can also notice a systematic trend in the behaviour of the estimated coefficients with respect to the measurement of capital.

Table 1

Alternative Estimates Of Cobb-Douglas Production Function For Indian Iron And Steel Industry

Equation No.	Measure of Capital Input	Regression Coefficients				Coefficient of Determination (R ²)	F-Ratio	Durbin Watson Statistic (DW)
		Constant term (A)	Labour Input (a)	Capital Input (b)	Exponential Trend Term (c)			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
<u>With Exponential Trend Term:</u>								
I	Depreciated book value	0.4861 (0.6551)	1.0562** (2.3270)	-0.0353 (0.1362)	0.0219	0.8578	34.18	2.4888
II	Net stock at 1960-61 prices	0.5487 (0.8165)	0.8354*** (1.8979)	0.1106 (0.4601)	0.0113 (0.5927)	0.8594	34.64	2.4501
III	Gross stock at 1960-61 prices	0.0811 (0.1745)	0.4118 (1.2776)	0.4357** (2.6584)	0.0189*** (1.9407)	0.9513	110.58	2.8518
<u>Without Exponential Trend Term:</u>								
I	Depreciated book value	0.5336 (0.7349)	0.8846** (2.4570)	0.1169 (1.1585)	-	0.8544	52.81	2.4904
II	Net Stock at 1960-61 prices	0.3472 (0.6101)	0.7675*** (1.8391)	0.2119 (1.2754)	-	0.8565	53.71	2.4780
III	Gross Stock at 1960-61 prices	-0.6123 (1.9178)	0.5187 (1.5203)	0.5308** (3.1591)	-	0.9405	142.17	2.6512

*Statistically significant at 1% level of significance

**Statistically significant at 5% level of significance

***Statistically significant at 10% level of significance

(Figures in brackets indicate the computed t-values)

Source: Appendix Table 3.

Thus, for instance, we find that the value of the estimated coefficient of capital input increases significantly as we proceed systematically to eliminate the preliminary errors of measurement of capital input --- the second measure marking an improvement over the first by making the necessary price adjustments, while the third one marking a further improvement over the second by making the necessary adjustments for arbitrary depreciation charges. Similarly, the coefficient of labour input shows a significant decline as we progressively adjust the capital input.

It is more interesting, however, to observe the trend in the computed t-values for the regression coefficients of labour and capital inputs, which indicate the extent of statistical significance of the corresponding coefficients. It can be easily seen that the t-value computed for the estimated coefficient of capital input consistently improves as we go on eliminating the errors in the measurement of capital input, and it accordingly diminishes in the case of the coefficient of labour input. Thus, as a result of the improvement in the measurement of capital input, the estimate of the elasticity of output with respect to capital systematically improves, while that of the elasticity of output with respect to labour consistently deteriorates, both in quantitative as well as qualitative terms. In fact, the estimated coefficient of capital input turns out to be negative and that of labour input exceeds unity, when we use the first measure (i.e., depreciated book value) of capital to estimate the production function with exponential trend term. This would imply, on the other hand, that there are increasing returns with respect to a single factor, viz., labour. It is obvious that both these results violate the standard neo-classical constraints on the factor elasticities, which define what is known as a normal or 'well-behaved' production function,^{*15} and thereby make the estimated relationship 'unrealistic' or implausible from the point of view of economic logic.

As against this, when we use the third measure (i.e., gross stock of capital at constant prices), we find that the estimated elasticities of output with respect to labour as well as capital turn out to be both positive and significantly less than unity. In other words, in this case we get an estimate of the production function that meets all the neo-classical constraints, and thereby brings the empirical estimate of the production function quite close to its theoretical counterpart.

Another interesting observation that can be made from Table 1 relates to the effect of errors in the measurement of capital input on the estimated rate of technical progress.

The first thing that can be noted in this connection is that the exponential trend term turns out to be positive in all the three equations. However, it is found to be statistically insignificant in the first two equations, while it turns out to be significant at 10% level in the third equation. Thus, the use of any of the first two measures of capital would warrant the conclusion that the Indian iron & steel industry has not experienced any significant technical progress during the period 1946-66; while the use of the third measure would indicate a statistically significant rate of technical progress at 1.87% per annum. It would not be implausible, therefore, to argue that one of the possible reasons why a number of studies on production functions in Indian manufacturing find no significant technical progress to have occurred in many industries, is that they invariably use one of the first two measures of capital.

Finally, we may also note that the overall explanatory power of the estimated equation also improves considerably when we use the third measure as against the first two. Thus, the use of the gross stock measure simultaneously improves the overall fit as well as the structure of the estimated production function by yielding a higher value of R^2 and a set of estimates of the regression coefficients that would be consistent with the economic logic. Moreover, it is satisfying to see that this happens irrespective of the inclusion or otherwise of the exponential trend term in the original functional form to be estimated. It follows therefore that the behaviour of R^2 and the estimated regression coefficients along with their respective t-values is not so sensitive to the inclusion or otherwise of the exponential trend term in the equation as to the measure of capital input used in estimating the equation.

V

The implications of the above findings for the analysis of the sources of output growth in Indian industries are quite obvious. The use of any one of the first two measures of capital would exaggerate the importance of labour by overstating its relative contribution to output growth to a considerable extent. Similarly, it would simultaneously underestimate the relative importance of capital and technical progress in the process of industrial growth. This is evident from the conclusions arrived at in a number of studies on the subject.*¹⁶

It would be of interest to examine in greater detail some of the possible explanations of the above findings. In most cases, the main explanation that is generally offered for the observed statistical insignificance of the exponential trend term, or the coefficient of capital, is the presence of multi-collinearity, especially the high degree of correlation between capital and time. It is customary, therefore, to drop the variable T (time) from the equation in order to avoid the problem of multi-collinearity. However, it is not uncommon to come across negative or statistically insignificant elasticity coefficients of capital even in those equations that do not contain the exponential trend term. In such cases, the phenomenon is then attributed to various factors such as under-utilisation of capacity, existence of uneconomic units in the industry, retention of outdated machinery, inter-firm sale of capital equipment at exorbitant prices induced by import restrictions etc.*¹⁷ In practice, it is quite possible, however, that elimination of the variable T may not be enough to remove multi-collinearity which might still be existing in the form of a much higher degree of correlation between labour and capital as compared to that observed between capital and output. From an econometric point of view, the observed imprecision of the estimated regression coefficients remains largely a consequence of the fundamental problem of high degree of inter-correlations among the explanatory variables. It is obvious, therefore, that the marked improvement observed in the case of equations estimated by using the third measure of capital, can be ultimately traced to the changes (brought about by the use of the third measure) in the basic structure of the correlation matrix itself. The relevant parts of the correlation matrix underlying the estimated production functions for Indian iron and steel industry have been presented in Table 2 to examine the validity of this contention.

It can be readily noticed from this table that the basic structure of the correlation matrix undergoes significant changes as we change the measure of capital input. The changes which are particularly noteworthy in this context are: (a) The correlation between K and Y turns out to be significantly lower than the correlation between K and T on the one hand and between K and L on the other, when we use the first two measures of capital input. However, the difference between the correlations of K with Y and with T is found to be much smaller in the case of the second measure as compared to that observed in the case of the first measure of capital; (b) The correlation of K with Y consistently improves as we eliminate the preliminary errors in the measurement of K while passing on from the first to the

Table 2
Correlation of Alternative Measures Of Capital Input With Output,
Labour Input And Time

Other Variables	Measure Of Capital Input (K)		
	Depreciated Book Value (I)	Net Stock at 1960-61 Prices (II)	Gross Stock At 1960-61 Prices (III)
Value Added (Y)	0.8975	0.9108	0.9658
Labour Input (L)	0.9379	0.9548	0.9614
Time (T)	0.9670	0.9256	0.8643

Note : Figures indicate the coefficient of correlation between log K on the one hand, and, log Y, log L and T respectively, on the other.

Source: Same as Table 1.

third measure; (c) Similarly, the correlation of K with T deteriorates considerably as we pass on from the first to the third measure; (d) consequently, when we use the third measure of capital, the correlation of K with Y turns out to be greater than the correlation of K with T or with L, though the correlation between K and L also turns out to be very high and quite close to the correlation between K and Y.

It is evident from the above observations that the use of the gross stock measure is instrumental in reducing the intensity of the serious problem of multi-collinearity that is generally found to plague the precision of the estimated regression coefficients of time-series production functions. The direct implication of this finding is that, to some extent, the problem of multi-collinearity arises on account of the errors in the measurement of capital input; and at least a part of it can, therefore, be avoided by eliminating the preliminary errors in the measurement of capital input.

It may be noted in this connection that the results of the third equation in Table 1 above indicate that the regression

coefficients of K and T are statistically significant, while the coefficient of L turns out to be statistically insignificant, though it shows the expected sign and magnitude. This phenomenon, as it would be evident from the figures given in Table 2 above, is clearly attributable to the very high correlation between K and L in the case of the third measure. Obviously, an extremely high correlation between K and L represents another aspect of the same problem of multi-collinearity, which can again be avoided by a further refinement of the underlying measure of capital input. After all, in a time-series analysis, the variables K and L are expected to have a high correlation, inasmuch as a high rate of capital formation is likely to generate additional demand for labour to a considerable extent over a long period of time. In fact, as Klein has argued, production functions with overall correlations much in excess of 0.95 can be well estimated with inter-correlations between explanatory variables as high as 0.80 to 0.90.*¹⁸ All that might be required, therefore, could as well be to reduce the extent of correlation between K and L from its very high degree (about 0.96 in the present case) to a reasonable limit (say below 0.85). And, this reduction can perhaps be achieved by a further improvement in the measure of capital input itself, all other things remaining the same. The further improvement that is still possible to make in the gross stock measure is obviously the one related to the degree of capacity utilisation. As Solow has explicitly pointed out, "What belongs in a production function is capital in use, not capital in place".*¹⁹ However, on account of the non-availability of the appropriate and reliable data on the year-to-year fluctuations in the degree of utilisation of capital in Indian iron and steel industry, we have not been able to conduct the required exercise to test the validity of the above contention.

VI

Finally, we may conclude the discussion by summarising the main findings of the study. The main conclusions which may be drawn from the present study are:

1. The estimates of time-series production functions for Indian Industries are highly sensitive to the measurement of capital input.
2. The use of depreciated book value of capital or the net stock of capital at constant prices generally yields the set of estimates in which the regression coefficients of capital input or time or both turn out to be either negative or statistically insignificant.

3. The use of gross stock of capital at constant prices leads to a remarkable improvement in the precision of the estimated coefficients of capital as well as time, and also, in the overall explanatory power of the estimated production function. It thereby reduces the gap between the theoretical concept of production function and its empirical counterpart.
4. The elimination of preliminary errors in the measurement of capital input alters significantly the estimates of the relative contributions made by various factors to the growth of output, inas much as it raises the relative importance of capital and technical progress, and, reduces the relative contribution of labour.
5. The elimination of errors in the measurement of capital input reduces the intensity of the problem of multi-collinearity arising frequently in the estimation of time-series production functions.

The implications of those findings are obvious. While estimating production functions on the basis of time series data, more attention needs to be paid to the problems of measurement of factor inputs, than has hitherto been paid in many cases; and, a great deal of precaution and care need to be exercised before drawing any definite conclusions or direct policy implications from the results of the production function exercise conducted at the industry level and based on the time-series data.

NOTES AND REFERENCES

1. A list of such studies would consist of the following:
 - 1) A.K. Sarkar: "Production Function For Indian Steel Industry", Indian Economic Journal, Vol.2, No.3, Jan.-March 1965; pp. 263-285.
 - 2) V.S.R.K. Sastry: "Measurement of Productivity and Production Function In Sugar Industry in India - 1951-1961", Indian Journal of Industrial Relations, Vol.2, No.1, July 1966; pp.70-94.
 - 3) Damodar Gujarati: "Sources of Output Growth in Indian Manufacturing", Indian Journal of Industrial Relations, Vol.3, No.1, July 1967; pp.41-48.
 - 4) G.S. Gupta: "Production Function and Factor Productivity In The Indian Cement Industry", Indian Journal of Industrial Relations, Vol.8, No.3, January 1973; pp.339-350.
 - 5) G.S.Gupta and Kirit Patel: "Production Function In Indian Sugar Industry", Indian Journal of Industrial Relations, Vol.11, No.3, January 1976; pp.315-337.
 - 6) S.S. Mehta: "Returns to Scale and Sources Of Growth of Output In Large Scale Indian Industries", Indian Journal of Industrial Relations, Vol.11, No.3, January 1976; pp.339-350.
2. Cf. A.K. Sarkar, Op. cit., pp.266-268.
See also G.S. Gupta: "Production Function And Factor Productivity In The Indian Cement Industry", Op.cit., p.368.
3. For further details regarding the measurement of real capital stock, see Bakul H. Dholakia: The Sources of Economic Growth In India, (Good Companions, Baroda, 1974); Ch.V, pp.138-140.
4. Cf. Joan Robinson: "The Production Function And The Theory Of Capital", in Collected Economic Papers, Vol.II, (Oxford; Basil Blackwell, 1960); p.115.
5. Cf. Harvard Economic Research Project; Estimates Of The Capital Stock Of American Industries, 1947 (Cambridge Mass., 1953); pp.21-22.

6. Cf. Tibor Barna: "On Measuring Capital", in The Theory of Capital, F.A. Lutz and D.C. Hague (Eds.), (London: McMillan, 1961); pp. 75-94.
7. For details, see Bakul H. Dholakia, Op. cit.
8. Of the six studies referred to in footnote 1 above, three studies, viz., No. 1 (Sarkar), No. 3 (Gujarati) and No.4 (Gupta) have used the first measure (i.e., depreciated book-value of capital stock); while the remaining three have used the second measure (i.e., depreciated values of capital stock at constant prices). It may also be mentioned that all the six studies have used the same functional form for the purpose of estimating the production function, viz., the well-known Cobb-Douglas form of production function. Since this function is linear in the double logarithmic scale, the coefficients of labour and capital indicate the elasticities of output with respect to labour and capital respectively; while the coefficient of trend term t , which is incorporated in the exponential form, indicates the overall rate of the so-called 'neutral technical progress'. It may be noted that the rate of technical progress so defined is a sort of 'catch-all' in as much as it covers all those factors that lead to a shift in the production function.

Cf. R.M. Solow: "Technical Change and the Aggregate Production Function", Review of Economics and Statistics, Vol.39, August 1957.

9. Some conclusive evidence in this regard is provided by the findings of the studies referred to above. Thus, for instance; we can see from the study made by Gujarati, which uses the first measure of capital and which examines the production function in 28 industries, that in as many as 24 out of 28 industries considered, estimated elasticity of output with respect to capital is either negative or statistically insignificant at 10 per cent level of significance (Cf. Gujarati, Op. cit., pp.44-46). Moreover, we can also notice that in 18 of the 28 industries examined by the study, the rate of technical progress (i.e., exponential trend term) turns out to be either negative or statistically insignificant; while in each of the remaining 10 industries where the rate of technical progress is found to be positive and significant, the elasticity of output with respect to capital is either negative or statistically insignificant.

Similarly, we may also notice from the study made by Mehta, which uses the second measure of capital and which covers 27 different industries, that in as many as 25 out of the 27 industries considered the elasticity of output with respect to capital is either negative or statistically insignificant at 10 per cent level of significance (S.S. Mehta, Op. cit., pp.345-346). Again, in 18 out of 27 industries examined in Mehta's study, the rate of technical progress turns out to be either negative or statistically insignificant, while in each of the remaining industries where the rate of technical progress is found to be positive and significant, it is the elasticity of output with respect to capital which turns out to be either negative or statistically insignificant. The studies made by Sarkar and Gupta also provide further evidence to support the broad observations made above.

10. Cf. A.K. Sarkar, Op. cit., pp.266-268.
See also, G.S. Gupta, "Production Function and Factor Productivity in the Indian Cement Industry", Op. cit., pp.368-371.
11. G.S. Gupta, "Production Function and Factor Productivity in the Indian Cement Industry", Op. cit., p.379. The same conclusion seems to have been arrived at again in G.S. Gupta and Kirit Patel: "Production Function in the Indian Sugar Industry", Op. cit., p.332.
12. It may be noted that when the first two measures of capital are being used, the relevant measure of value added would be the one that excludes depreciation, i.e., net value added. However, when we use the third measure of capital, the relevant measure of value added would be the one that does not exclude accounting depreciation. In other words it is the use of gross value added at constant prices which is consistent with the use of gross capital stock in the production function analysis. Hence, we have used the gross value added at 1960-61 prices as the dependent variable whenever capital input is measured as gross capital stock at 1960-61 prices, whereas in all other cases we have used the net value added at 1960-61 prices as the dependent variable.
13. CMI as well as ASI provide fairly detailed data on the quantity and value of various products and by-products which together constitute output and also on the various items which constitute the fuel, elasticity, etc.

and the materials consumed. We have, therefore, derived the price relatives for various products and by-products and also for the various items of input for each year (with 1960 as the base) from the detailed data thrown up by the CMI and the ASI. We have then obtained the overall price deflators, for total output and total input separately, as weighted averages of the various price relatives obtained for each category with the weights proportional to the share of the respective items in the value of total output or total input, as the case may be, in the base year 1960. The estimates of value added at 1960 prices are then derived by subtracting the value of total input at 1960 prices from the value of total output at 1960 prices so obtained for each year in the period under consideration. The estimates of total output, total input and value added at constant 1960-61 prices derived by this method are presented in Appendix Table 1.

14. The time-series of total man-hours worked per annum is derived by assuming that the average number of hours worked by employees other than workers is the same as the average number of hours worked by the workers. This assumption is necessary because the data on the number of hours worked by employees other than workers are not available.
15. The neo-classical constraints on the elasticity of output with respect to factor inputs, which define the normal or well behaved production function are two-fold, viz., the factor elasticities must be non-negative and each of them taken separately should be less than unit. For details, see R.G.D. Allen: Macro-Economic Theory: A Mathematical Treatment (London: McMillan, 1968 ch. 3).
16. For a list of some of the studies on the subject, see footnote 1 above.
17. Cf. A.K. Sarkar, Op.cit., and G.S. Gupta: "Production Function and Factor Productivity in the Indian Cement Industry", Op. cit.
18. Cf. L.R. Klein: An Introduction to Econometrics, (New Delhi, Prentice-Hall of India, 1965), p.101.
19. R.M. Solow, Op. cit.

Appendix

METHODOLOGICAL NOTE ON THE DERIVATION OF THE ALTERNATIVE MEASURES OF CAPITAL STOCK IN INDIAN IRON & STEEL INDUSTRY

Of the three measures of capital stock considered in our analysis, the first, viz., depreciated book value of capital stock, is readily available from the annual reports of CMI and ASI. The second measure, viz., depreciated value of capital stock at constant 1960-61 prices (also referred to as the net capital stock at 1960-61 prices), is derived from the reported figures by making the necessary price adjustments. For this purpose, three separate price indices are used for the three main types of capital assets, viz., buildings & construction (including land improvements), plant, machinery & equipment, and inventories consisting of the stocks of raw materials, semi-finished products and finished products. The price indices (with 1960=100) for the first two categories for the period 1948 to 1966 have been obtained directly from Bakul H. Dholakia: The Sources of Economic Growth in India, Good Companions, Baroda, 1974; (p.196). The figures for the year 1948 are then carried backwards to cover the two preceding years 1947 and 1946 with the help of the index numbers of the prices of manufactured articles (with 1948=100). Similarly, the price index for the assets acquired before 1946 is obtained by adjusting the figures for the year 1948 with the help of the weighted average value of the overall price deflator for the period 1912 to 1945. The figures of the price deflator (with 1948=100) for each year in the period 1912-1945 were obtained from M. Mukherjee: National Income Of India - Trends And Structure, Statistical Publishing Society, Calcutta, 1969 (p.94). For the purpose of deriving the weighted average, the entire period was divided into three parts, viz., 1912-1920, 1921-1939 and 1940-45 and the average value of price deflator obtained for each of these three periods was assigned the weight of 0.50, 0.25 and 0.25 respectively, the weights indicating roughly the likely age profile of the assets acquired before 1946 so far as the iron & steel industry is concerned. The set of price deflators, so derived for the period before 1948, are given below:

Year/Period	Price Index with 1960-61=100	
	Buildings and Construction	Plant, machinery & Equipment
Period Before 1946	32.4	27.2
1946	60.8	51.0
1947	67.6	56.7
1948	81.6	68.5

Having obtained the required price deflators for fixed capital assets, we have derived the estimates of annual additions to the net stock of buildings & construction and machinery & equipment at 1960-61 prices by deflating the corresponding figures at current prices with the help of their respective price deflators. The time series of net fixed capital stock is then obtained by using the following identity:

$$NK_t = NK_{t-1} + NA_t$$

where NK_t and NK_{t-1} are the net capital stock (at 1960-61 prices) at the end of the year t and $t-1$ respectively while NA_t represents the net addition (at 1960-61 prices) to the capital stock during the year t . NK_0 in this case would represent the net stock of capital assets acquired before 1946, valued at 1960-61 prices.

The price deflators for inventories are derived by computing the weighted average of the separate price indices for finished & semi-finished products (output) and raw materials (materials consumed), the weights representing the respective shares of the stocks of finished & semi-finished products on the one hand, and the raw materials on the other, in the total value of the stock of inventories in the year 1960. (For the details regarding the derivation of the price indices for output and raw materials, see footnote No.13). The price index for inventories, so derived, is then used to deflate the series of reported figures to obtain the required time series of the stock of inventories at 1960-61 prices.

To obtain the estimates pertaining to the third measure of the capital stock, i.e., the gross stock of capital at 1960-61 prices, we have relied upon the information available from the balance sheets of about 1000 firms covered by ASI for the year 1960, collected by the Reserve Bank of India, and reproduced in S.R. Hashim and M.M. Dadi: Capital-Output Relations In Indian Manufacturing, The M.S. University Economic Series No.2, Baroda, 1973; pp.14-15. Analysis of these balance sheets, classified at the 3-digit level according to the standard ASI classification, yields the required ratios of the gross value to the net value of fixed capital assets at purchase prices by three main categories for the year 1960. The gross-net ratios for the iron & steel industry turn out to be 1.3297 for buildings and construction, 1.5777 for plant & machinery and 1.5778 for other fixed capital equipment. By using these figures, we have worked out the gross stock of fixed capital, valued at the purchase prices, in iron & steel industry for the year 1960. As the next

step, we have obtained the reported annual gross additions (i.e., net addition plus depreciation) to the fixed capital assets for each year in the period under consideration. The category-wise break-up of depreciation required for this purpose is obtained by assuming, on the basis of available information, that the average rate of depreciation charged on buildings & construction while calculating the accounting depreciation is 7% of the net value of stock in each year, the remaining part of total reported depreciation representing the accounting depreciation charged on plant, machinery & other equipment.

The category-wise annual gross additions to fixed capital assets derived by the above method are then deflated by their respective price deflators (with 1960-61=100) to arrive at the gross additions at 1960-61 prices. Furthermore, the gross value at purchase prices of the assets acquired before 1946 is obtained by deducting the cumulative total value of annual gross additions at purchase prices made during the period 1946-1960 from the estimated gross stock at purchase prices existing in 1960. The value so obtained is also deflated by the corresponding deflators given above to arrive at the gross value of fixed capital assets acquired before 1946 at 1960-61 prices. Finally, the required time series of gross fixed capital stock at 1960-61 prices is derived by using the following identity.

$$GK_t = GK_{t-1} + GA_t - AD_t$$

where, GK_t and GK_{t-1} indicate the gross stock at 1960-61 prices existing at the end of the years t and $t-1$ respectively; while GA_t represents the value of gross additions to the capital stock at 1960-61 prices during the year t , and AD_t represents the given value (at 1960-61 prices) of assets discarded or scrapped (actually) during the year t . The latter is obtained by assuming, on the basis of the information available from a recent study on capital-output relations in Indian manufacturing (Hashim and Dadi, Op. cit., p.20), that the average annual rate of discarding old assets works out at 4.47% of the gross value of assets acquired before 1946 (at 1960-61 prices).

The time series of gross fixed capital assets (category-wise) and net fixed capital assets valued at constant 1960-61 prices, obtained through the above method, are presented in Appendix Table 2; while the three alternative measures of total capital stock (which include the value of inventories also) are presented in Appendix Table 3.

Appendix Table 1

Growth Of Output, Material Input And Value Added In Iron And Steel Industry, 1946-1966
 (in million at 1960-C1 prices)

Year	Gross Output	Inputs Consumed			Total*	Depreciation	Gross Value Added	Net Value Added
		Fuels, Electricity, & Lubricants	Various types of Materials					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
1946	93.89	9.90	46.60	56.54	4.03	37.35	33.32	
1947	102.62	11.83	50.77	63.05	3.63	39.57	35.93	
1948	93.24	11.48	46.58	58.55	3.11	34.69	31.58	
1949	112.09	12.84	54.45	67.89	2.98	44.20	41.22	
1950	117.56	14.14	58.26	71.47	3.31	46.09	42.78	
1951	123.44	15.23	64.33	80.01	2.71	43.43	40.72	
1952	129.09	14.68	70.10	84.05	2.40	45.04	42.64	
1953	125.71	13.18	65.65	78.37	2.83	47.34	44.51	
1954	123.87	13.01	60.29	75.48	3.52	48.39	44.87	
1955	127.26	13.05	67.30	81.16	4.06	46.10	42.03	
1956	134.28	12.58	65.71	79.17	5.26	55.10	49.84	
1957	128.46	13.80	59.45	74.01	8.00	54.46	46.46	
1958	123.52	15.42	63.54	79.72	13.17	43.80	30.63	
1959	167.68	15.70	73.66	91.15	15.27	76.53	61.26	
1960	174.12	18.18	85.37	116.18	11.32	57.95	46.62	
1961	194.24	19.27	87.14	114.35	17.14	79.89	62.72	
1962	307.96	31.51	136.39	180.37	49.94	127.59	77.65	
1963	359.80	33.80	166.97	217.52	41.16	142.28	101.12	
1964	369.42	28.67	163.91	205.12	40.00	164.30	124.30	
1965	396.62	33.02	147.73	193.07	47.99	203.55	155.55	
1966	398.75	31.07	180.61	227.23	45.52	171.53	126.01	

* Also includes some miscellaneous inputs in addition to fuels, electricity lubricants and various types of materials consumed.

See Footnote No.12.

Appendix Table 2

Estimates Of Fixed Capital Stock In Iron And Steel Industry According To Different Measures of Capital
(Figures in Rs. crores)

Year	Gross Stock of Fixed Capital			Total	Net stock Of Fixed Capital At 1960-61 Prices	Net Stock Of Fixed Capital At Purchase Prices*
	At 1960-61 Prices	Structures and Construction	Plant, Machinery and Other Equip- ment			
1946	9.32	164.36	173.68	58.86	23.49	
1947	10.33	159.44	169.77	59.26	23.87	
1948	10.76	155.31	166.07	60.39	24.75	
1949	11.59	150.24	161.83	61.10	25.39	
1950	11.63	149.32	160.95	64.85	28.15	
1951	14.84	142.06	153.90	63.03	26.62	
1952	11.90	135.28	147.18	61.85	25.53	
1953	11.21	134.69	145.90	65.67	28.90	
1954	14.50	135.29	149.79	73.98	35.90	
1955	15.04	136.64	151.68	72.74	40.88	
1956	16.63	145.98	162.61	93.35	52.90	
1957	21.23	172.36	193.59	124.26	84.06	
1958	21.17	230.44	251.61	178.05	134.06	
1959	25.27	277.23	302.50	220.61	173.64	
1960	32.53	277.37	309.90	244.62	177.98	
1961	35.28	285.22	320.50	246.02	179.50	
1962	101.32	736.28	837.60	701.12	681.69	
1963	124.56	775.57	897.13	727.42	710.70	
1964	143.84	803.78	947.62	745.85	732.16	
1965	165.11	788.15	953.26	711.44	749.71	
1966	187.44	855.32	1042.76	763.36	879.18	

* The reported figures representing depreciated book values of all types of fixed capital assets taken together

Source: See the Appendix.

Appendix Table 3

Time Series Of Labour Input And The Alternative Measures Of Capital Input In Iron And Steel Industry, 1946-1966

Year	LABOUR INPUT		CAPITAL INPUT		
	(1)	(2)	(3)	(4)	(5)
	(Total No. of Man-hours worked in lakhs)	Net Stock of Capital at Purchase Prices* (Rs. crores)	Net Stock of Capital at Constant 1960-61 Prices** (Rs. crores)	Net Stock of Capital at Constant 196 Prices*** (Rs. crores)	Gross Stock of Capital at Constant 196 Prices*** (Rs. crores)
1946	17.91	31.50	92.24	207.06	
1947	19.05	33.12	97.08	207.59	
1948	19.93	36.41	99.02	204.70	
1949	20.53	39.47	103.81	204.54	
1950	20.53	51.71	111.69	207.79	
1951	21.78	56.38	120.89	211.76	
1952	20.90	54.76	116.44	201.77	
1953	19.89	62.52	127.98	208.21	
1954	22.26	70.19	124.60	200.41	
1955	23.23	77.91	129.99	201.93	
1956	23.67	94.02	133.84	203.10	
1957	23.53	120.80	162.98	232.31	
1958	23.73	182.84	227.68	302.24	
1959	21.20	213.60	279.13	361.02	
1960	23.78	218.27	282.01	367.28	
1961	24.37	225.32	284.72	379.20	
1962	45.47	776.31	824.60	961.08	
1963	43.35	822.05	867.41	1037.12	
1964	47.31	881.57	898.08	1099.85	
1965	47.12	939.07	850.46	1092.28	
1966	48.52	1089.63	965.29	1244.69	

*represents the reported figures of depreciated book values of fixed capital assets plus the value of inventories at current prices.

**represents the depreciated value of fixed capital assets plus the value of inventories - both at constant