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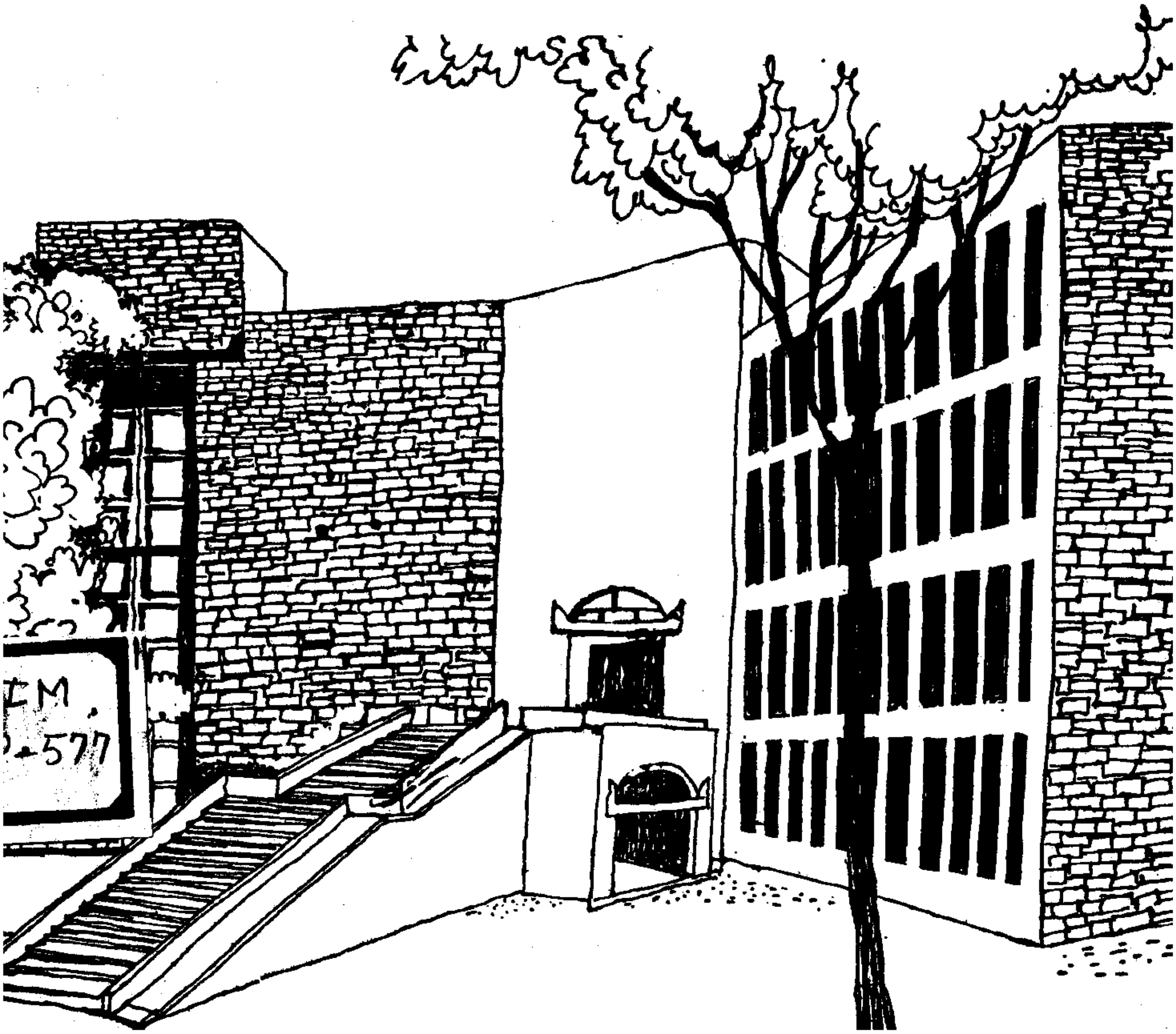

A MIXED INTEGER LINEAR PROGRAMMING MODEL FOR SELECTION OF ENERGY SYSTEMS AND ALLOCATION OF ENERGY TO MULTIPLE END-USES

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A MIXED INTEGER LINEAR PROGRAMMING MODEL FOR SELECTION OF ENERGY SYSTEMS AND ALLOCATION OF ENERGY TO MULTIPLE END-USES

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ABSTRACT

This paper presents a Mixed Integer Linear Programming model for selection of energy system and allocation of energy that satisfies energy needs of multiple end-uses at a location and minimizes the sum of annual fixed and variable costs. The energy generation and distribution is modelled as two stages of energy transfer, i.e. the energy generated at first stage energy systems is distributed sometimes via second stage energy systems to the ultimate end-uses. The model includes several time periods during a year to realistically accommodate different efficiencies of energy systems and varying energy demand during different periods of a year. Application of the model to four villages is discussed. Sensitivity analysis is presented indicating the sensitive areas which need attention of developmental agencies and which can contribute towards reducing the energy costs and encourage utilization of local energy resources.

1. Introduction

In developing countries like India, decentralized energy systems are used to satisfy significant portion of rural energy needs. Efficient decentralised energy systems which use local energy resources are encouraged by the government to conserve the nationally scarce fuels and centralized electricity. Optimization models for energy planning have been reported [9] and [7] both for micro and macro level planning and even with multiple objectives [10]. Simulation model with uncertainty and considering non-conventional energy systems also have been attempted [6]. The problem considered in this paper and the optimization model proposed are pertinent to the developing countries. The problem considered is to select energy systems and distribute energy to multiple end-uses at a location with minimum cost. The model is essentially for making optimal micro level and decentralized decisions; however, sensitivity analysis using the model can provide insight and important implications for macro level planning for energy.

The model provides optimal decisions for a location. Energy demand for the location is considered to be aggregate of demands for four end-uses, namely (i) cooking, (ii) hot water, (iii) irrigation, and (iv) electricity. For each end-use, energy is supplied by various energy systems. Energy systems are classified in two types (i) primary energy systems which generate energy, and (ii) secondary energy systems which are required to transfer, in some cases, the energy produced by the primary systems to end-uses. Various energy systems are available in several discrete sizes. Primary energy systems used are: (1) Solar cooker, (2) Wood stove, (3) Kerosene stove

(4) Biogas plant, (5) Solar hot water system, (6) Wind mill, (7) Diesel Engine, (8) Gasifier engine, (9) Photovoltaic cells, (10) Centralized Electricity (to the extent available).

Secondary energy systems used are: (1) Biogas burner, (2) Dual-Fuel engine, (3) Pumpset, (4) Generator set, (5) Motor pumpset.

2. Model

Energy systems are available in discrete sizes, e.g. diesel engines are available in sizes 5 kilowatt (KW), 10 KW, 25 KW, etc. with higher sizes offering economies of scale. Each energy system of a given size has a known fixed cost. Variable costs of running the various energy systems are significantly different. Energy system efficiencies and demand for various end-uses are different during different time periods in a year. For example, biogas system during winter months operates at sixty per cent of its normal efficiency during rest of the year. Similarly, irrigation demand during monsoon season is different compared to rest of the year. Hence the model should accommodate several periods of a year. Also the model should include constraints on resources, especially local resources wood and gobar (i.e. dung or any other biomass usable as input to a biogas plant) which have restricted availability.

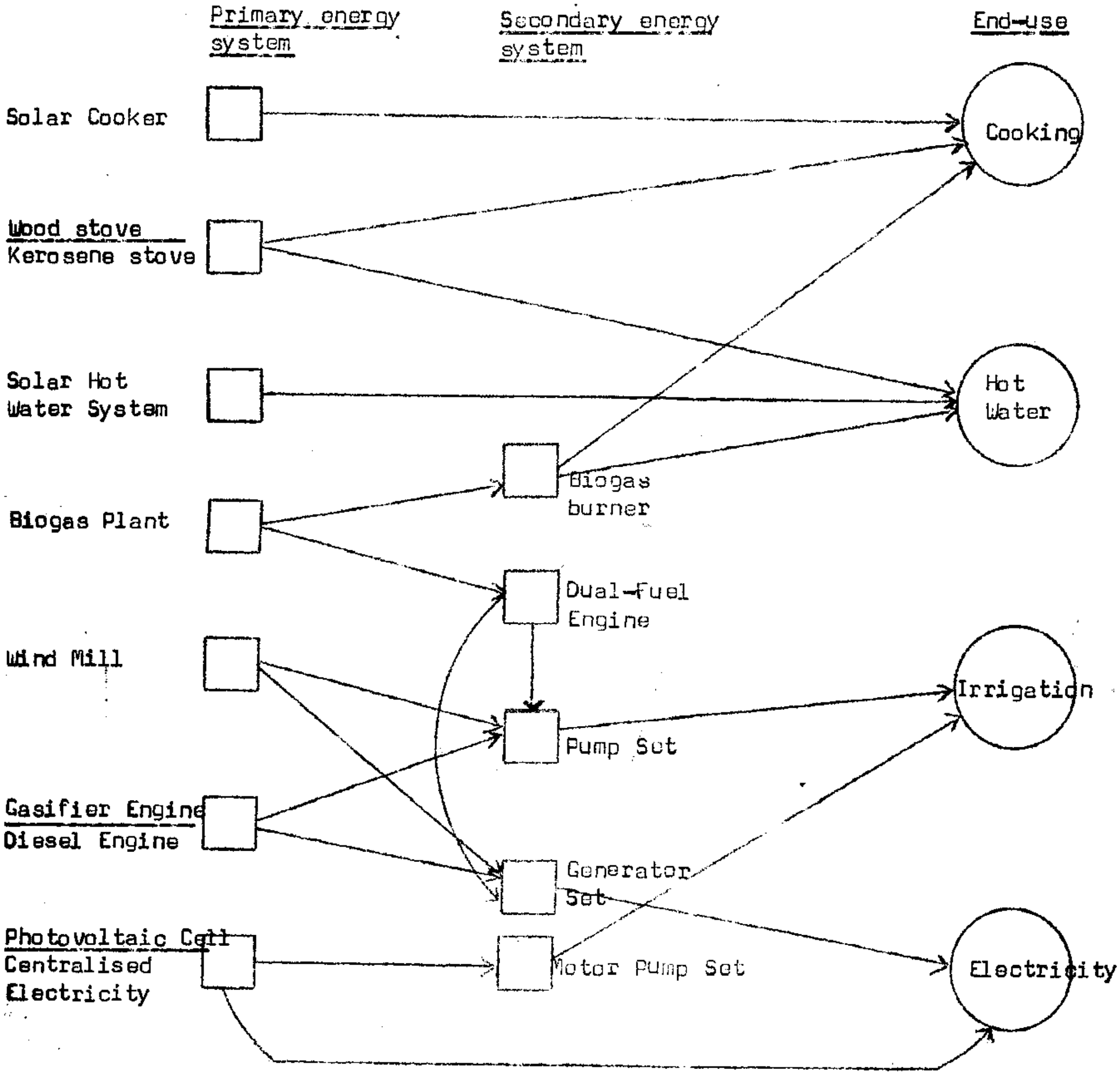
The problem modelled thus is to select primary and secondary energy systems of appropriate sizes to satisfy energy needs of a location in every time period using resources no more than available in a given time period and allocate energy to various end-uses in different time periods so as to minimize total annual costs, i.e. the sum of annual fixed costs and variable costs. As the fixed and variable costs are involved, the problem is modelled similar to a fixed charge problem like location-allocation problem with Mixed Integer Linear Programming (MILP) formulation [1] [2] [8]. As the supply of energy from an energy system to an end-use is sometimes via a secondary system, the energy generation and distribution is modelled as a two stage system as illustrated in Figure-1, which is comparable in a plant location-allocation problem to a situation of supplying multiple commodities from a plant to different markets via warehouses located at some intermediate distribution centres [3] [10].

2.1 Model Assumptions

The model is based on following assumptions:

1. A year is divided into several time periods to account for different efficiencies of some energy systems and different energy demands for each end-use in different time period of a year.
2. Each primary and secondary energy system has specified capacity during a period.
3. For each energy system, the amount of raw materials used and variable costs are proportional to the amount of energy supplied.
4. Given the energy supplied by each energy system, the total cost and energy supplied are sums of costs and energy supplied by each source.

The energy generation and distribution to the village is as illustrated in Figure-1 below.



Energy Generation and Distribution System

Figure-1

5. Fixed cost of a primary energy system is charged if and only if the system supplies positive amount of energy.

6. Fixed cost of a secondary energy system is charged if and only if any primary energy system supplies positive energy to an end-use via that secondary energy system.

7. The demand for each end-use during each period must be satisfied.

2.2 Legend and variable and co-efficient definitions

p = **Number of primary energy systems.**

i = Index number for primary energy system - $i = 1,2,....,p$

n_i = Number of sizes of i^{th} primary energy system.

j = Index number for sizes of each primary system. For i^{th} primary system $j = 1,2,....,n_i$.

d = Number of End-uses requiring energy.

k = Index number for end-uses - $k = 1,2,...,d$.

e = Number of time periods in a year.

t = Index number for time period - $t = 1,2,...,e$.

q = Number of secondary energy systems.

l = Index number for secondary energy system - $l = 1,2,...,q$.

m_l = Number of sizes of l^{th} secondary energy system.

s = Index number for sizes of each secondary system. For the l^{th} secondary system - $s = 1,2,...,m_l$.

h = Number of raw materials used by energy systems.

r = Index number of raw materials - $r = 1,2,\ldots,h$.

Y_{ij} = Number of units 0 primary energy system i of size j.

F_{ij} = Fixed cost (annual) of installing a primary energy system i of size j.

Z_{ls} = Number of units of secondary power system of type l and size s.

G_{ls} = Fixed cost (annual) of installing a secondary energy system of type l and size s.

x_{ikt} = Amount of energy supplied annually by a primary energy system i to end-use k in a period t.

C_{ikt} = Variable cost of using a unit of energy from energy system i for end-use k in time period t.

U_{ijt} = Capacity of primary energy system of type i of size j in time period t.

V_{lst} = Capacity of secondary energy system of type l of size s in time period t.

a_{iktr} = Raw material of type r required to supply a unit of energy by primary energy system i to end-use k in period t.

A_{rt} = Availability of raw material of type r in time period t.

D_{kt} = Energy demand of end-use k in period t.

The problem of optimal selection of energy system and allocation of energy to several end-uses is formulated as under:

2.3 Formulation

Minimize Annual Cost

$$Z = \sum_{i} \sum_{k} \sum_{t} C_{ikt} X_{ikt} + \sum_{i} \sum_{j} F_{ij} + \sum_{l} \sum_{s} G_{ls} Z_{ls} \qquad \ldots\ldots (1)$$

Subject to:

Primary Energy System capacity constraints

$$\sum_{k} X_{ikt} - \sum_{j} U_{ijt} Y_{ij} \leq 0; \quad i = 1,2,\ldots,n; \quad t = 1,2,\ldots,e \qquad \ldots\ldots (2)$$

Secondary Energy System capacity constraints

$$\sum_{i \in I} \sum_{k \in K} X_{ikt} - \sum_{s} V_{lst} Z_{ls} \leq 0; \quad l = 1,2,\ldots, q; \quad t = 1,2,\ldots, e. \qquad \ldots\ldots (3)$$

Where

$K = \{k : k =$ all indices of end-uses which can be supplied by a secondary system $l\}$.

$I = \{i : i =$ all indices of primary energy system which can supply to a corresponding $k \in K$ via secondary energy system $l\}$.

Raw Materials availability constraints

$$\sum_{i} \sum_{k} a_{iktr} X_{ikt} \leq A_{rt}; \quad r = 1,2,\ldots,h; \quad t = 1,2,\ldots,e. \qquad \ldots\ldots (4)$$

Demand Constraints:

$$\sum_{i} X_{ikt} \geq D_{kt}; \quad k = 1,2,\ldots, d; \quad t = 1,2,\ldots, e. \qquad \ldots\ldots (5)$$

Non-negativity and Integer Constraints:

$$X_{ikt} \geq 0; \quad i = 1,2,\ldots,m; \quad k = 1,2,\ldots,d; \quad t = 1,2,\ldots,e. \qquad \ldots\ldots (6)$$

$$Y_{ij} \geq 0; \text{ and free integer}; \quad i = 1,2,\ldots,n; \quad j = 1,2,\ldots,n_i. \qquad \ldots\ldots (7)$$

$$Z_{ls} \geq \ ; \text{ and free integer}; \quad l = 1,2,\ldots, q; \text{ and } s = 1,2,\ldots, m_l. \qquad (8)$$

3. Application

An application of the model is made to four villages in India. Energy Demand for these villages was estimated by a survey. For each village, energy demand was estimated for four end-uses, namely cooking, hot water, irrigation, and electricity. For this application, twenty one primary energy system-size combinations and six secondary energy system-size combinations are considered which are represented with twenty seven integer variables. A year is divided into three time periods nearly coinciding with three major seasons. Energy allocation in this application requires forty two real variables. The problem was solved for each village using a MILP computer code based on branch-and-bound algorithm [4] [5] on VAX 11/730 computer. The average CPU time for a computer run was 1 minute 30 seconds.

3.1 Summary of Optimal Results

The analysis of optimal solution given by the model for four villages revealed that:

(1) Solar cookers were selected for all villages.

(2) For all four villages, the local energy resources were found to be inadequate, i.e. constraint relating to these resources had no slack at optimality.

(3) Gasifier based systems and dual-fuel engines which use wood and gobar respectively were not selected in any village.

(4) Cost per unit of energy was significantly lower (up to twenty five percent) for a village having relatively higher availability (i.e. in proportion to energy demand) of local energy resources compared to a village having relatively lower availability of local energy resources.

4. Sensitivity Analysis

Analysis of optimal results for four villages suggested that the energy costs are sensitive to the availability of local energy resources.
A sensitivity analysis with regard to the availability of local energy resources was first attempted. The problem was solved for each village by relaxing the resource availabilities to different levels. The results of sensitivity analysis were significant and suggested the following:

(1) The energy costs are quite sensitive to the availability of local biomass resources.

(2) If wood is made freely available at current market price, then the unit cost of energy can be reduced upto twenty percent.

(3) Even when wood is freely available, wood based gasifier systems were not selected for any village at optimality.

(4) If gobar is made freely available then energy system selection changes almost totally such that for every end-use only biogas based systems are selected at optimality and cost per unit of energy is reduced upto forty percent.

4.1 Implications for national planning

Micro level data was used in applying our model to four villages. The sensitivity analysis, however, has implications for macro level national planning. Results of sensitivity analysis clearly suggest that governmental authorities and various developmental agencies in the country dealing with energy programs must make efforts to develop suitable local biomass resources. Such efforts can offer substantial financial benefits and at the same time can encourage the decentralised energy systems. Sensitivity analysis also suggests that gasifier based systems, with the present cost structure, are not selected at optimality. However, since gasifier systems use wood efficiently and can save substantial quantities of nationally scarce energy sources such as diesel or centralized electricity, government agencies have shown interest in subsidizing such systems. In fact, government has been subsidizing several non-conventional energy systems in the country. Hence, sensitivity runs were made at different levels of subsidy on gasifier unit to assess the minimum amount of subsidy that can ensure the selection of gasifier based systems both for irrigation and electricity needs. If wood availability is constrained at the present level then the gasifier is not selected at any level of subsidy, since the available wood is allocated at optimality mainly for cooking needs. But when wood is made freely available at current market price, then the gasifier systems are selected both for irrigation and electricity needs at forty percent subsidy. This analysis suggests that the economical selection of gasifier system would require the government action to make wood available (e.g. through fuel-plantations) and by providing at least forty percent subsidy to gasifier. Sensitivity analysis with gobar availability clearly demonstrates that significant savings can be made by improving the present gobar collection method and more so by developing alternate biomass usable as input to biogas plants. The sensitivity analysis using the model thus is very helpful in indicating the various areas requiring attention of national planners and the type of governmental actions needed in these areas.

5. Conclusions

The MILP model developed in this paper gives selection of energy systems and allocation of energy to multiple end-uses during different periods of a year which minimizes annual cost of meeting energy needs of a location. The model is general and can accommodate any number of energy system-size combinations, end-uses and time periods during a year. The application of the model is made to four villages. The analysis of results of this application and sensitivity analysis using the model indicates important areas such as development of biomass resources, government subsidy to some energy systems etc., which can be most beneficial.

At present further work is being done to formulate a model for the problem of multi-period dynamic selection of energy system and allocation of energy as well as for developing a goal programming model. In India, the coming five year plan envisages a major effort in promoting non-conventional and decentralized energy systems. The operations research models thus are expected to contribute to the national effort planned in the energy field in India.

REFER

[1] Cooper, L., "Location-Allocation Problems", Operations Research, 11, 1963, 331-343.

[2] Ellewin, L.B., and Gray, P., "Solving Fixed-Charge Location - Allocation Problems with Capacity and Configuration Constraints", AIEE Transactions, 3(4), December 1971, 290-298.

[3] Geoffrian, A.M., and Greaves, G.W., "Multi-commodity Distribution System Design by Benders Decomposition", Management Science, 20(5), Jan. 1971, 822-844.

[4] Land, A.H., and Doig, A.G., "An Automatic Method for solving Discrete Programming Problems", Econometrica, 28(3), 1960, 496-520.

[5] Land, A.H., and Powell S., "Fortran Codes for Mathematical Programming", Wiley (London), 1973.

[6] Martin Brian, and Diesendorf Mark, "The Economics of Large-Scale Wind Power in the UK - A Model of an Optimally Mixed CEGB Electricity Grid", Energy Policy, 11 (3) (1983), 259-266.

[7] Meier Peter, and Muyobi Vinod, "Modelling Energy Economic Interactions in Developing Countries : A Linear Programming Approach", EJOR, 13 (1983), 41-59.

[8] Murty, K.G., "Linear and Combinatorial Programming", Wiley, (N.Y.), 1976.

[9] Samoulidis, J. Emmanuel, and Berahas, Solomen A., "Energy Policy Modelling in Developing and Industrializing Countries", EJOR, 13 (1983), 2-11.

[10] Schuly Volkhard, and Stenfest Herald, "Regional Energy Supply Optimization with Multiple Objectives", EJOR, 17 (1984), 302-312.

[11] Sweeney, D.J., and Tatham, R.R., "Improved Long Run Model for Multiple Warehouse Location", Management Science, 22(7), March 1976, 748-758.